Solutions for Extra Ch. 3 Problems

1) There are three equally likely outcomes in the reduced sample space: (4,6), (5,5), and (6,4). Two of these have at least one 6, and so the desired conditional probability is simply 2/3.

2) Letting W be the event the first two are white, and B be the event the last two are black, the desired probability is

$$\begin{aligned} P(W \cap B) &= P(W)P(B|W) \\ &= \frac{\binom{9}{2}}{\binom{15}{2}} \frac{\binom{6}{2}}{\binom{13}{2}} \\ &= \left(\frac{36}{105}\right)\left(\frac{15}{78}\right) \\ &= 6/91. \end{aligned}$$

Alternatively, letting W_1 be the event the 1st ball is white, W_2 be the event the 2nd ball is white, B_1 be the event the 3rd ball is black, and B_2 be the event the 4th ball is black, the desired probability is $P(B_2|B_1W_2W_1)P(B_1|W_2W_1)P(W_2|W_1)P(W_1) = (5/12)(6/13)(8/14)(9/15) = 6/91$.

3) Letting B be the event that Barbara hits the duck, and D be the event that Dianne hits the duck, the desired probability is

$$\begin{aligned} P(B \cap D|B \cup D) &= \frac{P(B \cap D)}{P(B \cup D)} \\ &= \frac{P(B \cap D)}{P(B) + P(D) - P(B \cap D)} \\ &= \frac{P(B)P(D)}{P(B) + P(D) - P(B)P(D)} \\ &= \frac{p_1p_2}{p_1 + p_2 - p_1p_2}. \end{aligned}$$

4) Letting B_1 be the event the 1st ball is black, B_2 be the event the 2nd ball is black, W_1 be the event the 3rd ball is white, and W_2 be the event the 4th ball is white, the desired probability is

$$P(B_1B_2W_1W_2),$$

which is equal to

$$P(W_2|B_1B_2W_1)P(W_1|B_1B_2)P(B_2|B_1)P(B_1) = (6/16)(4/14)(8/12)(6/10) = 3/70 \doteq 0.0429.$$

5) Letting F be the event *the student is female* and C be the event *the student is majoring in computer science*, the desired probability is

$$P(F|C) = \frac{P(F \cap C)}{P(C)} = 0.02/0.06 = 1/3 \doteq 0.333.$$

6) Letting N_i be the event that exactly i of the first three balls drawn are new, and E be the event that the single ball drawn is new, the desired probability is

$$\begin{aligned} P(E) &= \sum_{i=0}^{3} P(E|N_i)P(N_i) \\ &= \sum_{i=0}^{3} \left(\frac{9-i}{15}\right) \frac{\binom{9}{i}\binom{6}{3-i}}{\binom{15}{3}} \\ &= (9/15)(20/455) + (8/15)(135/455) + (7/15)(216/455) + (6/15)(84/455) \\ &= 12/25 \\ &= 0.48. \end{aligned}$$

7) Bayes's formula can be used. Letting H be the event the coin results in heads, and W be the event the ball selected is white, the desired probability is

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\begin{aligned}
P(H^C|W) &= \frac{P(W|H^C)P(H^C)}{P(W|H)P(H) + P(W|H^C)P(H^C)} \\
&= \frac{(3/15)(1/2)}{(4/12)(1/2) + (3/15)(1/2)} \\
&= \frac{(1/5)(1/2)}{(1/3)(1/2) + (1/5)(1/2)} \\
&= \frac{(1/5)}{(1/3) + (1/5)} \\
&= 3/8 \\
&= 0.375.
\end{aligned}
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