Solutions for Extra Ch. 3 and Ch. 4 Problems

1) The answer is no. \( E \) is a proper subset of \( F \), so that \( F \cap E = E \). This gives us that \( P(F|E) = P(F \cap E)/P(E) = P(E)/P(E) = 1 \), which is not equal to \( P(F) \) and \( P(E|F) \), which are less than 1.

2) The desired probability is 1/2, by symmetry.

3) The desired probability is 1. (Since \( A \) is a subset of \( B \) we have \( B \cap A = A \), which gives us \( P(B|A) = P(B \cap A)/P(A) = P(A)/P(A) = 1 \).

4) \{1, 2, 3, 4, 5, 6\}

5) Clearly \( P(X \geq 0) = 1 \) (since \( X \) can’t be less than 0). For \( X \) to take a value of at least 1, player 1 must have the higher number (compared to player 2) in his first comparison. So, by symmetry, \( P(X \geq 1) = 1/2 \). For player 1 to win his first two comparisons, he needs the highest number among the first three players. So, again by symmetry, \( P(X \geq 2) = 1/3 \). Similarly, \( P(X \geq 3) = 1/4 \) and \( P(X \geq 4) = 1/5 \). Now clearly \( P(X = 4) = P(X \geq 4) = 1/5 \) (since \( X \) cannot assume a value greater than 4). We also have \( P(X = 3) = P(X \geq 3) - P(X \geq 4) = 1/4 - 1/5 = 1/20 \), \( P(X = 2) = P(X \geq 2) - P(X \geq 3) = 1/3 - 1/4 = 1/12 \), \( P(X = 1) = P(X \geq 1) - P(X \geq 2) = 1/2 - 1/3 = 1/6 \), and \( P(X = 0) = P(X \geq 0) - P(X \geq 1) = 1 - 1/2 = 1/2 \).

To summarize, we have

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P(X = 0) = 1/2, \quad P(X = 1) = 1/6, \quad P(X = 2) = 1/12, \quad P(X = 3) = 1/20, \quad P(X = 4) = 1/5.
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