**CLIM 750 Geophysical Fluid Dynamics, Problem Set 4**

1. In class, we calculated the form drag due to internal waves generated by topography. We also found that the "waves" are evanescent if $\omega > N$ or $\omega < f$, and hence no waves are generated.

(a) For the evanescent case, if

$$w = We^{-mz}e^{i(kx-\omega t)} \tag{1}$$

what is the pressure $p'$ (assuming constant $N$)?

(b) Calculate the form drag for such an evanescent wave?

2. Consider an internal wave in a fluid with constant $N$ and $f = 0$. The wave has vertical velocity given by

$$w = w_0 \cos(kx + mz - \omega t) \tag{2}$$

(a) By analyzing the propagation of lines of constant phase, show that the magnitude of the phase speed is given by

$$|\vec{c}_p| = \omega/\sqrt{k^2 + m^2} \tag{3}$$

(b) For the internal wave mentioned above, what is $|\vec{c}_p|$ as a function of $k$, $m$, and $N$?

(c) For the same internal wave, what is the magnitude of the group velocity, $|\vec{c}_g|$ as a function of $k$, $m$, and $N$?

(d) What is the ratio of the group speed and phase speed as a function of $\theta$, the angle the wave number makes to the horizontal?

3. Consider an incompressible, non-rotating fluid flowing with constant "background" velocity $u = U\hat{x}$ over a series of ridges given by surface height $h = h_0 \sin(kx)$, where $k = 2\pi/\lambda_H$. For $N/Uk = .5$, plot the streamlines of the complete flow—including any disturbance to the flow induced by the topography—in the frame of reference in which the topography is stationary. Assume that $h_0$ is small enough to be able to find a linear solution. Be sure to include expressions for $u(x, z, t)$, $w(x, z, t)$, and $x(t)$ and $z(t)$, all in terms of parameters $h_0$, $\lambda_H$, $U$, and $N$.

4. Given the internal wave dispersion relation

$$\omega^2 = f^2 \sin^2\theta + N^2 \cos^2\theta \tag{4}$$

calculate the minimum and maximum possible values of $\omega$ and the propagation angle $\theta$ at which these occur.