

Let $\zeta_{0.05}$ and $\zeta_{0.95}$ denote the 5th and 95th percentiles of the distn of $\hat{\theta} - \theta$. Then

$$\begin{aligned} 0.9 &= P(\zeta_{0.05} < \hat{\theta} - \theta < \zeta_{0.95}) \\ &= P(-\hat{\theta} + \zeta_{0.05} < -\theta < -\hat{\theta} + \zeta_{0.95}) \\ &= P(\hat{\theta} - \zeta_{0.05} > \theta > \hat{\theta} - \zeta_{0.95}) \end{aligned}$$

gives us that

$(\hat{\theta} - \zeta_{0.95}, \hat{\theta} - \zeta_{0.05})$
is a 90% c.i. for θ .

Since $\zeta_{0.05}$ and $\zeta_{0.95}$ are unknown, in practice we can estimate them with estimates of the 5th and 95th percentiles of the bootstrap distn of $\hat{\theta}^* - \hat{\theta}$, which are $\hat{\theta}^{*(.05)} - \hat{\theta}$ & $\hat{\theta}^{*(.95)} - \hat{\theta}$.

This gives us that an approximate 90% c.i. for θ is

$$\begin{aligned} &(\hat{\theta} - [\hat{\theta}^{*(.95)} - \hat{\theta}], \hat{\theta} - [\hat{\theta}^{*(.05)} - \hat{\theta}]) \\ &= (2\hat{\theta} - \hat{\theta}^{*(.95)}, 2\hat{\theta} - \hat{\theta}^{*(.05)}) \end{aligned}$$

which is equivalent to (13.9) on p. 174 of E & T.

(Note: The above is equivalent to solving Problem 13.5 on p. 177 of E & T, except I choose to use different notation.)