CLIM 750 Geophysical Fluid Dynamics, Problem Set 3

1. For a shallow water gravity wave in rotating coordinates, we showed

$$< E_K + E_P >_t + \vec{\nabla} \cdot < \rho H \eta \vec{u} >= 0. \tag{1}$$

Show that this can be rewritten

$$< E_K + E_P >_t + \vec{\nabla} \cdot (\vec{c_g} < E_K + E_P >) = 0. \tag{2}$$

2. In class we outlined how to derive the vertical mode wave solutions to a 2-layer system. Here we fill in the details

a) Show that if pressure is hydrostatic,

$$\frac{1}{\rho_1} \vec{\nabla} p_1 = g \vec{\nabla} \eta \tag{3}$$

$$\frac{1}{\rho_2} \vec{\nabla} p_2 = g \frac{\rho_1}{\rho_2} \vec{\nabla} \eta + g \frac{\Delta \rho}{\rho_2} \vec{\nabla} h \tag{3}$$

b) Derive the equation in μ and the another equation relating μ and α that are a condition for finding solutions to the 2-layer system subject to $\vec{u}_2 = \alpha \vec{u}_1$ and $h = \mu \eta$. Use $\epsilon \equiv \Delta\rho/\rho_2$ and $\gamma \equiv H_1/H_2$.

c) Solve the equations you derived in (b) for μ, α, and $He \equiv H_1/(1-\mu)$, in the limit $\epsilon \ll 1$. When you solve, rather than solving the full equation and taking the small-ϵ limit, approximate the equations before solving. For one solution, approximate the equations assuming $\mu \leq O(1)$. For the other solution, approximate the equations assuming $\mu \geq O(1/\epsilon)$.

3. Evolution of initial shallow water disturbances.

a) All quantities depend on x only (no y-dependence), no rotation, uniform density, mean depth a constant H. At $t = 0$, $\eta(x) = \eta_0(1 - (x/a)^2)$ for $|x| < a$ and $\eta(x) = 0$ for $|x| \geq 0$, with $\eta_0 \ll H$ and $a \gg H$. Initial u and w is zero. Calculate how η evolves over time from this initial condition.

(b) Same as above, but two layers of thickness H_1 and H_2 and density diference $\Delta\rho$. Initial $\eta(x) = 0$, initial $h(x)$ has same form as initial η in (a) except substitute h_0 for η_0. Initial velocity is zero. Calculate how η and h evolves over time from this initial condition.

(c) Like (a) but with f constant (nonzero), wall at $y = 0$ and with initial $\eta(x, y)$ with same x-dependence as in (a), but only for $0 < y < b$, with $\eta(x) = 0$ for $y \geq b$. Initial velocity is zero, and a and b are both on the same order as $\sqrt{gH}/f$. Calculating the evolution of this initial condition is somewhat complicated. Rather than carrying out the calculation, explain how you would set up the calculation to solve it and what kind of response you would expect.