Ocean Circulation, Heat Transport, and Kinematics

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Oceanic Heat Transport

“heat content” or “internal energy” per volume can be approx:

\[ E = c_p \rho \theta \]

\( \rho \) = density of seawater \( \approx 1030 \text{ kg/m}^3 \)
\( \theta \) = potential temperature
\( c_p \) = specific heat \( \approx 4000 \text{ J kg}^{-1} \text{ K}^{-1} \)

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See for instance Gill (1982)
Derive “heat budget” from temperature equation

\[ \theta_t + \nabla \cdot (\vec{u} \theta) = (\kappa_V \theta_z)_z + \nabla_H \cdot (\kappa_H \nabla_H \theta) \]

\( \vec{u} \) = velocity

\((\kappa_V, \kappa_H)\) = eddy diffusivity coefficients

Multiply by \( c_p \rho \) and integrate over volume of some ocean region:

\[
\int c_p \rho \theta_t dV + \int_{\text{all}} c_p \rho \theta \vec{u} \cdot \vec{n} dA = \int \int c_p \rho \kappa_V \theta_z \big|_{\text{TOP}} dxdy + \int_{\text{sides}} c_p \rho (\kappa_H \nabla_H \theta) \cdot \vec{n} dA
\]

Physical interpretation of terms:

\[ H = \int_{\text{all}} c_p \rho \theta \vec{u} \cdot \vec{n} dA = \text{lateral advective heat transport} \]

\[ Q = \int \int c_p \rho \kappa_V \theta_z \big|_{\text{TOP}} dxdy = \text{heat exchange with atmosphere} \]

\[ D = \int_{\text{sides}} c_p \rho (\kappa_A \nabla_H \theta) \cdot \vec{n} dA = \text{lateral eddy heat transport} \]

\[ \int E_t dV + H = Q + D \]
\[
\int E_t dV + H = Q + D
\]

Several possibilities: including
1. all terms \(= 0\) (thermally passive ocean)
2. \(H, D \) small, \(Q = \int E_t \ dV\) (ocean is thermal flywheel)
3. \(\int E_t dV\) small, \(Q = H - D\) (ocean moves heat around)

1 \(\Rightarrow\) good approx global and annual average
2 \(\Rightarrow\) good approx some places for timescales \(\leq\) seasonal
3 \(\Rightarrow\) good approx annual average

What determines \(Q(x,y)\)?
Need to consider both atmosphere dynamics and
global oceanic \textit{velocity} and \textit{temperature} field.

\[
H = \int_{\text{all}} c_p \rho \theta \vec{u} \cdot \vec{n} dA
\]

Similar expressions exist for transport of salt and other tracers.
Annual Avg Net Heat Flux

From COADS, W/m²
Net Annual-Average Heat Flux into Ocean

General pattern:
  absorbs heat near equator, loses heat at high latitudes
  (as expected)
Typical values $O(50 \text{ W/m}^2)$
Some interesting details:
  heat gain largely in east, heat loss in west
  Pacific heat gain especially big
  Atlantic heat loss especially big
When calculating $H$, we often choose domain so that most of the lateral boundaries are “walls”, except for a 1 or 2 zonal sections (as shown). $u \cdot n = 0$ on bottom, and if we can neglect $u \cdot n$ through the top, $H$ is simply meridional heat transport.
\[ H = \int \int c_p \rho \theta v dxdz \]

This expression not too meaningful unless

\[ \int \int c_p \rho v dxdz = 0. \]

Why? Let \( \theta' = \theta + \theta_0 \) (Kelvin vs. Celsius). Then

\[ H' = \int \int c_p \rho \theta' v dxdz \]

\[ H' = \int \int c_p \rho \theta v dxdz + \theta_0 \int \int c_p \rho v dxdz \]

We want \( H = H' \).
Ocean and Atmospheric Meridional Heat Transport

Ocean clearly important in tropics
Latest observations ➔ smaller ocean role at high latitudes
(still significant uncertainties)

Trenbirth and Caron, 2001: J Clim, 3433-3443
Coupled models indicate: significant atmospheric response to oceanic circulation change

**FIG. 22.** Difference in surface air temperature (degrees Celsius) between experiments I and II.

Manabe and Stouffer, 1988: J Clim; Stouffer *et al.*, 2006: J Clim;
Let $U(x, y) = \int u \, dz$, $V(x, y) = \int v \, dz$ and $\vec{U} = U \hat{x} + V \hat{y}$

It can be shown that we can find “potential” $\Phi(x, y)$ and “stream-function” $\Psi(x, y)$ such that

$$U = \Phi_x - \Psi_y$$

$$V = \Phi_y + \Psi_x$$

Vorticity and Divergence are then

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{U}) = V_x - U_y = \nabla^2 \Psi$$

$$\vec{\nabla} \cdot \vec{U} = U_x + V_y = \nabla^2 \Phi$$

No vorticity $\rightarrow$ “Potential Flow” $(\Psi = 0)$.

No divergence $\rightarrow$ “Barotropic Streamfunction”: $(\Phi = 0)$.

See for instance Batchelor (1969), Kundu (1990)
Can Find Streamfunction in Other Dimensions:

Let \( V(y, z) = \int v \, dx \), \( W(y, z) = \int w \, dx \) and \( \bar{U} = V \hat{y} + W \hat{z} \)

If \( \nabla \cdot U \equiv V_y + W_z = 0 \) then can define

\[
V = -\Psi_z \\
W = \Psi_y
\]

where \( \Psi(y, z) \) is “Meridional Overturning Streamfunction”

No divergence \( \Rightarrow \) no flow from eastern or western boundaries of basin.

Could also define “Zonal Overturning Streamfunction” but hasn’t been useful in oceanography.

To calculate streamfunction:
1) Set [arbitrary] value at some boundary
2) Calculate either \( \Psi = -\int V \, dz \) or \( \Psi = \int W \, dy \)
Example: Near-global numerical model (HYCOM), climatological forcing

Annual Avg B-T Stream Function, Years 86-90

c.i. = 25 (>25) and 5 (<25) Sv

Klinger et al (2005), unpublished
Meridional Overturning Streamfunctions (same model)
Not necessarily clean relationship between streamfunction and heat transport

Temperature field is similar

Nonlinear contour intervals for $v$ ($\pm 0.125 \times 2^{[0:7]} \text{ cm/s}$) and $\sigma_\theta$

same model as previous pages
Meridional Overturning Streamfunction and Heat Transport

For basin of width $L$ at a given latitude, overturning is:

$$\bar{v}(y, z) = \frac{1}{L} \int v dx$$

$$v = \bar{v} + v'$$

Use this to analyze $H$ (integral over $x$ and $z$):

$$\int v \theta dA = \int (\bar{v} + v')(\bar{\theta} + \theta') dA$$

$$= \int (\bar{v} \bar{\theta} + v' \theta') dA.$$ 

Now average the “gyre” residual in the vertical:

$$\tilde{v'} = \frac{1}{D} \int_{-D}^{0} v' dz$$

$$v' = \tilde{v'} + v''$$

Substituting, we find

$$\int v \theta dA = \int \bar{v} \bar{\theta} dA + \int \tilde{v'} \bar{\theta} dA + \int v'' \theta'' dA$$
We can define 2 kinds of overturning streamfunction

**Z-Coordinate Overturning**

\[ V(z) = \int v(x, z) \, dx \]

(integrate along constant \( z \))

\[ \Psi_z = - \int V \, dz \]

(Subscripts not derivatives here)

**\( \theta \)-Coordinate Overturning**

\[ V(\theta) = \int v(x, \theta)(\partial z / \partial \theta) \, dx \]

(integrate along constant \( \theta \))

\[ \Psi_\theta = - \int V \, d\theta \]
Temperature-coord overturning
Immediately shows net diabatic and adiabatic components of flow

Atlantic Isotherms (WOA94, 30° W) and Idealized Transport
Simple Example

- 2 layer flow
- temperatures: $\theta_1$ and $\theta_2$
- mass transports: $\Psi_1$ and $\Psi_2 (= -\Psi_1)$

$$H = c_p \rho (\theta_1 \Psi_1 + \theta_2 \Psi_2) = c_p \rho \Psi_1 (\theta_1 - \theta_2)$$

Can generalize to continuous profiles
Let

\[ H_0 = H / (c_p \rho) = \int_B^T \int_W^E v \theta \, dx \, dz. \]

If we think of everything as a function of \((x, \theta)\) rather than \((x, z)\), then

\[ H_0 = \int_{\theta_B}^{\theta_T} \int_W^E v(x, \theta) \theta \frac{\partial z}{\partial \theta} \, dx \, d\theta. \]

The volume transport per temperature interval is

\[ V(\theta) = \int_W^E v z_\theta \, dx \]

and it can be shown that

\[ H_0 = \int_{\theta_B}^{\theta_T} V \theta \, d\theta \]

The meridional overturning streamfunction in temperature coordinates \(\Psi_\theta\) can be defined though

\[ V = - \frac{\partial \Psi_\theta}{\partial \theta} \]

Assuming that \(\Psi_\theta = 0\) at the top and bottom, integration by parts gives

\[ H_0 = \int_{\theta_B}^{\theta_T} \Psi_\theta \, d\theta \]

\[ \textbf{Klinger and Marotzke (2000, JPO)} \]
IMPORTANT: In general, $v(z)$ can look very different from $v(\theta)$, not just “stretched”.  

$$H \approx c_p \rho K \Delta \Psi \Delta \theta$$
Heat Transport Measurements

Three main approaches:
Measure surface heat flux and integrate
  advantage: focus on relatively accessible data
  disadvantage: how accurate are bulk formulae?
Measure top-of-atmosphere radiation and subtract atmos transport
  advantage: accurate radiation and lots of atmos data
  disadvantage: problems with atmos models, errors in resid
Directly measure ocean $v$, $\theta$
  advantage: can relate transport to what ocean is doing
  disadvantage: ocean data only in some regions

All methods have substantial error bars,
Useful to compare different ways of calculating same quantity

See Bryden and Imawaki, Ocean Circulation and Climate sec. 6.1
Meridional Heat Transport from reanalysis surface heat fluxes, Feb 1985 – Apr 1989

Trenbirth and Caron (2001)
"Direct" Measurement of Ocean Heat Transport (based on hydrography, Ekman transport, tracer distributions)

Ganachaud and Wunsch (2000, Nature)
Another Direct Heat Transport Measurement

Talley (2003, JPO)
Talley (2003) 

Trenberth and Caron (2001) 

Graphs showing ocean temperatures and salinity across different latitudes and oceanic regions.
Observed Mass Transport as function of $\theta$ bins and density bins

(Roemmich et al. (2001, JGR)

based on repeat sections