

Solutions for Extra Ch. 1 Problems

- 1) If we list the jobs in a fixed order, each of the $20!$ possible permutations of the names of the workers will provide a different way of matching the jobs and the workers, and these are all of the ways of matching one worker to each job, since any 1:1 matching will correspond to one of the $20!$ permutations. So the answer is simply $20! \doteq 2.43 \times 10^{18}$.
- 2) $7!/(2! \cdot 2!) = 5040/4 = 1260$
- 3) Each time a student is chosen for an award there are 20 possible choices, no matter how the choices are made for any of the other awards. So the answer is just $20^5 = 3,200,000$.
- 4) $20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 = 1,860,480$
- 5) The answer is just the number of possible pairs of people, which is $\binom{10}{2} = 45$.
- 6) There are $\binom{16}{2}$ possibilities if math is chosen, $\binom{7}{2}$ if science is chosen, and $\binom{4}{2}$ if economics is chosen. So the total number of possibilities is $\binom{16}{2} + \binom{7}{2} + \binom{4}{2} = 120 + 21 + 6 = 147$. (Note: The addition of the three values is just an application of the result of Theoretical Exercise 2 on p. 18 of Ross (which is the 10th problem of this assignment).)
- 7) If we call the feuding friends 1 and 2, there are $\binom{16}{5}$ possibilities that include neither of the feuding friends, $\binom{16}{4}$ that include 1 but not 2, and $\binom{16}{4}$ that include 2 but not 1 (since if 2 is invited and 1 is not, we must include 4 of the other 16 friends). So the total number of possibilities is $\binom{16}{5} + \binom{16}{4} + \binom{16}{4} = 4368 + 2(1820) = 8008$. (Alternatively, we could start by considering all $\binom{18}{5}$ sets of 5 and then subtract off the $\binom{16}{3}$ sets that include both feuding friends, as was done *Example 4b* on p. 6 of Ross, giving us $8568 - 560 = 8008$.)
- 8) We can apply Proposition 6.2 on p. 13 of Ross, giving us $\binom{9+4-1}{4-1} = \binom{12}{3} = 220$.
- 9) We can apply Proposition 6.1 on p. 13 of Ross, giving us $\binom{9-1}{4-1} = \binom{8}{3} = 56$.
- 10) The total number of outcomes for the sequence of two experiments is the number for the two experiments having the first experiment result in outcome 1, plus the number for the two experiments having the first experiment result in outcome 2, plus the number for the two experiments having the first experiment result in outcome 3, etc., giving us $\sum_{i=1}^m n_i$.
- 11) $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 10!/3! = 604,800$
- 12) The 3 upward moves can be made at any 3 of the 7 possible steps, so the answer is the number of ways three numbers can be chosen from the set $\{1, 2, \dots, 7\}$, which is $\binom{7}{3} = 35$.
- 13) To get from A to the circled point, 2 upward moves can be made at any 2 of the 4 possible steps, so there are $\binom{4}{2} = 6$ paths from A to the circled point. To get from the circled point to B, 1 upward move can be made at any of 3 possible steps, so there are $\binom{3}{1} = 3$ paths from the circled point to B. Using the basic principle of counting, altogether there are $6 \cdot 3 = 18$ valid paths.
- 14) $52!/(13!)^4 \doteq 5.36 \times 10^{28}$ (applying the result indicated in the box on p. 10 of Ross)
- 15) $4^8 = 65,536$
- 16) $8!/(2!)^4 = 2520$ (applying the result indicated in the box on p. 10 of Ross)