Instructional Design Document
Logarithms Tutorial

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Introduction

"Telecommunications Engineering" (ECE 540) is a core course for students in the M. A. Telecommunications program (MATP) in George Mason University. The MATP is an interdisciplinary program from the College of Arts and Sciences. Students in this program have to take four out of five core courses. "Telecommunications Engineering" is a popular choice among the students. This is because it is the only technical course among the core courses and it is a required course for students specializing in the telecommunications systems track.

The " Telecommunications Engineering" course has been taught since the program began in 1988. It is only taught in the summer semesters in George Mason University. The syllabus includes topics such as telecommunications mathematics, analog and frequency modulation, sampled signals, Fourier analysis, digital signaling, fiber-optic communication, and satellite communications. Many of these topics are physics and mathematics oriented.

Students' comments on this course have indicated that the subject matter is complex and any supplementary assistance would be useful. The purpose of this project is to take one topic from this course, analyze the learners' needs in relation to this topic; develop a strategy to conduct a tutorial and evaluate how effective and useful it is towards the Telecommunications Engineering course. The topic selected is telecommunications mathematics. Other topics in the course may be added after evaluating its effectiveness.

Needs Analysis

The needs analysis will be done in three ways. They are: using a questionnaire to collect information from students, observation of class activity, and reviewing students' evaluation forms and profiles.

1) **Questionnaire** (See Appendix A)

A questionnaire was designed to collect information on students' background and the level of comprehension on the topic of mathematics taught in the class. The questionnaire consisted of seven questions. The three questions that would indicate the learning situation are as follows:

- When reading the math refresher section of the course, do you understand the concepts readily?
- In general, when you work on the math assignments, do you find them easy?
- When you have completed the math assignments, how confident are you that you have gotten the right answer?

Seventeen students who are currently taking this course at George Washington University were given the questionnaire. The same questionnaire was also emailed to twenty other students who

had taken the same course in George Mason University last summer.

It was found that 66% of the students surveyed took a mathematics course more than five years ago; 30% of the students do not understand the mathematical concepts readily; 40% of the students find the math assignments easy some of the time, and 73% of the students are in favor of supplementary help for this course.

2) **Observation of Class Activity**

Observation is done through in-class participation and attending small study group discussions. Students themselves form study groups. Each group ranges from two to six typically.

In-Class

It is observed that other topics taught in this course use many of the telecommunications mathematics concepts. Students who have yet to master these mathematical principles tend to lose track of how the instructor solves certain problems. After a while, they appear to lose concentration or become disinterested.

Study Groups

Discussions with participants in these groups revealed that a proportion of learners had not used high school mathematics for a long time. They were often unsure if they did their assignments correctly. Answer keys were provided only upon request. Those who were weak in this area welcomed more practice and discussion of these assignments.

Reviewing Evaluation Documents

It was found that students find this course work-intensive and the subject matter complex. They also requested for supplementary assistance.

Actual

Students enrolled in this course come from a mixed variety of backgrounds that range from liberal arts to engineering. Although telecommunications mathematics is not complex, a number of students still have a difficult time working on the assignments. Moreover, many students took a mathematics course more than five years ago.

Optimal

Ideally, the students should be proficient in applying the telecommunications mathematics theories and procedural rules to the assignments given to them. These theories include: a) Logarithms; b) Power of ten notations; c) Decibels; and d) Prefixes and units. Once they have achieved this proficiency, it will also help them solve problems in other areas in the Telecommunications Engineering course.

Gap

Students are still uncertain in their understanding of telecommunications mathematics.

They learned the mathematical theories and procedures but they are unable to apply them in problem solving effectively.

Instructional Goal

Through information gathered in the needs analysis, it was decided that a tutorial will help students improve their understanding of the mathematics component in the Telecommunications Engineering course.
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The goal of this tutorial is to help students attain proficiency in applying learned mathematical principles and procedures in this course to solving related problems. This document will focus on designing instruction for one principle: Logarithms (log).

When given a problem to compute on Logarithms, the student should be able to give the computation using its principles and procedures.

The Learning Environment

Organization
The class meets twice or thrice a week over the summer semester. This may include a Saturday morning. The schedule depends largely on the availability of the instructor, Dr. Robert Finn, who has taught this class since its inception.

Instructor
Dr. Finn is supportive to developing additional instructional material to help learners.
He has offered to help in any way possible and he will also be the content expert for this product.
He is easy to work with and is open to suggestions and comments on how to help students prepare for this course.

Decision on the selection of instructional materials for this project will be left mostly to the recommendation of the instructional designer.

Instructional Medium
The instructor uses overhead slides for most of his lectures. Assignments and notes are presented in print format.

Learner Analysis

The targeted learners for this instruction are the students of the Telecommunications Engineering course. As mentioned before, the students in this course have diverse backgrounds. Their prior knowledge on this topic varies greatly. Students who have liberal arts degrees tend to be weaker in mathematics whereas students with engineering degrees exhibit stronger understanding in this area. The following is a summary of the students' cognitive, physiological, and psychosocial characteristics obtained from the questionnaire and observations in class:

Cognitive Characteristics	Physiological Characteristics	Psychosocial Characteristics
Students read and write at college level.	Generally, age ranges from 20s to early 50s.	Motivated to learn and obtain high grades.
Mathematics skills range from low to very high.	Usually more male students than female students.	About 60% of students work full-time in the day and attend classes at night. Many are tired when they attend class.
Students learned principles and rules for logarithms.	A mix of learning and thinking styles.	High anxiety levels because past students have indicated that this is a difficult course and has a high failure rate.
Medium to high computer literacy.	No obvious physiological characteristics that will impede learning.	Students have very mixed backgrounds. Undergraduate majors range from liberal arts to engineering.

Design Implications

Since the principles and procedures of using logarithms are taught in the course, it can be assumed that the students have a base understanding of these knowledge.

It is important to note that mathematics skills of the students vary. It will give students flexibility in selecting specific areas they wish to learn more about if the tutorial is designed in a modular format.

The exercises in the tutorial should vary in difficulty to accommodate the different skill levels among the students.

Task Analysis

Goal Statement

After going through the tutorial, the students would demonstrate effective computational skills to solve problems in logarithms.

Types of Learning Outcomes

The two types of learning outcomes for this instruction are intellectual skills and cognitive strategies. Intellectual skills are required for students to apply logarithmic principles to solve problems not encountered previously. Specifically, procedural outcomes will be the main

learning experience for the students. This will be in the form of a set of learned procedures to compute logarithmic problems.

Cognitive strategies will be used to enhance retention and exercise selective recall of these procedures.

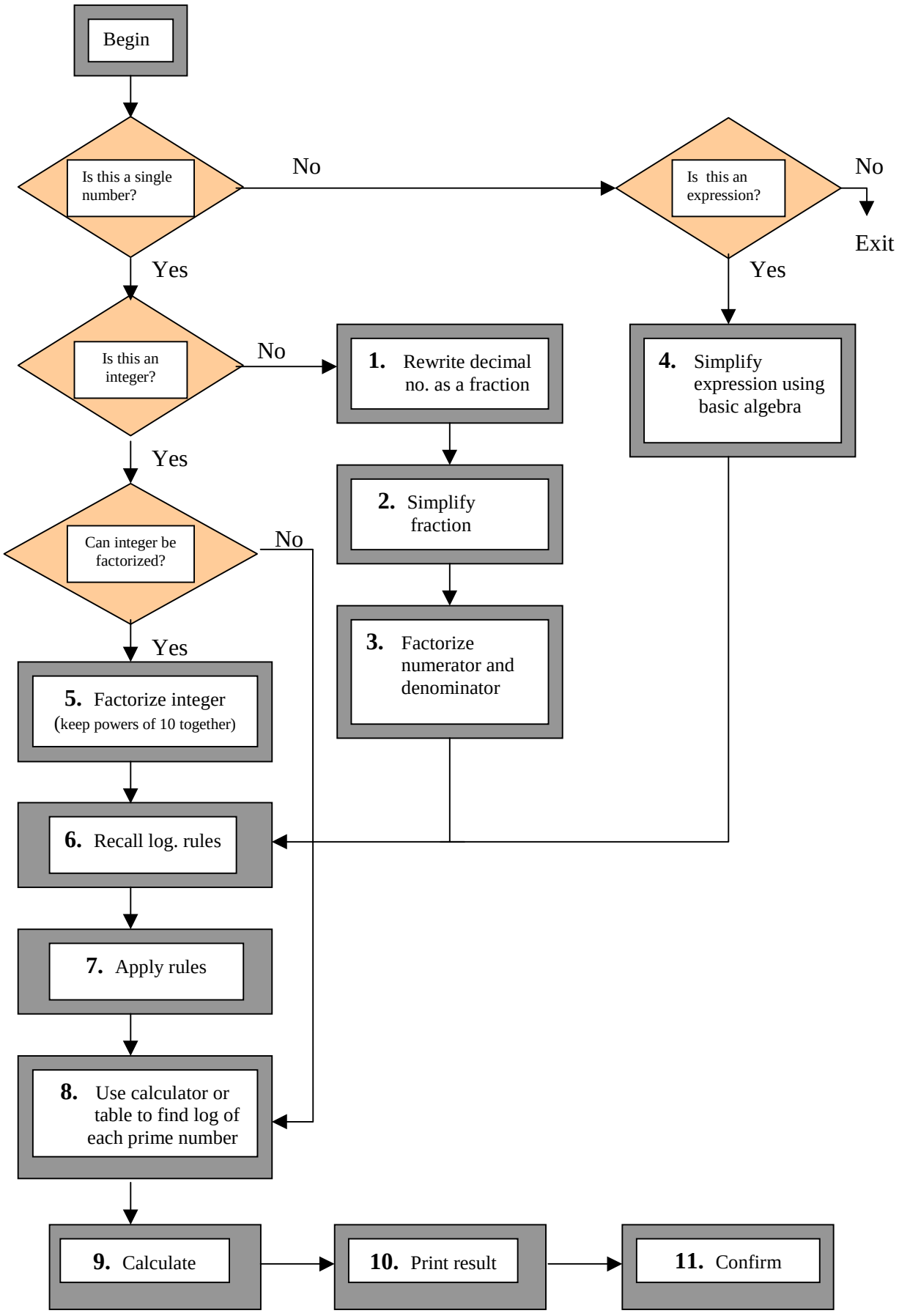
Information-Processing Analysis

Please see the following flow chart A.

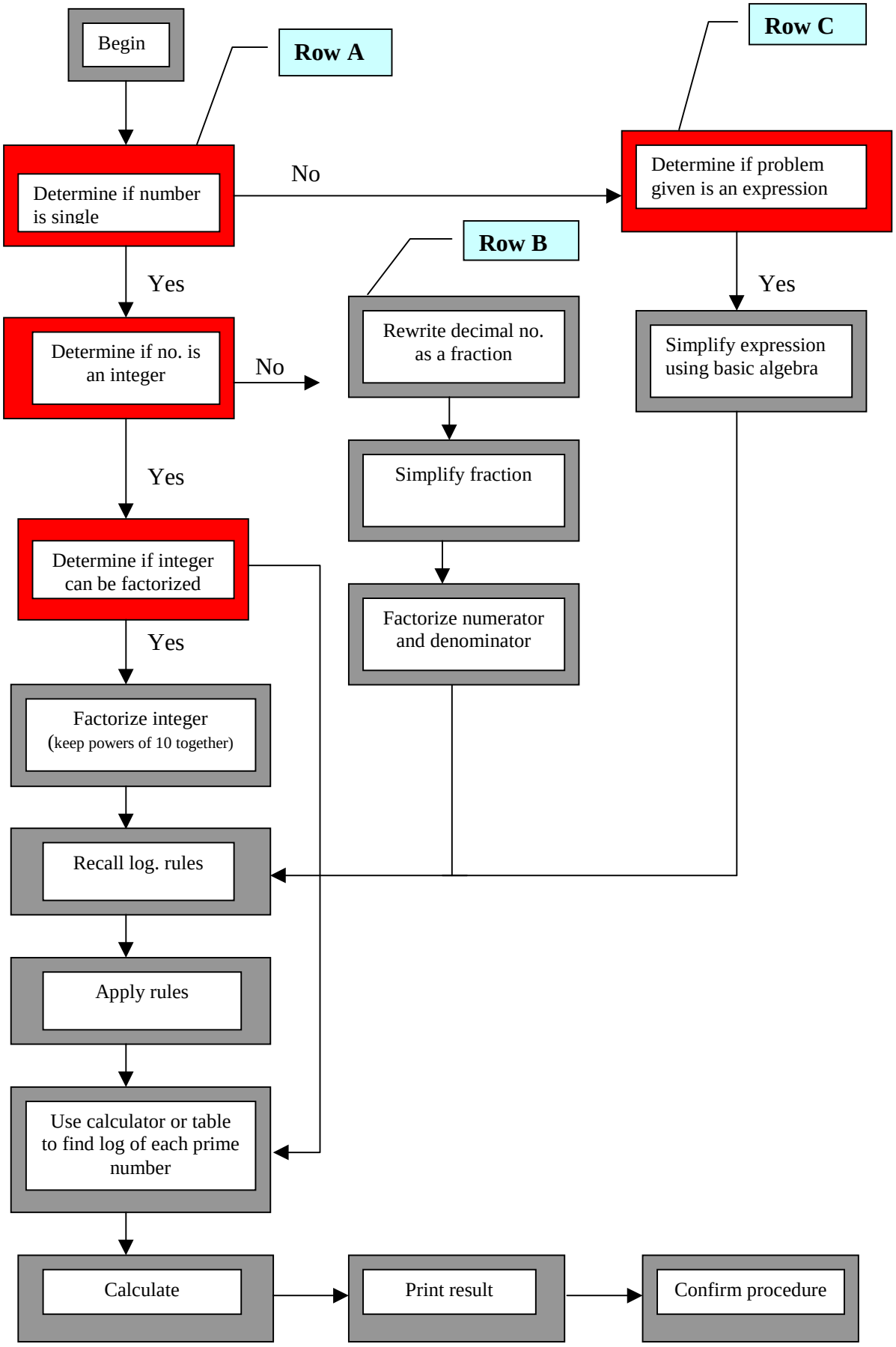
Prerequisite Analysis

Please see flow chart B (Part I) and Part II.

Information-Processing Analysis—Flow Chart A



Prerequisite Analysis—Flow Chart B (Part I)



Prerequisite Analysis—Part II

Row A

1) Determine if number is single
   - Student can recognize single numbers

2) Determine if number is an integer
   - Student can recognize integers

3) Determine if integer can be factorized
   - Student knows if an integer can be factorized

4) Factorize integer (keep powers of 10 together)
   - Student knows how to factorize integers
   - Student can identify powers of ten

5) Recall log rules
   - Student can recall all the log rules
   - Student can list all the log rules

6) Apply rules
   - Student recognizes which rule to apply

7) Use calculator or table to find log of each prime number
   - Student knows how to use a calculator to find logs
   - Student know how to use a log table to find logs
   - Student can identify prime numbers

8) Calculate
   - Student can define the term calculate

9) Print result
   - Student can define the term print

10) Confirm procedure
   - Student can define the term confirm

Row B

1) Rewrite decimal number as a fraction
   - Student knows how to convert a decimal number to a fraction
   - Student can define the terms decimal number and fractions

2) Simplify fraction
   - Student can define the term simplify
   - Student knows how to simplify fractions

3) Factorize numerator and denominator
   - Student can define the term numerator
   - Student can define the term denominator
   - Student knows how to factorize

Row C

1) Determine if this is an expression
   - Student can define the term expression
   - Student can differentiate expressions from other numbers

2) Simplify expression using basic algebra
   - Student knows basic algebra

Performance Objectives

Performance objectives for the tutorial are listed as follows:

1) Upon completion of the tutorial, the student will be able to rewrite a decimal number as a fraction.

2) Upon completion of the tutorial, the student will be able to simplify a fraction.

3) Upon completion of the tutorial, the student will be able to factorize the numerator and denominator of a fraction.

4) Upon completion of the tutorial, the student will be able to simplify an expression using basic algebra when the number given is not a single number.

5) Upon completion of the tutorial, the student will be able to factorize a given integer keeping powers of ten together.

6) Upon completion of the tutorial, the student will be able to recall the appropriate logarithm rules to the single number or expression.

7) Upon completion of the tutorial, the student will be able to know how to apply the recalled logarithm rules to the single number or expression.

8) Upon completion of the tutorial, the student will be able to use the calculator or the logarithm table to find the values of each prime number.

9) Upon completion of the tutorial, the student will be able to calculate the results of the given single number or expression using the logarithm values.

10) Upon completion of the tutorial, the student will be able to determine the results of the problem and print them out accordingly.

11) Upon completion of the tutorial, the student will be able to go through the problem again and confirm that the procedures applied are correct.

Assessment

The format of assessment used for the logarithm tutorial will be in the form of pen-and-pencil assessments. This format is selected because students use a set of procedures to obtain the correct results and therefore by using assessment types such as short answers, the instructor is able to check if the student has applied the appropriate procedures for the problem.

The pen-and-pencil assessments will be designed to solicit a constructed answer from the students. The student must not only demonstrate that they can recall the procedures, they must also show the ability to apply these procedures in a variety of unfamiliar situations.

The following is a table showing each assessment item matching the performance objectives mentioned earlier:

Performance Objective	Task	Assessment
1. Upon completion of the tutorial, the student will be able to rewrite a decimal number as a fraction.	Given a decimal number, rewrite it as a fraction.	Student will be able to rewrite a given decimal number as a fraction and display answer to the board or on paper.
2. Upon completion of the tutorial, the student will be able to simplify a fraction.	After rewriting into fraction, simplify to its lowest terms.	Student will be able to simplify fraction to its lowest terms and display answer to the board or on paper.
3. Upon completion of the tutorial, the student will be able to factorize the numerator and denominator of a fraction.	After simplifying to its lowest terms, factorize the numerator and denominator of the fraction.	Student will be able to factorize the numerator and denominator of the fraction and display answer to the board or on paper.
4. Upon completion of the tutorial, the student will be able to simplify an expression using basic algebra when the number given is not a single number.	Given an expression, simplify to its lowest terms.	Student will be able to simplify expression to its lowest terms and display answer to the board or on paper.

Performance Objective	Task	Assessment
5. Upon completion of the tutorial, the student will be able to factorize a given integer keeping powers of ten together.	Given an integer, factorize keeping powers of ten together.	Student will be able to factorize the integer keeping powers of ten together and display answer to the board or on paper.
6. Upon completion of the tutorial, the student will be to recall the appropriate logarithm rules to the single number or expression.	After simplifying or factorizing, recall appropriate logarithm rules for it.	Student will be able to recall appropriate logarithm rules and display answer to the board or on paper.
7. Upon completion of the tutorial, the student will be able to apply the recalled logarithm rules to the single number or expression.	After recall the appropriate logarithm rules, apply them to the problem.	Student will be able to apply recalled logarithm rules to the problem and display answer to the board or on paper.
8. Upon completion of the tutorial, the student will be able to use the calculator or the logarithm table to find the values of each prime number.	After apply the logarithm rules to the problem, use the calculator or logarithm table to find values of each prime number.	Student will be able to use the calculator or logarithm table to find the value of the given prime numbers and display answer to the board or on paper.
9. Upon completion of the tutorial, the student will be able to calculate the results of the given single number or expression.	After applying the logarithm rules to the problem and finding the values of each prime number, calculate the results.	Student will be able to calculate the results of the problem and display answer to the board or on paper.
10. Upon completion of the tutorial, the student will be able to determine the results of the problem and print them out accordingly.	After calculating, determine the results and print out the answer.	Student will be able to determine the results and print out the answer to the board or on paper.
11. Upon completion of the tutorial, the student will be able to go through the problem and confirm that the procedures applied are correct.	After printing out the answer, go through problem again and check if procedures are applied correctly.	Student will be able to go through problem again and check to make sure procedures are applied correctly.

Instructional Strategy

The instructional strategy used for this tutorial will be mainly supplantive. This strategy is selected because 1) the students need to master the use of logarithms quickly in order to use the knowledge in other areas of the Telecommunications Engineering course; using supplantive strategies enables the students to learn more in a shorter period of time; 2) most students have not taken advance mathematics or they have taken them long ago, as such, much of the learning may be new to them, supplantive strategy is used to avoid cognitive overload; 3) it is necessary to ensure that students obtain the same level of learning outcomes.

The instructional strategy developed for the logarithm tutorial will follow Gagne's Nine Events of Instruction. The details of this instructional strategy are as follows:

Gaining Attention

The instructor will play the music of Beethoven's Fifth Symphony at various volumes and ask the students to guess the loudness of each play in decibels. The instructor will prompt the students with a list of possible answers. After the students finish guessing, the instructor will give the correct answers and explain the relation between logarithm and decibels (the unit for the measurement of sound). The instructor will also underline the fact that research found that humans actually perceive only the logarithm of the physical intensity of a sound and not the intensity itself.

Informing Learner of Instructional Purpose/ Stimulating Learner's Attention

The instructor will inform students of the purpose of the tutorial at the beginning of the lesson. The students will be informed that at the end of the tutorial, they will be able to apply the rules of logarithm and solve logarithm problems when a single number or an expression is given.

After informing students of the purpose of the tutorial, the instructor will stimulate their attention by telling them briefly the following:

a) How Information Theory (this is an important theory in Telecommunications) use logarithms to measure the quantity of information in a message.
b) Why logarithms are useful— they "transform" products in sums and this was historically important—for sailors!
c) Where logarithms are generally used—in measuring the loudness of sound, in the Richter scale of earthquake magnitudes, and in the astronomical scale of stellar brightness.

Stimulating Recall of Prior Knowledge

The instructor will review some basic algebraic operations that will be used in this tutorial. He/she will also recapitulate the definition of a logarithm and the rules for taking logarithms. Finally, the instructor will get students' participation in recalling logarithm values for number one to ten.

Presenting the Stimulus Material

The instructor will go through the procedure for solving logarithm problems and show students how to apply these procedures to solving logarithm problems.

Providing Learning Guidance

The instructor will provide examples on how to apply the procedure and stop at each step of the procedure to show students how to work through each step. The instructor will ensure that students understand each step before continuing with the instruction. The instructor will also assist students while they are doing their exercises.

Eliciting the Performance

The instructor will set aside time for students to practice logarithm problems in class. The instructor will be in class to help any student who has difficulty in solving the given problems. The instructor will also ask students to explain selected problems to the rest of the class so that the students can reinforce their learning and share them with other students.

Providing Feedback/ Assessing Performance

The instructor will provide feedback to students while helping them individually and when the students present their answers to the class.

When the students present their answers, the instructor will also assess their performance and monitor their progress.

Enhancing Retention and Transfer

At the end of the tutorial, the instructor will ask each student to recall the set of procedures in solving logarithms by writing them down on a three by five inches card. Writing down the procedures by the students will help reinforce understanding and retention. They are asked to look at it while waiting for a bus or a friend or while doing exercises at home.

Media Choice

Initially, the medium decision was to develop a CD-ROM or an asynchronous long distance tutorial over the Internet. This was considered then because most of the students own a computer at home and they work full-time; in view of these factors, the initial decision was to select a medium that can be accessed at any time. However, after careful consideration, the instructional designer decided it is more important for students to have an instructor physically present to answer questions and assess their work on the spot. For example, if a student is unable to come up with the right answer even after using the procedures provided, the instructor can look through the student's work and check which part of the procedure the student has done incorrectly. In a CD-ROM or Internet tutorial, it will be difficult to incorporate this form of supervision and feedback.

The medium for this instruction will be an instructor-led classroom tutorial. This will be held on four Saturday mornings for about two hours each. In order not to overload the student's schedule, the tutorial will begin when logarithms are introduced in the main course.

The student will be provided with printed supplementary material in class. A sample of these materials is provided in Appendix B.

Formative Evaluation

Expert Review of Materials

Once a draft of the instructional materials is ready, they will be given to Professor Finn to review for content accuracy and completeness. Professor Finn is the instructor who teaches the Telecommunications Engineering course and the mathematics component in this course. Besides accuracy and completeness, Professor Finn will be asked to check if examples and practice exercises given are adequate and the difficulty level suitable for the students. He will also be asked to review the relevance of the examples and exercises to the Telecommunications Engineering course as a whole.

Learner Validation of Materials

Once the instructional designer has completed the draft of the instruction materials, the designer will test out the instructional materials with two or three potential students of this tutorial. The designer will note down any problems or misunderstandings the students have on the materials. After obtaining some initial feedback on the materials, the designer will hand the materials to the assigned instructor to conduct a trial session with a group of potential students to get additional evaluation on the materials. At the end of the trial session, the students will be asked to complete an attitude questionnaire that will be used to fine tune the weak areas of the instruction materials.

Once all the feedback is collected, the instructional designer will update and revise the instructional materials accordingly.

Summative Evaluation

After the tutorial has been used for two semesters, a summative evaluation will be conducted. For the subjective part of the evaluation, the instructor for the tutorial and Professor Finn will be interviewed to obtain feedback on the effectiveness and usefulness of the tutorial. For the objective part of the evaluation, a questionnaire will be distributed to students who have taken the tutorial to see how they have benefited from it.

Appendix A – Learner Analysis Questionnaire

Thank you for taking a few moments to answer these questions. The answers to these questions will be used in a pilot project to design a mathematics tutorial module for this class.

1. To help us evaluate your answers, please indicate by checking...

• Student status	Full-time student	Part-time student
• Computer experience	1 year or less	2 to 3 years
	4 to 5 years	more than 5 years
• Computer at home	Yes	No

2. When was the last time you took a mathematics course?

less than 2 years ago	Course name : _____
2 to 3 years ago	Course name : _____
4 to 5 years ago	Course name : _____
more than 5 years ago	Course name : _____

3. What is your major?

Engineering
Telecommunications
Computer Science
Other, please specify _____

4. When reading the math refresher section of the course, do you understand the concepts readily?

All of the time
Often
Some of the time
Rarely
Never

5. In general, when you work on the math assignments, do you find them easy?

All of the time
Often
Some of the time
Rarely
Never

6. When you have completed the math assignments, how confident are you that you have gotten the right answers?

Very confident
Often confident
Sometimes confident
Rarely confident
Never confident

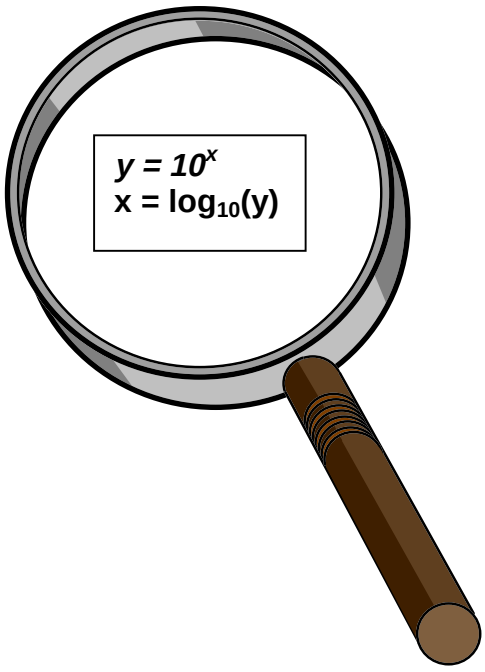
7. Are you in favor of supplementary instruction for this course?

Yes No

Appendix B - Sample Production

A tutorial on…

LOGARITHMS



Introduction

The logarithm is a useful mathematical concept in sciences. An understanding of them is essential to an understanding of many scientific ideas. When we represent data in scientific notation, our problem is simplified. The exponent of 10 gives us a simple way to rank different scales.

What is a LOGARITHM?

The logarithm ("log") of a number *y* is the power of 10 that gives that number.
In equation form, this is:

$$y = 10^x$$

$$x = \log_{10}(y)$$

Examples:

$$\log_{10}(1000) = 3$$

or

$$\log_{10}(10^3) = 3$$

Rules for Taking Logarithms

$\log (A \times B) = \log(A) + \log(B)$

$\log (A/B) = \log(A) - \log(B)$

$\log (A^x) = x \log(A)$

$\log (AB/CD) = \log(A) + \log(B) - \log(C) - \log(D)$

Some Common Logarithm Values

Number (n)	Log (n)
$10{,}000 = 10^4$	**4**
$1{,}000 = 10^3$	**3**
$100 = 10^2$	**2**
$10 = 10^1$	**1**
$1 = 10^0$	**0**
$0.1 = 10^{-1}$	**-1**
$0.01 = 10^{-2}$	**-2**
$0.001 = 10^{-3}$	**-3**
1	**0**
2	**0.301**
3	**0.477**
4	**0.602**
5	**0.699**
6	**0.778**
7	**0.845**
8	**0.903**
9	**0.954**
10	**1**

Steps for Solving Logarithms

1. Check if the number given is a single number or it is an expression.
2. If it is a single number, check if it is an integer, if it is not a single number, simplify the expression.
3. If it is an integer, check to see if it can be factorized. If not, rewrite the decimal number as a fraction and simplify it.
4. Factorize the integer, the fraction (numerator and denominator) or the expression.
5. Recall the appropriate rules for the factorized integer, fraction or expression.
6. Apply the recalled rules to the integer, fraction or expression.
7. Use the calculator or logarithm table to find the logarithm of each prime number.
8. Apply these values and calculate accordingly.
9. Write down your answer and check your work.

Exercises

1. Explain the idea of a "logarithm."

2. Use the basic definition of a logarithm to write the following logs.

   Examples: **$\log(10^5) = 5$** **$\log(10^{20}) = 20$**

   a) $\log(10^{-4}) =$

   b) $\log(10^{18}) =$

   c) $\log(10^{-1/2}) =$

   d) $\log(10^{2/3}) =$

3. What are the following logarithms? (Very important)

   Examples: **$\log(1) = 0$** **$\log(4) = 0.602$**

   a) $\log(2) =$

   b) $\log(10) =$

   c) $\log(1/2) =$

   d) $\log(1/10) =$

4. If the log(N) = x, what is the log (1/N)? (Very important)

5. A composite number has factors other than itself and one. To estimate values of

logarithms, it is usually convenient to begin by looking for factors of 10 and 2 in a composite number. Express the following as products of their factors, emphasizing 2s and 10s.

Examples: **4 = 2 x 2** **80 = 2 x2 x 2 x 10**

a) 8 =

b) 64 =

c) 40 =

d) 640 =

e) 4000 =

f) 80,000 =

g) 64,000,000 =

6. Write the rule that gives the value of log(A x B) when the values of log(A) and log(B) are known.

7. Remembering that log(2) ≅ 0.3, apply the rule stated in question 6.

Examples : **$\log(2000) = \log(2) + \log(10^3) = 3.3$**

$\log(16) = \log(2^4) = 4\log(2) = 1.204$

a) $\log(4 \times 10^{-6}) =$

b) log (0.004) =

c) log(8000) =

d) $\log(8 \times 10^{-6}) =$

8. Write the rule that gives the value of log(A/B) when the values of log(A) and log(B) are known.

9. Remembering that log(2) ≅ 0.3, show how to apply the rule stated in question 8.

   Examples : **$\log(10{,}000/2) = \log(10^4) - \log(2) = 3.7$**

   $\log(10^{15}/10^6) = 15 - 6 = 9$

   a) $\log(4/10^{-4}) =$

   b) $\log[(2 \times 10^5 \times 8 \times 10^{-7} \times 10^{18})] =$

   c) $\log(8{,}000/2) =$

   d) $\log[(6 \times 10^5 \times 4 \times 10^{-7} \times 10^{20})] =$

10. For the following examples, find the log of the result using a calculator, then find the log by applying (several times, as necessary) the rules used above.

    Examples: **$\log[(2 \times 200)/(8 \times 10^{-3}) = 4.699$ (calculator solution)**

    $= 0.3+2.3-0.9+3 = 4.7$ (Log rule ssolution)

    a) $\log[(8 \times 4)/(10 \times 2)] =$

    b) $\log[(10^5 \times 10^4 \times 10^{-2})/(10^{20} \times 10 \times 10^3)] =$

11. Occasionally, it is convenient to express a fraction as the ratio of two integers (for example, 2.5 = 5/2 or 6.667 = 2 x 10/3). To find the log of the fraction, it is enough to find the difference between the logs of the integers.

Use this approach to find:

a) log (5) =

b) log (7.5) =

c) log (22.5) =

d) log (1.667) =

e) log (3.333) =

12. Use the values of the logs of the integers and the various approaches to finding logs to find the values for the following:

Examples: **$\log (4 \times 10^3) = 2 (0.3) + 3 = 3.602$**

$\log (5 \times 10^7) = 0.699 + 7 = 7.699$

a) log (150×10^9) =

b) log $[(2 \times 10^8)/25000]$ =

c) log (48) =

d) log$[(4 \times 10^4)/(4 \times 10^{-20})$ =

e) log (2 x 9 x 8) =

f) log (48) =

g) log (25×10^{-6}) =

Acknowledgements:

Prof. Robert Finn for use of his logarithm exercises in the sample production.