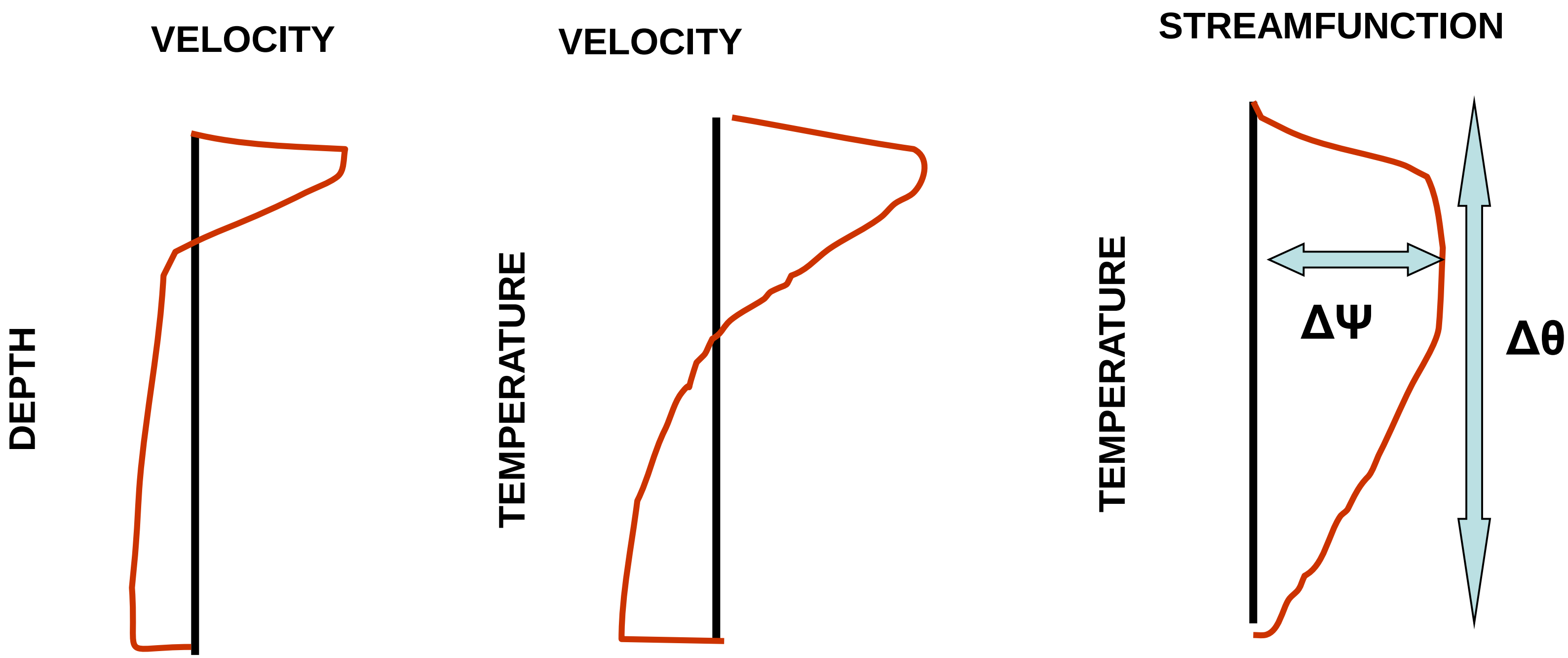
George Mason University
CLIM 752 Ocean General Circulation

Heat Transport & Streamfunctions



Reading: *Ocean Circulation in Three Dimensions,* Chapter I

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Spring 2017

Oceanic Heat Transport

"heat content" or "internal energy" per volume can be approx:

$$E = c_p \rho \theta$$

ρ = density of seawater $\approx 1030 \text{ kg/m}^3$
θ = potential temperature
c_p = specific heat $\approx 4000 \text{ J kg}^{-1} \text{ K}^{-1}$

p	T	cp
0	-2	3989
0	31	4002
5000	-2	3844
5000	2	3854

See for instance Gill (1982)

Derive **"heat budget"** from temperature equation

$$\theta_t + \vec{\nabla} \cdot (\vec{u}\theta) = (\kappa_V \theta_z)_z + \vec{\nabla}_H \cdot (\kappa_H \vec{\nabla}_H \theta)$$

$\vec{u}$ = velocity

(κ_V, κ_H) = eddy diffusivity coefficients

Multiply by $c_p\rho$ and integrate over volume of some ocean region:

$$\int c_p\rho\theta_t dV + \int_{\text{all}} c_p\rho\theta\vec{u}\cdot\vec{n}dA = \int\int c_p\rho\kappa_V\theta_z|_{\text{BOT}}^{\text{TOP}} dxdy + \int_{\text{sides}} c_p\rho(\kappa_H\vec{\nabla}_H\theta)\cdot\vec{n}dA$$

Physical interpretation of terms:

$$H = \int_{\text{all}} c_p\rho\theta\vec{u}\cdot\vec{n}dA = \text{lateral advective heat transport}$$

$$Q = \int\int c_p\rho\kappa_V\theta_z|_{\text{BOT}}^{\text{TOP}} dxdy = \text{heat exchange with atmosphere}$$

$$D = \int_{\text{sides}} c_p\rho(\kappa_A\vec{\nabla}_H\theta)\cdot\vec{n}dA = \text{lateral eddy heat transport}$$

$$\int E_t dV + H = Q + D$$

Some Special Cases are Instructive

$$\int E_t dV = Q - H + D$$

Several possibilities: including

1. all terms = 0 (thermally passive ocean)
   good approx. for global & annual avg

2. H, D small, $Q = \int E_t \, dV$ (ocean is thermal flywheel)
   good approx. for timescales ≤ seasonal, in some places
   global average over multiple years (global warming, etc.)

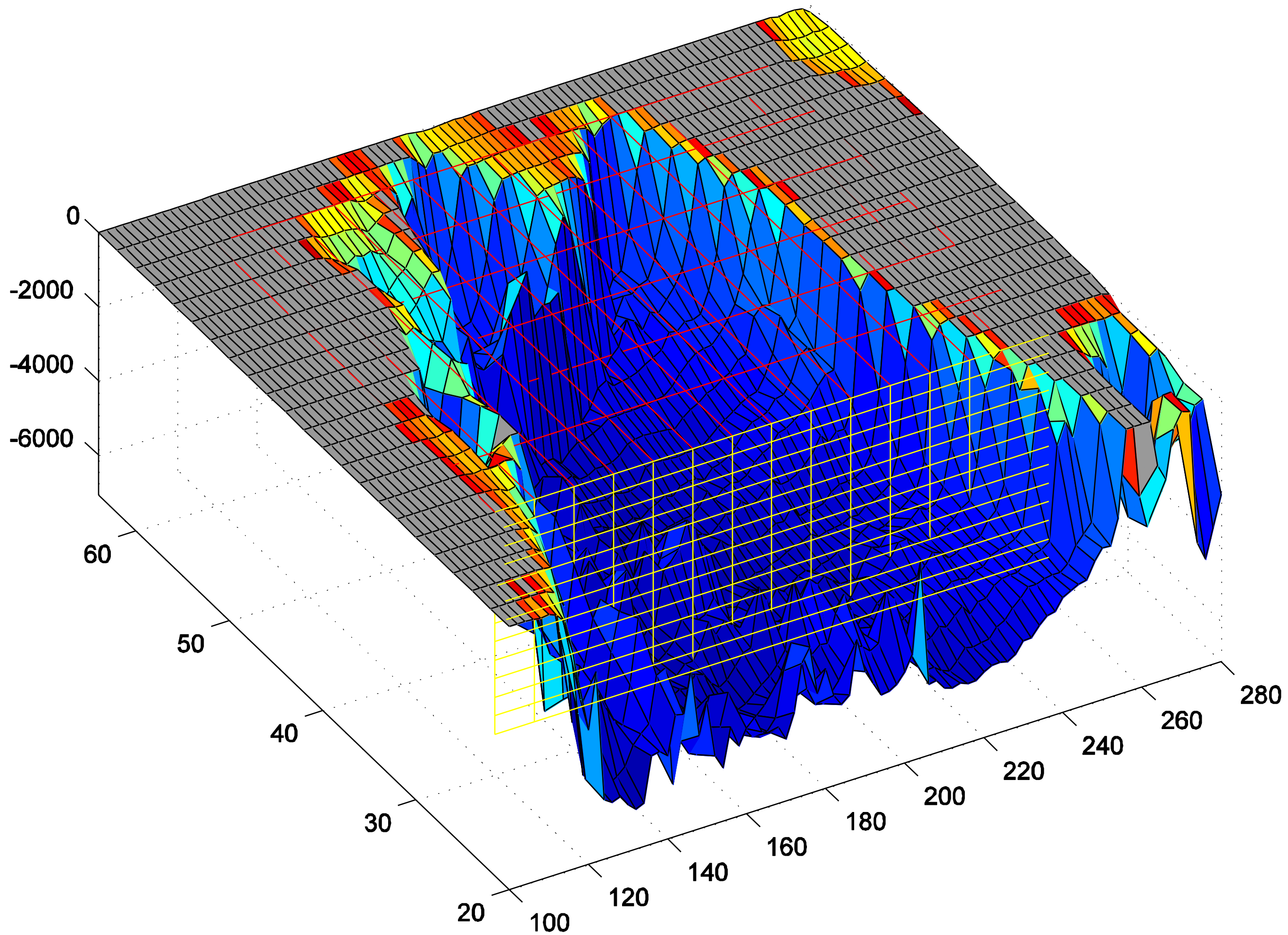
3. $\int E_t \, dV$ small, Q = H - D (ocean moves heat around)
   good approx. for annual average

What determines Q(x,y)?
Need to consider both atmosphere dynamics and global oceanic velocity and temperature field.

$$H = \int_{\text{all}} c_p \rho \theta \vec{u} \cdot \vec{n} \, dA$$

Similar expressions exist for transport of salt and other tracers.

When calculating *H*, we often choose domain so that most of the lateral boundaries are "walls", except for a 1 or 2 zonal sections (as shown). u●n = 0 on bottom, and if we can neglect u●n through the top, *H* is simply meridional heat transport.



Warning About "Temperature Transport"

$$H = \int\int c_p \rho \theta v dx dz$$

This expression not too meaningful unless

$$\int\int c_p \rho v dx dz = 0.$$

Why? Let $\theta' = \theta + \theta_0$ (Kelvin vs. Celsius). Then

$$H' = \int\int c_p \rho \theta' v dx dz$$

$$H' = \int\int c_p \rho \theta v dx dz + \theta_0 \int\int c_p \rho v dx dz$$

We want $H = H'$.

Streamfunctions

Let $U(x,y) = \int u dz,\ V(x,y) = \int v dz$ and $\vec{U} = U\hat{x} + V\hat{y}$

It can be shown that we can find "potential" $\Phi(x,y)$ and "stream-function" $\Psi(x,y)$ such that

$$U = \Phi_x - \Psi_y$$

$$V = \Phi_y + \Psi_x$$

Vorticity and Divergence are then

$$\vec{z} \cdot (\vec{\nabla} \times \vec{U}) = V_x - U_y = \nabla^2 \Psi$$

$$\vec{\nabla} \cdot \vec{U} = U_x + V_y = \nabla^2 \Phi$$

No vorticity $\rightarrow$ **"Potential Flow"** $(\Psi = 0)$.

No divergence $\rightarrow$ **"Barotropic Streamfunction"**: $(\Phi = 0)$.

See for instance Batchelor (1969), Kundu (1990)

Can Find Streamfunction in Other Dimensions:

Let $V(y,z) = \int v dx,\ W(y,z) = \int w dx$ and $\vec{U} = V\hat{y} + W\hat{z}$

If $\vec{\nabla} \cdot U \equiv V_y + W_z = 0$ then can define

$$V = -\Psi_z$$

$$W = \Psi_y$$

where $\Psi(y,z)$ is **"Meridional Overturning Streamfunction"**

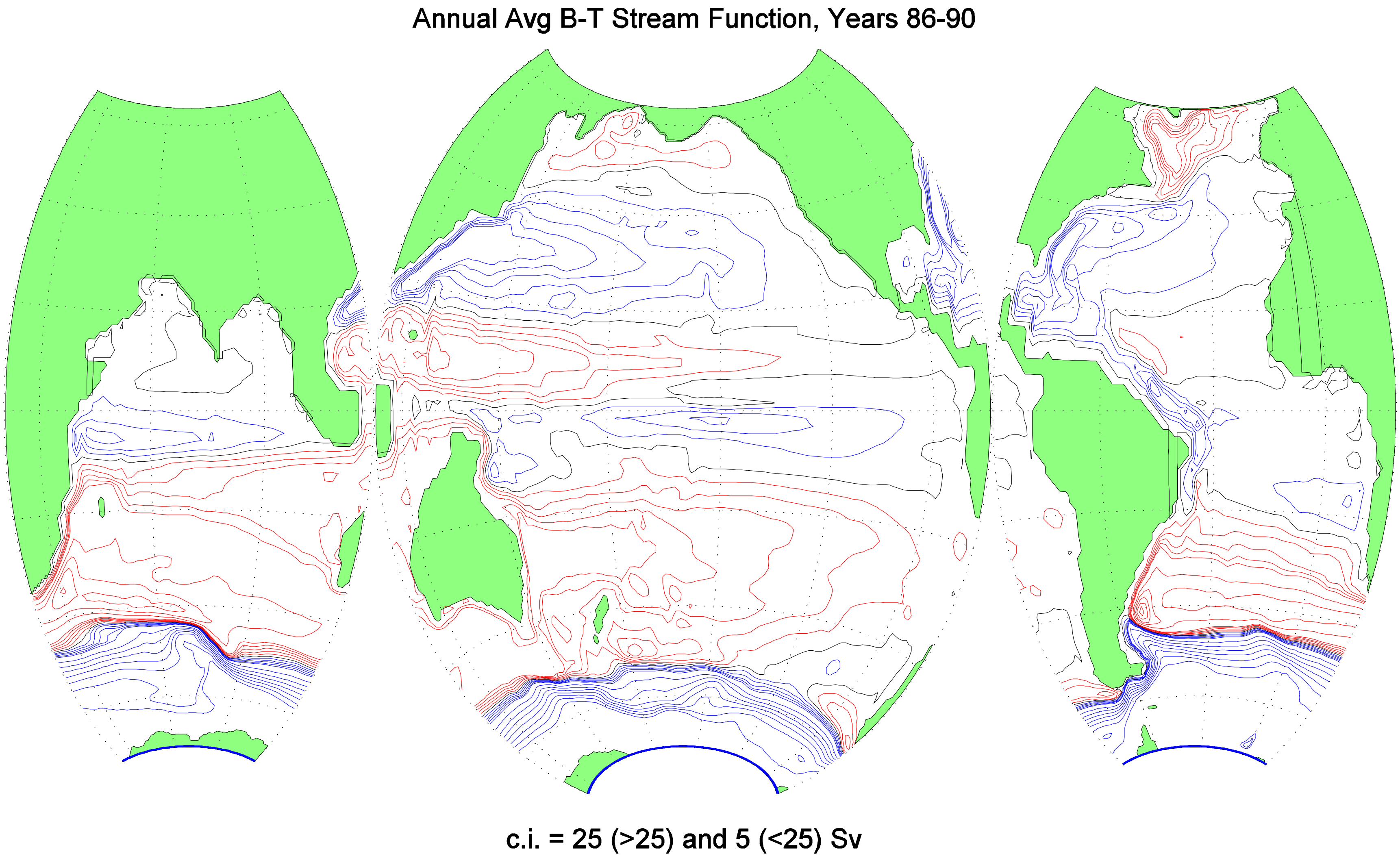
No divergence ➔ no flow from eastern or western boundaries of basin.

Could also define "Zonal Overturning Streamfunction"
but hasn't been useful in oceanography.

To calculate streamfunction:

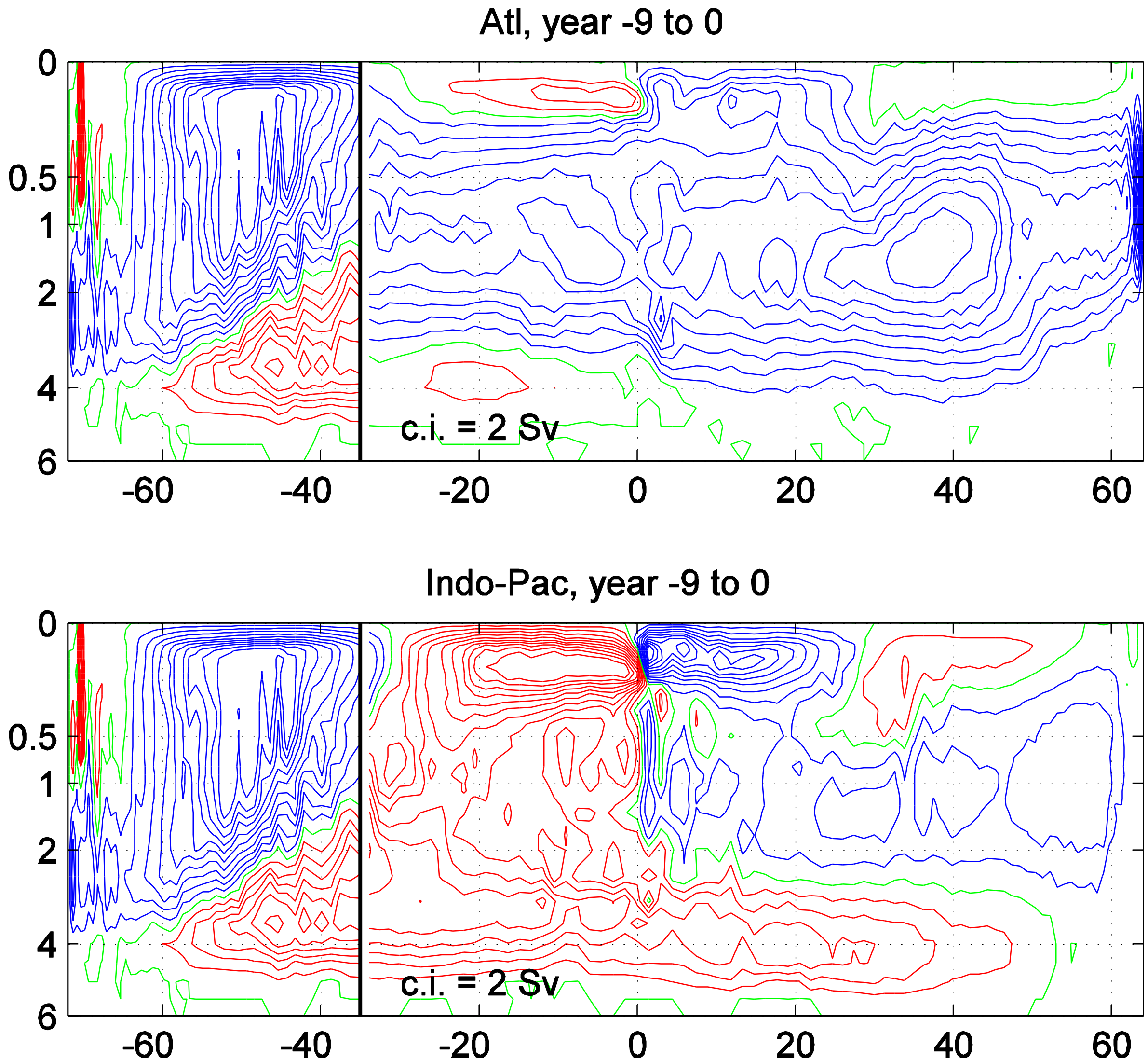
1) Set [arbitrary] value at some boundary
2) Calculate either $\Psi = -\int V\, dz$ or $\Psi = \int W\, dy$

Example: Near-global numerical model (HYCOM), climatological forcing

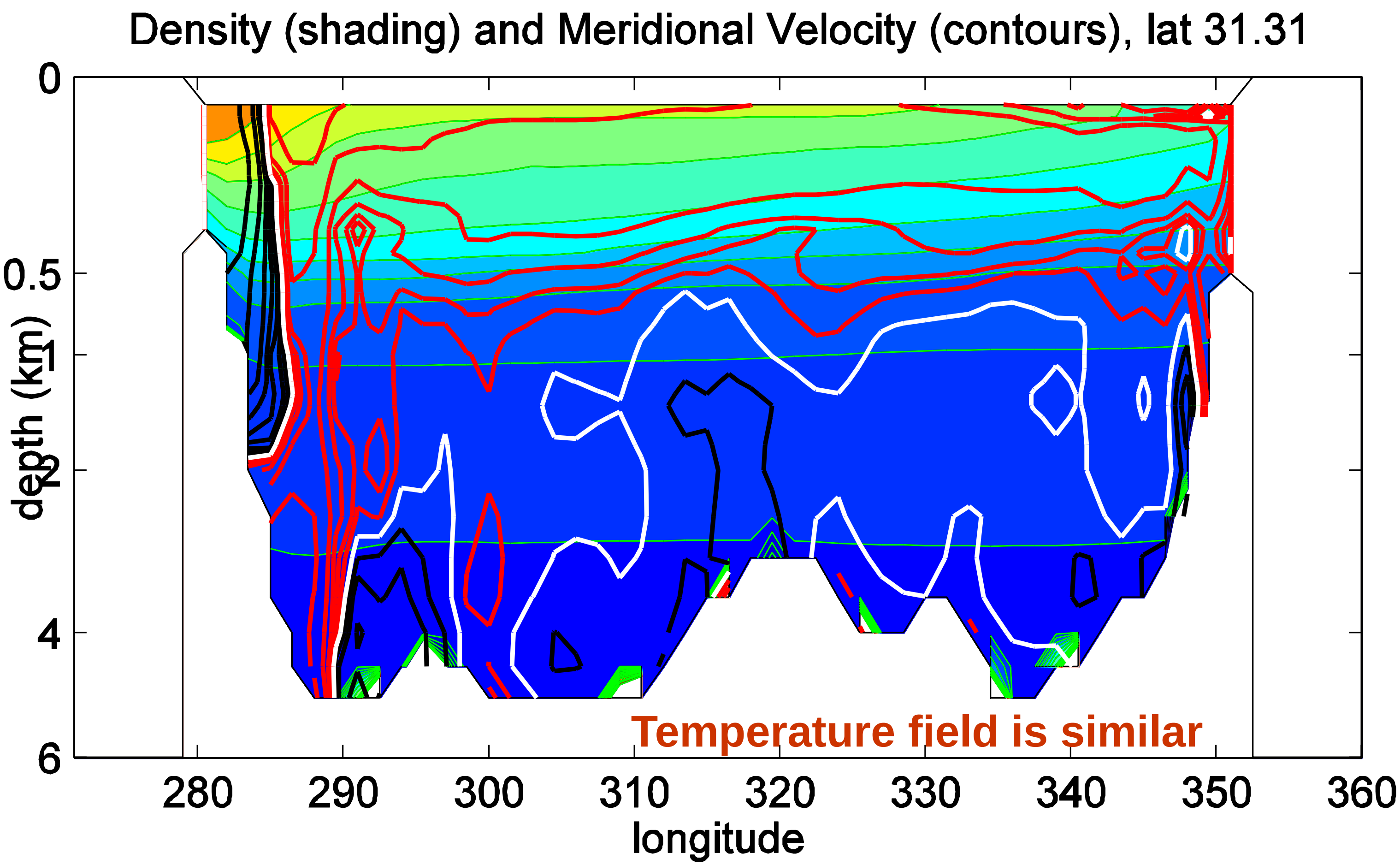


Klinger et al (2005), unpublished

Meridional Overturning Streamfunctions (same model)



Not necessarily clean relationship between streamfunction and heat transport



Nonlinear contour intervals for v ($\pm$ .125 $2^{[0:7]}$ cm/s) and σ_θ

same model as previous pages

Meridional Overturning Streamfunction and Heat Transport

For basin of width L at a given latitude, overturning is:

$$\overline{v}(y,z) = \frac{1}{L}\int v dx$$

$$v = \overline{v} + v'$$

Use this to analyze H (integral over x and z):

$$\int v\theta dA = \int (\overline{v} + v')(\overline{\theta} + \theta')dA$$

$$= \int (\overline{v}\overline{\theta} + v'\theta')dA.$$

Now average the "gyre" residual in the vertical:

$$\tilde{v}' = \frac{1}{D}\int_{-D}^{0} v' dz$$

$$v' = \tilde{v}' + v''$$

Substituting, we find

$$\int v\theta dA = \int \overline{v}\overline{\theta} dA + \int \tilde{v}'\tilde{\theta}' dA + \int v''\theta'' dA$$

We can define 2 kinds of overturning streamfunction

Z-Coordinate Overturning

let $V(z) = \int v(x,z)\, dx$

(integrate along constant z)

$\Psi_z = -\int V\, dz$

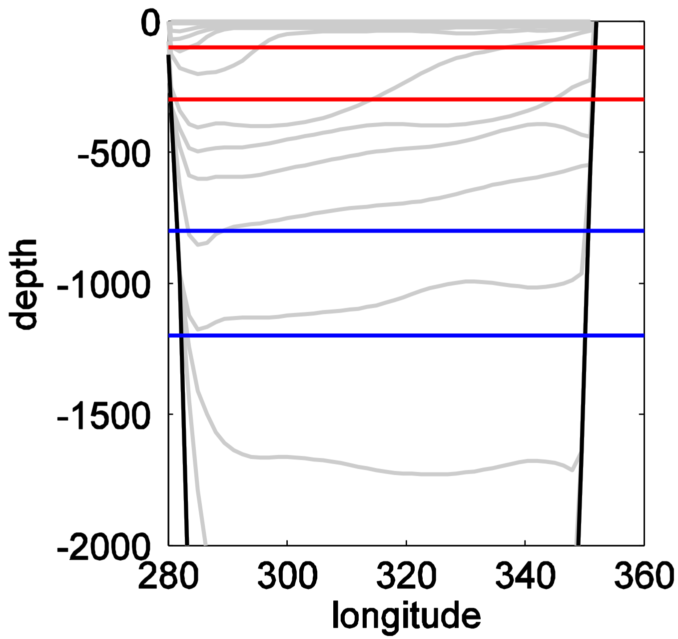
θ-Coordinate Overturning

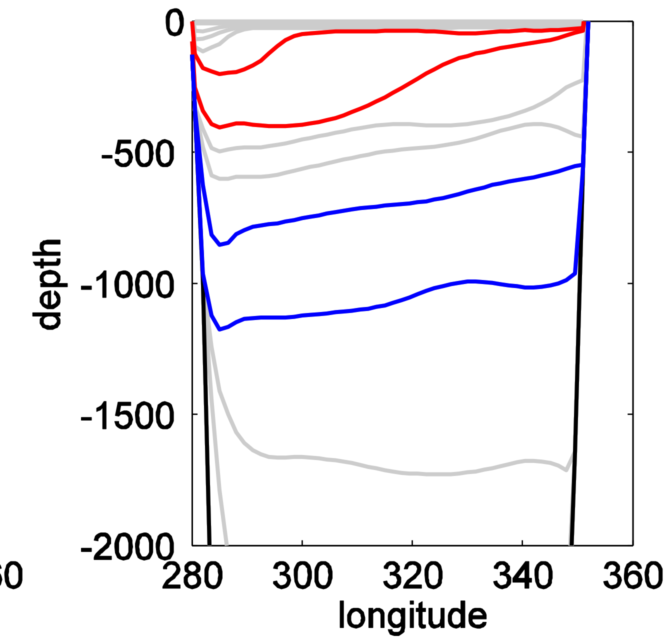
let $V(\theta) = \int v(x,\theta)(\partial z/\partial\theta)dx$

(integrate along constant θ)

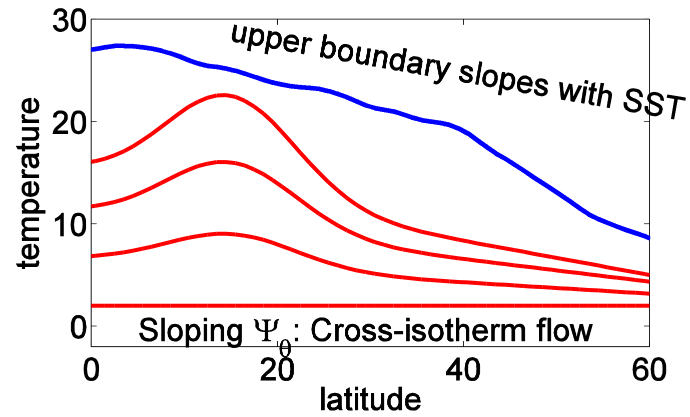
$\Psi_\theta = -\int V\, d\theta$

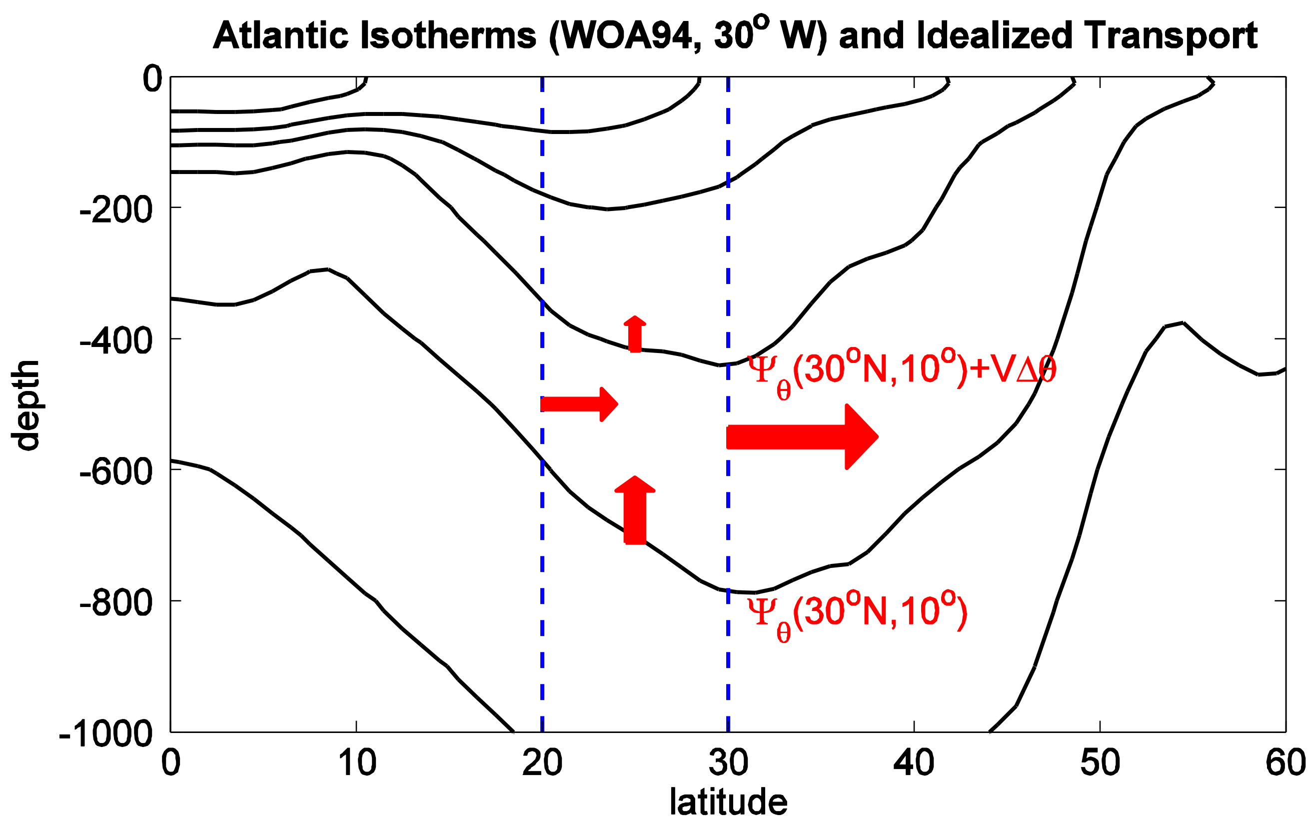
(Subscripts **not** derivatives here)





Temperature-coord overturning Immediately shows net diabatic and adiabatic components of flow



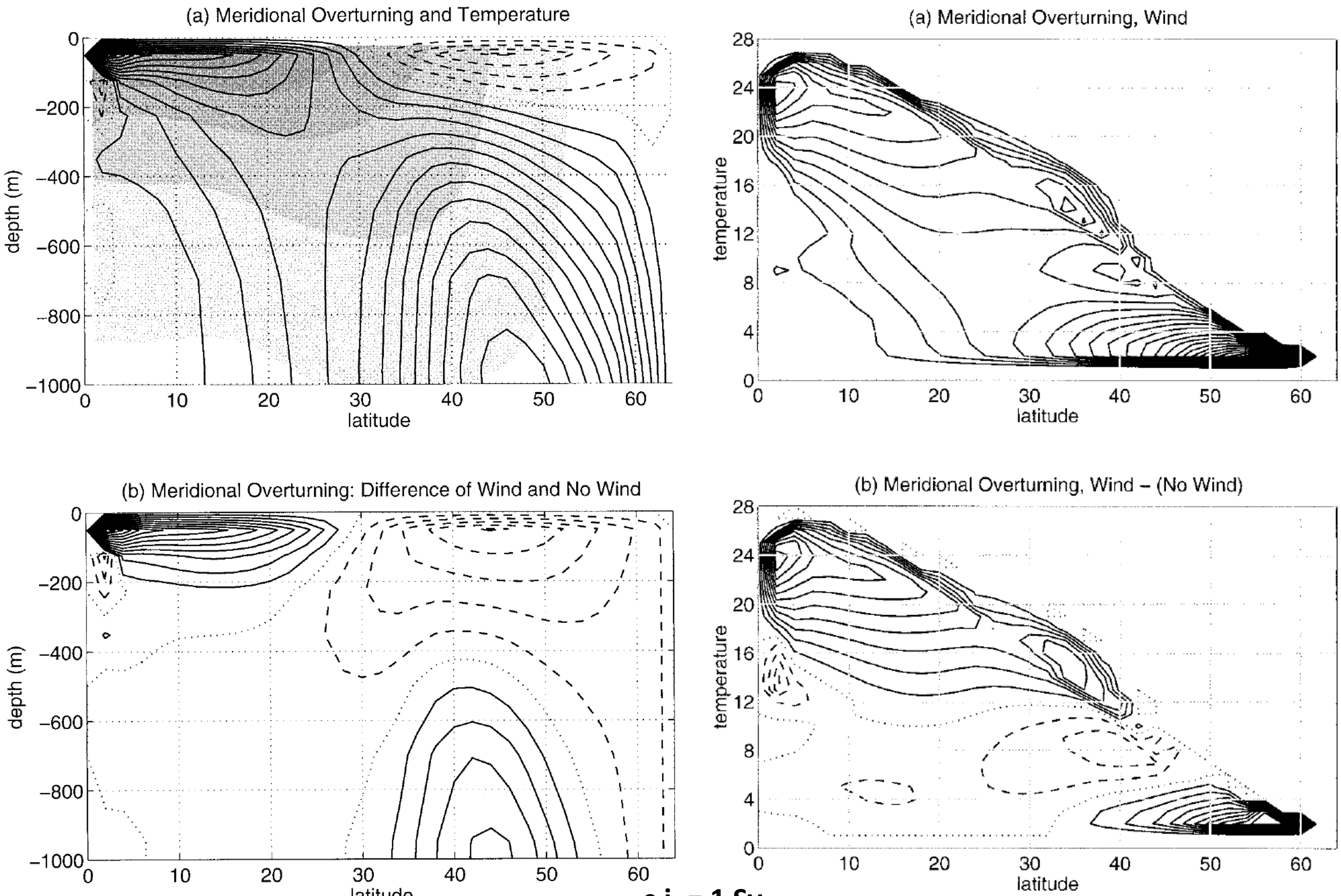


Numerical Model Example of Qualitative Differences between Ψ_z and Ψ_θ

Z-coordinate overturning | **ρ-coordinate overturning**

Overturning, buoyancy & wind forcing ➔

Overturning Difference, ➔ (buoy & wind run) minus (buoy-only run)



c.i. = 1 Sv

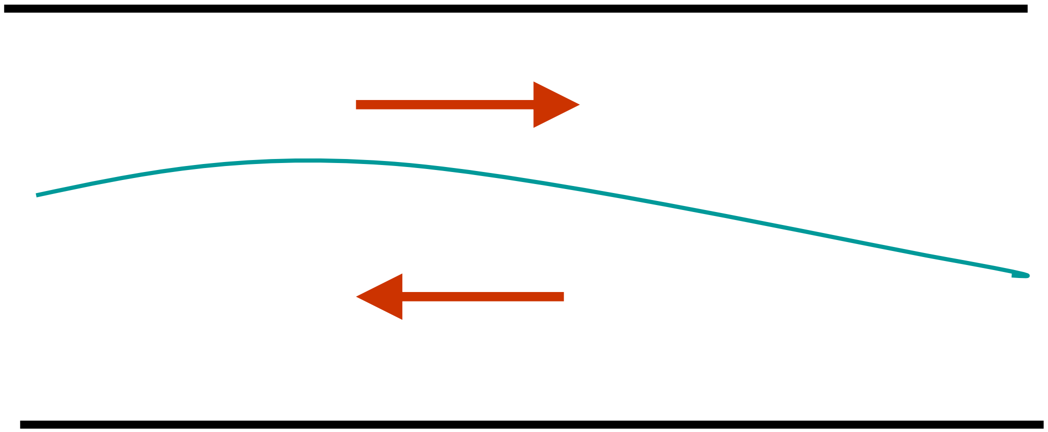
Klinger & Marotzke (2000, *JPO*)

Θ-Coordinate Overturning convenient for heat transport

Simple Example

- 2 layer flow
- temperatures: θ_1 and θ_2
- mass transports: Ψ_1 and $\Psi_2 (= -\Psi_1)$

➔ $H = c_p\rho(\theta_1\Psi_1 + \theta_2\Psi_2) = c_p\rho\, \Psi_1\, (\theta_1 - \theta_2)$



Can generalize to continuous profiles

Let

$$H_0 = H/(c_p\rho) = \int_B^T \int_W^E v\theta dx dz.$$

If we think of everything as a function of (x, θ) rather than (x, z), then

$$H_0 = \int_{\theta_B}^{\theta_T} \int_W^E v(x, \theta)\theta \frac{\partial z}{\partial \theta} dx d\theta.$$

The volume transport per temperature interval is

$$V(\theta) = \int_W^E v z_\theta dx$$

and it can be shown that

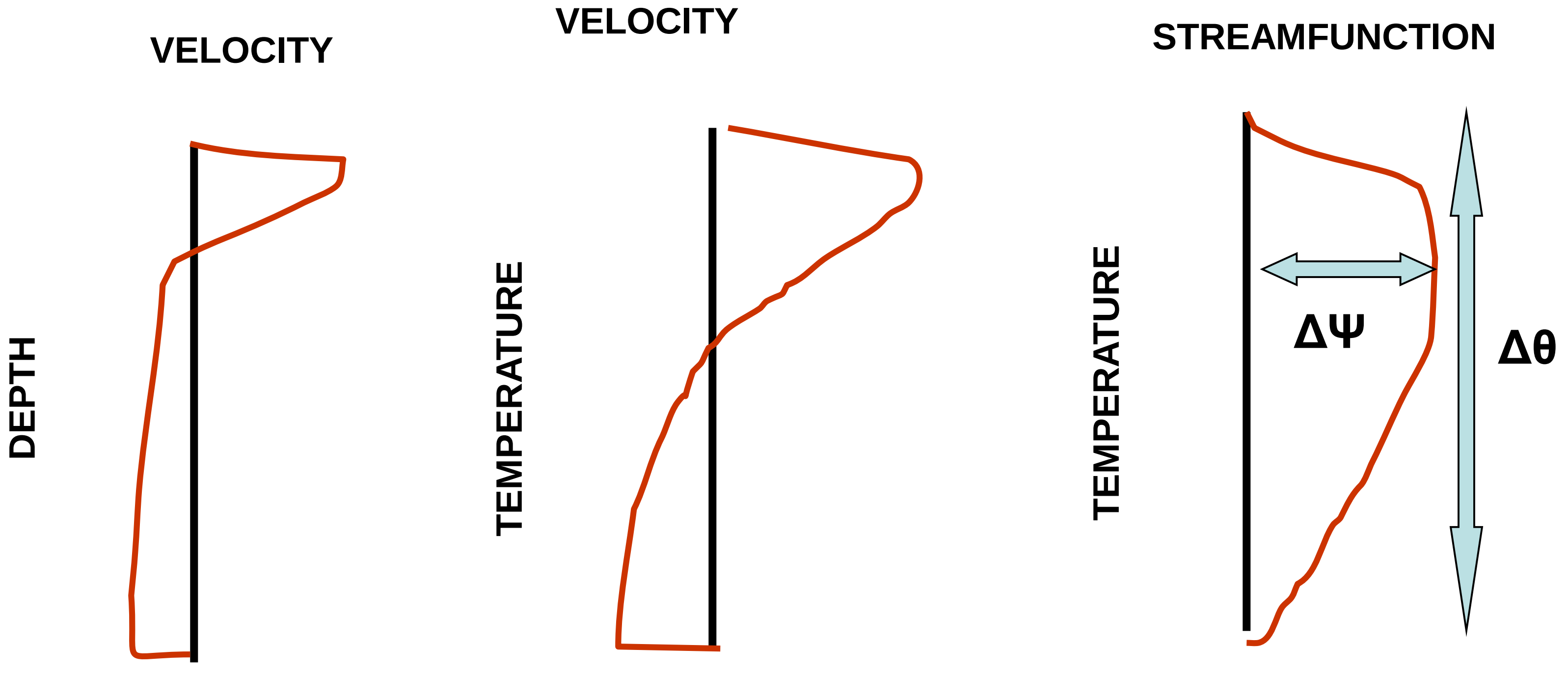
$$H_0 = \int_{\theta_B}^{\theta_T} V\theta d\theta$$

The meridional overturning streamfunction in temperature coordinates Ψ_θ can be defined though

$$V = -\frac{\partial \Psi_\theta}{\partial \theta}$$

Assuming that $\Psi_\theta = 0$ at the top and bottom, integration by parts gives

$$\boxed{H_0 = \int_{\theta_B}^{\theta_T} \Psi_\theta d\theta}$$



$$H \approx c_p \rho K \, \Delta\Psi \, \Delta\theta$$

IMPORTANT: In general, v(z) can look very different from v(θ), not just "stretched".

Heat Transport Measurements

Three main approaches:

Measure surface heat flux and integrate

advantage: focus on relatively accessible data

disadvantage: how accurate are bulk formulae?

Measure top-of-atmosphere radiation and subtract atmos transport

advantage: accurate radiation and lots of atmos data

disadvantage: problems with atmos models, errors in resid

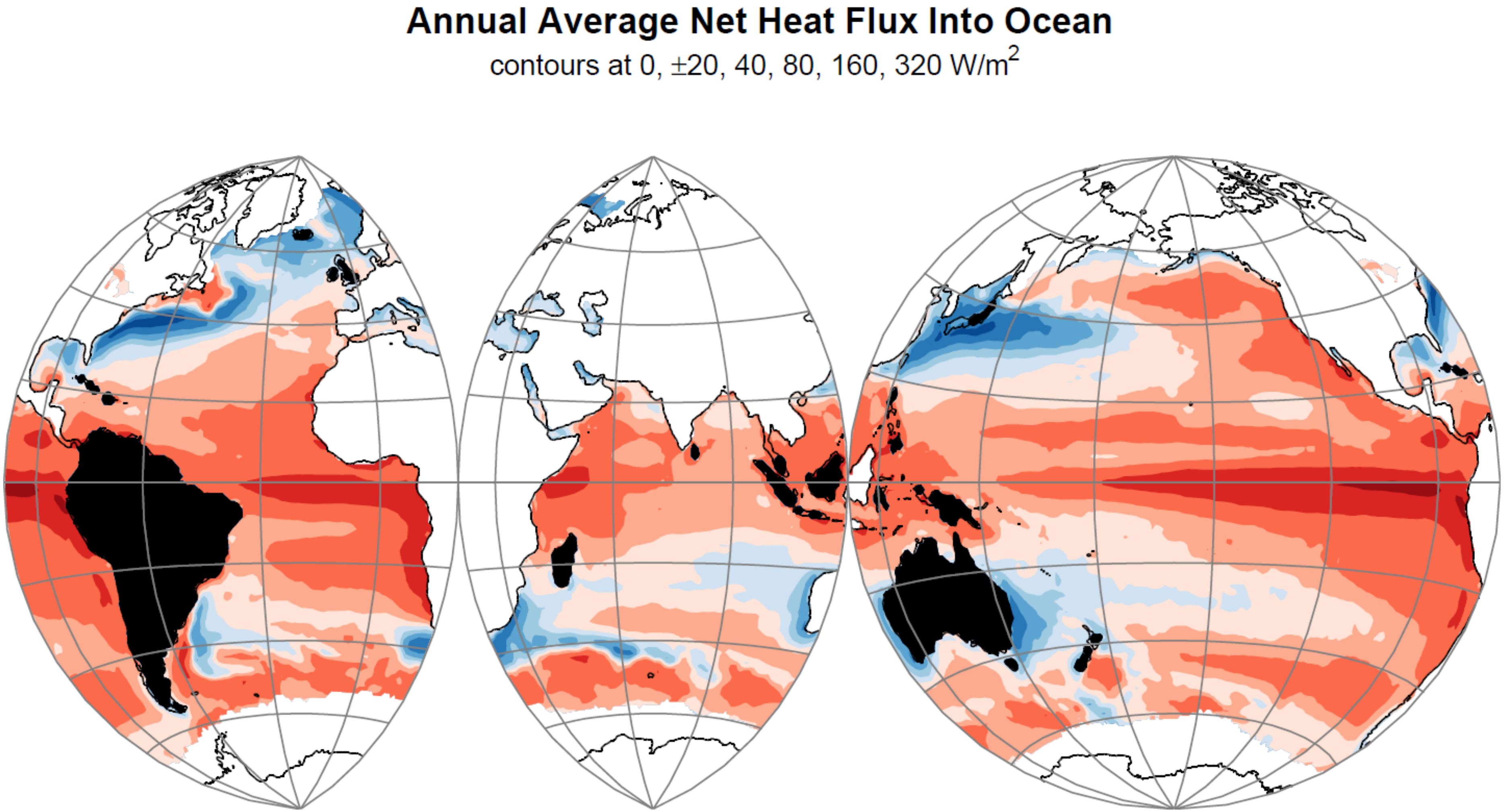
Directly measure ocean v, θ

advantage: can relate transport to what ocean is doing

disadvantage: ocean data only in some regions

All methods have substantial error bars,
Useful to compare different ways of calculating same quantity

See Bryden and Imawaki, *Ocean Circulation and Climate* sec. 6.1



Heat exchange with atmosphere indicates ocean influence on climate

- Heat flux magnitudes comparable to solar irradiance.
- Ocean absorbs heat at equator, releases heat at mid-high latitudes
- Also zonal structure: heat absorption in east, emission in west
- Differences between oceans and between N and S hemispheres

Net Annual-Average Heat Flux into Ocean

General pattern:

absorbs heat near equator, loses heat at high latitudes

(as expected)

Typical values O(50 W/m^2)

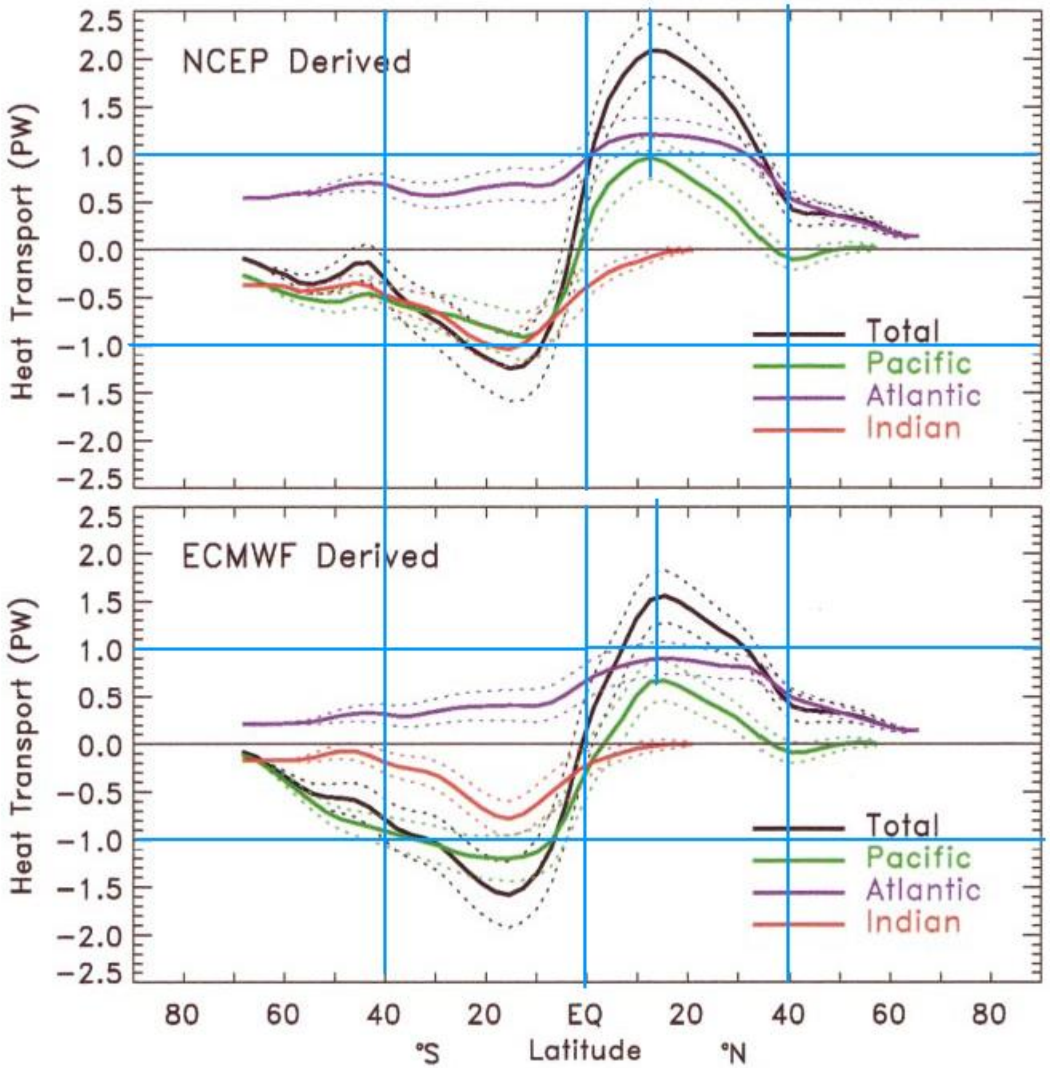
Some interesting details:

heat gain largely in east, heat loss in west

Pacific heat gain especially big

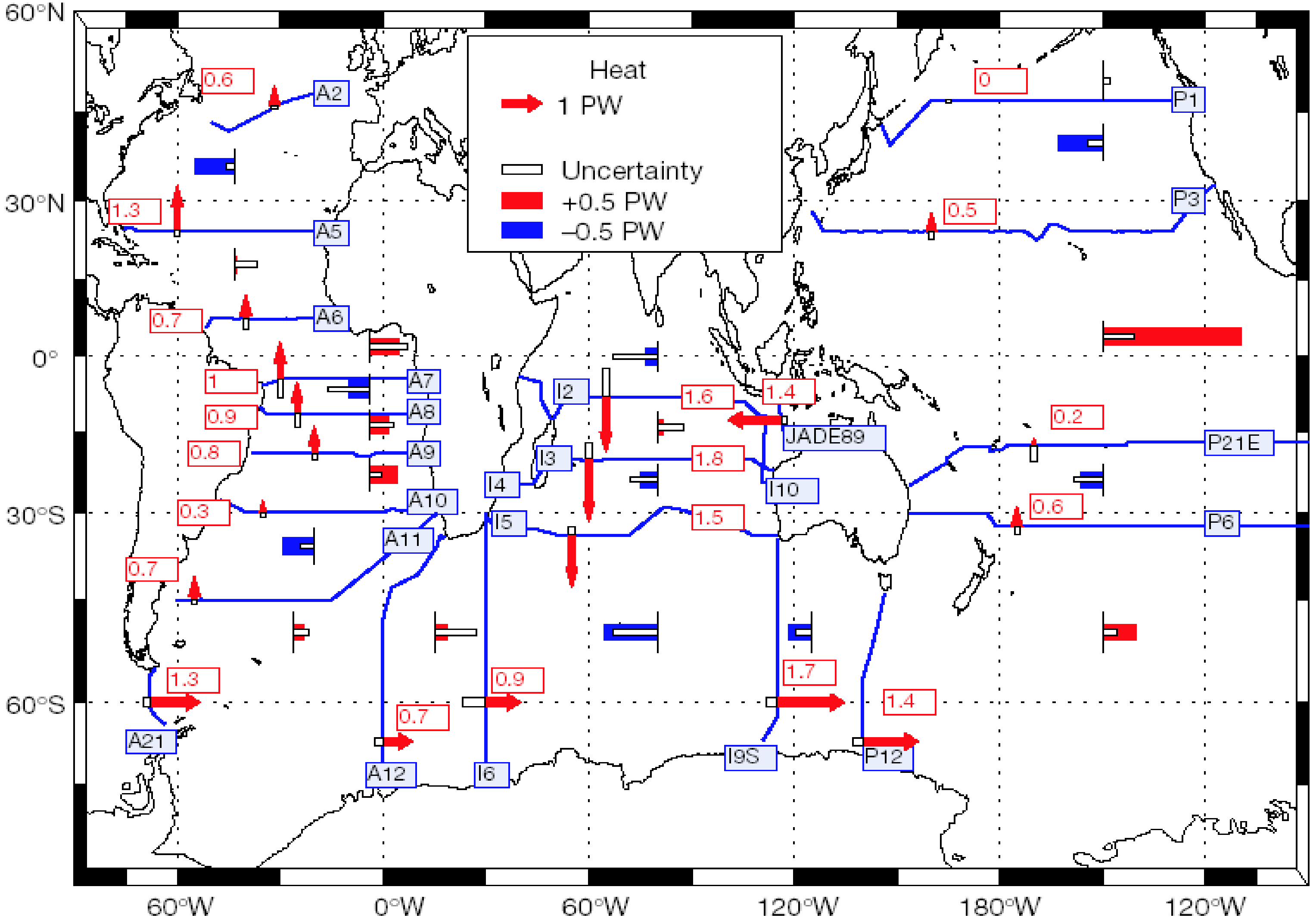
Atlantic heat loss especially big

Meridional Heat Transport from reanalysis surface heat fluxes,
Feb 1985 – Apr 1989



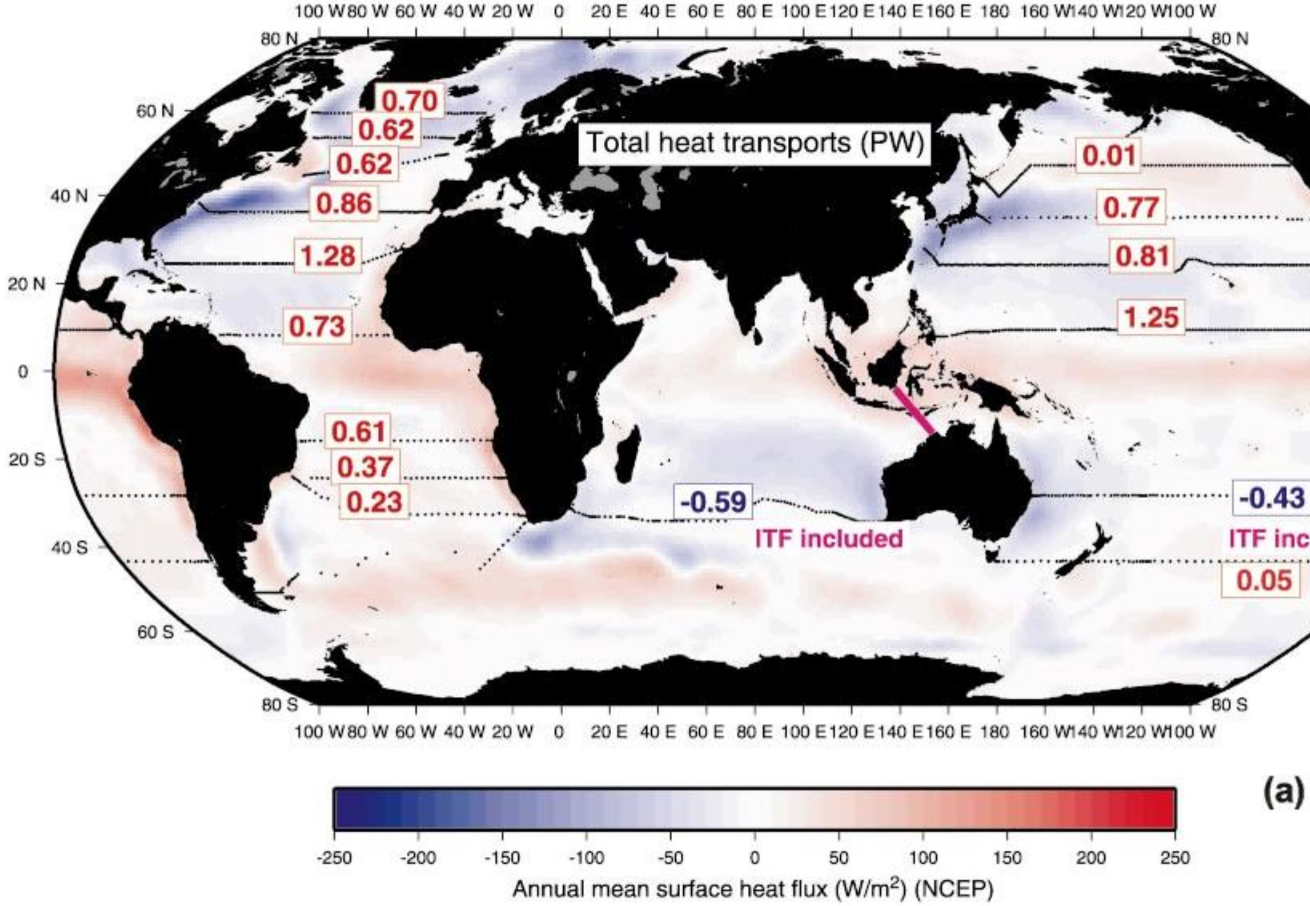
Trenbirth and Caron (2001)

“Direct” Measurement of Ocean Heat Transport
(based on hydrography, Ekman transport, tracer distributions)



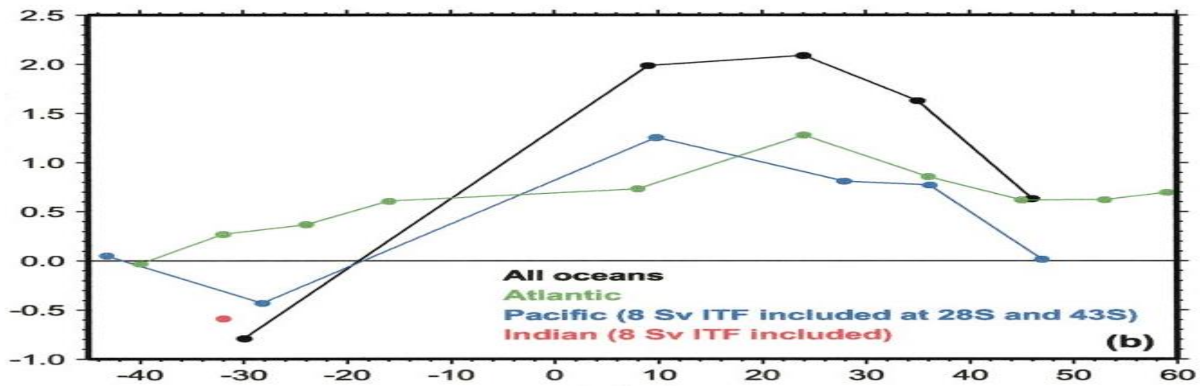
Ganachaud and Wunsch (2000, Nature)

Another Direct Heat Transport Measurement

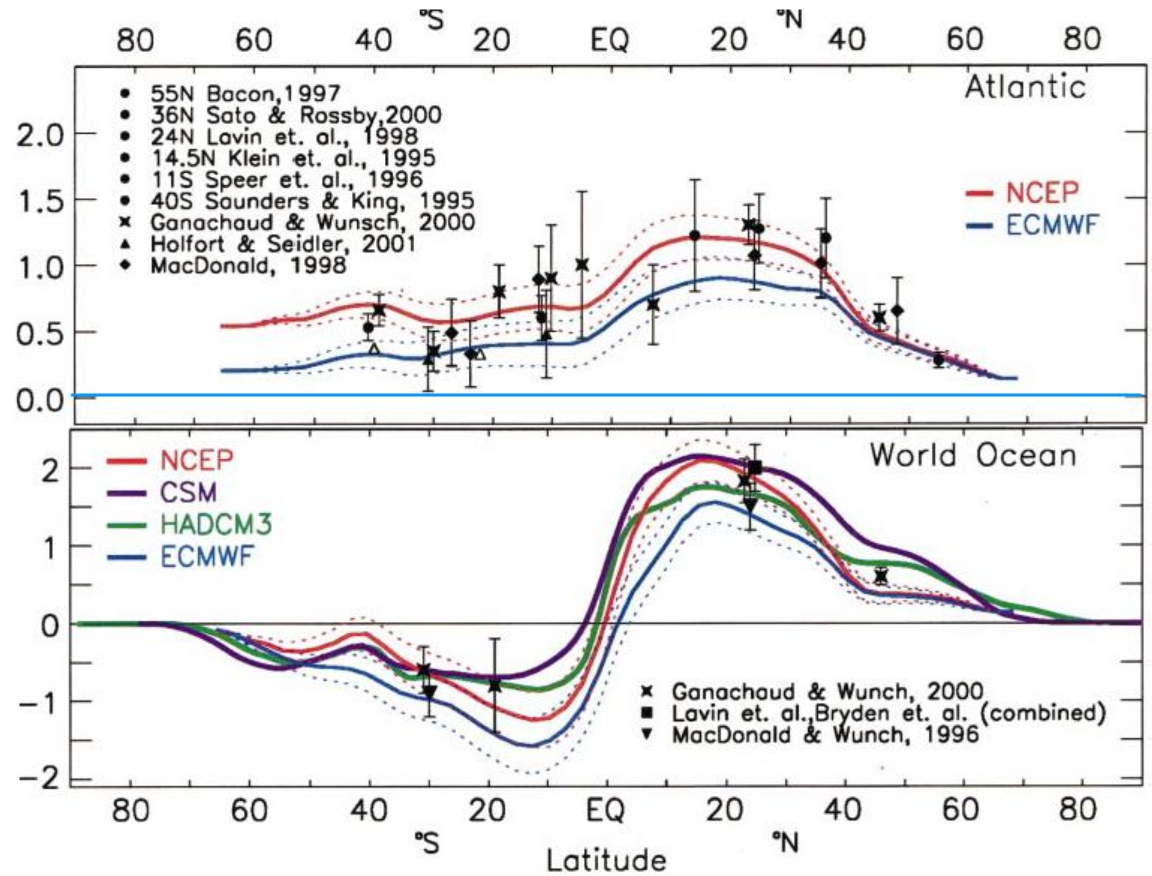


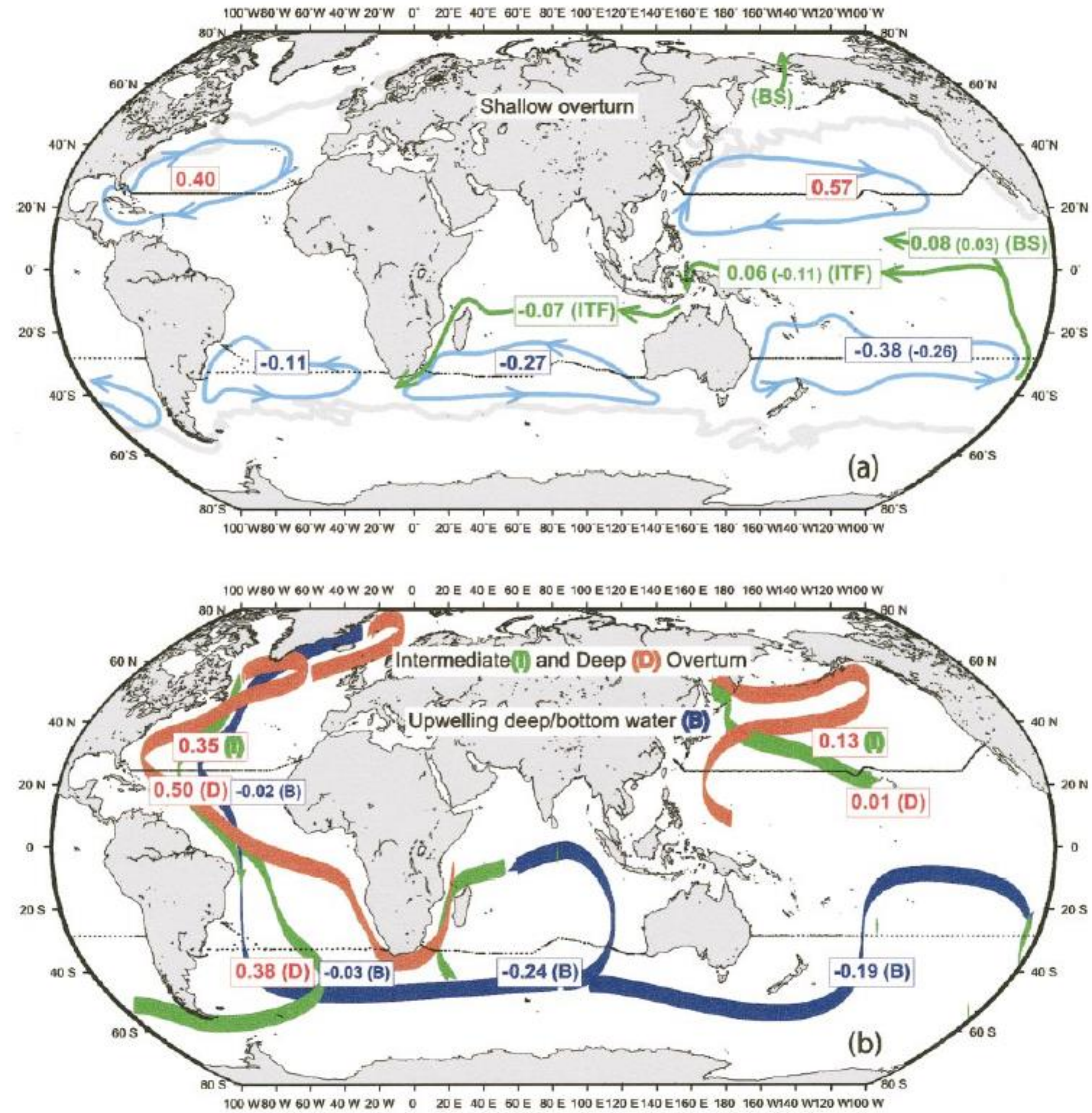
Talley (2003, JPO)

Talley (2003) ➔



Trenberth and Caron (2001) ➔




Shallow overturn
(BS)
0.40
0.57
0.08 (0.03) (BS)
0.06 (-0.11) (ITF)
-0.07 (ITF)
-0.11
-0.27
-0.38 (-0.26)
(a)
Intermediate (I) and Deep (D) Overturn
Upwelling deep/bottom water (B)
0.35 (I)
0.13 (I)
0.50 (D)
-0.02 (B)
0.01 (D)
0.38 (D)
-0.03 (B)
-0.24 (B)
-0.19 (B)
(b)

Observed Mass Transport as function of θ bins and density bins

(Roemmich *et al.* (2001, JGR)

based on repeat sections