Solutions for Extra Ch. 3 and Ch. 4 Problems

1) The answer is *no*. E is a proper subset of F, so that $F \cap E = E$. This gives us that $P(F|E) = P(F \cap E)/P(E) = P(E)/P(E) = 1$, which is not equal to $P(F)$ and $P(F|E^C)$, which are less than 1.

2) The desired probability is 1/2, by *symmetry*.

3) The desired probability is 1. (Since A is a subset of B we have $B \cap A = A$, which gives us $P(B|A) = P(B \cap A)/P(A) = P(A)/P(A) = 1$).

4) $\{1, 2, 3, 4, 5, 6\}$

5) Clearly $P(X \geq 0) = 1$ (since X can't be less than 0). For X to take a value of at least 1, player 1 must have the higher number (compared to player 2) in his first comparison. So, by symmetry, $P(X \geq 1) = 1/2$. For player 1 to win his first two comparisons, he needs the highest number among the first three players. So, again by symmetry, $P(X \geq 2) = 1/3$. Similarly, $P(X \geq 3) = 1/4$ and $P(X \geq 4) = 1/5$. Now clearly $P(X = 4) = P(X \geq 4) = 1/5$ (since X cannot assume a value greater than 4). We also have $P(X = 3) = P(X \geq 3) - P(X \geq 4) = 1/4 - 1/5 = 1/20$, $P(X = 2) = P(X \geq 2) - P(X \geq 3) = 1/3 - 1/4 = 1/12$, $P(X = 1) = P(X \geq 1) - P(X \geq 2) = 1/2 - 1/3 = 1/6$, and $P(X = 0) = P(X \geq 0) - P(X \geq 1) = 1 - 1/2 = 1/2$. To summarize, we have

$$P(X = 0) = 1/2, \qquad P(X = 1) = 1/6, \qquad P(X = 2) = 1/12, \qquad P(X = 3) = 1/20, \qquad P(X = 4) = 1/5.$$