Problem 12.4 (p. 167)

(a) \[ g(X) = g(M_X) + g'(M_X)(X - M_X) = c + g'(M_X)X \]
\[ \Rightarrow \text{Var}(g(X)) \approx [g'(M_X)]^2 \text{Var}(X) \]

(b) Letting \( \theta = M_X \), we have
\[ \text{Var}_\theta(g(X)) \approx [g'(\theta)]^2 \text{Var}_\theta(X). \]

If (12.26) holds and \( g'(\theta) = \frac{1}{\sqrt{\text{Var}(X)}} \), clearly the above gives us that \( \text{Var}_\theta(g(X)) \approx 1. \)

(c) \[ X_i - \bar{X} \Rightarrow 2 = \text{Var}(\frac{X_i}{\theta}) = \frac{\text{Var}(X_i)}{\theta^2} \Rightarrow \text{Var}(X_i) = 2\theta^2. \]
So \( \text{Var}(\hat{\theta}) = \text{Var}(X) = \frac{2\theta^2}{n} \Rightarrow s(\theta) = \sqrt{\frac{2}{n}} \theta. \)

From (12.26) we need \( g(x) \) such that \( g'(x) = \frac{\sqrt{2}}{\theta}. \)
Clearly \( g(X) = \sqrt{\frac{2}{\theta}} \log \theta \) will work.