Problem 12.4 (p. 167)

(a) $g(X) \approx g(\mu_x) + g'(\mu_x)(X-\mu_x) = c + g'(\mu_x)X$

$\Rightarrow Var(g(X)) \approx [g'(\mu_x)]^2 Var(X)$

(b) Letting $\theta = \mu_x$, we have

$$Var_\theta(g(X)) \approx [g'(\theta)]^2 \; Var_\theta(X).$$

If (12.26) holds and $g'(\theta) = \frac{1}{\sqrt{Var_\theta(X)}}$, clearly the above gives us that $Var_\theta(g(X)) \approx 1$.

(c) $\frac{X_i}{\theta} \sim \chi^2_1 \Rightarrow 2 = Var(\frac{X_i}{\theta}) = \frac{Var(X_i)}{\theta^2} \Rightarrow Var(X_i) = 2\theta^2$.

So $Var(\hat{\theta}) = Var(\bar{X}) = \frac{2\theta^2}{n} \Rightarrow s(\theta) = \sqrt{\frac{2}{n}}\,\theta$.

From (12.26) we need $g(x)$ such that $g'(x) = \sqrt{\frac{n}{2}}\frac{1}{\theta}$.

Clearly $g(X) = \sqrt{\frac{n}{2}} \log\theta$ will work.