Solutions for Extra Ch. 2 Problems

1) There are a total of 48 children in the 20 families, and 15 of them come from a family having 3 children. So the desired probability is just $15/48 = 5/16 = 0.3125$.

2) Letting R be the event that all of the balls are red, B be the event that all of the balls are blue, and G be the event that all of the balls are green, the desired probability is

$$\begin{aligned} P(R \cup B \cup G) &= P(R) + P(B) + P(G) \\ &= \frac{\binom{5}{3}}{\binom{19}{3}} + \frac{\binom{6}{3}}{\binom{19}{3}} + \frac{\binom{8}{3}}{\binom{19}{3}} \\ &= 10/969 + 20/969 + 56/969 \\ &= 86/969 \qquad (\doteq 0.08875). \end{aligned}$$

(*Note*: Even though the statement of the problem didn't refer to events R, B, and G, for the midterm exam it'll good to define events for problems such as this one so that your solutions will be easier to follow.)

3) The desired probability is

$$\frac{\binom{12}{3}\binom{16}{2}\binom{18}{2}}{\binom{46}{7}} = \frac{(220)(120)(153)}{53524680} = \frac{4039200}{53524680} = \frac{3060}{40549} \qquad (\doteq 0.07546).$$

4) There are $N!$ ways the people can be arranged in a line. If A and B are next to one another, they can be in any of $N-1$ pairs of adjacent positions $((1,2),(2,3),(3,4),\ldots,(N-2,N-1),(N-1,N))$, and given they are in a pair of adjacent positions, either A can come before B or B can come before A, and the other $N-2$ people can be arranged in any of $(N-2)!$ ways. So, altogether, there are $(N-1)(2)[(N-2)!] = 2[(N-1)!]$ arrangements with A and B next to one another, and so the desired probability is $2[(N-1)!]/N! = 2/N$.