CLIM 750 Geophysical Fluid Dynamics, Problem Set 2

1. An alternate derivation of rotating, shallow-water gravity waves. Instead of putting all the equations together into one big equation, we can directly solve the original linearized, shallow-water equations. To do this, assume that the solution is of the form

$$u = u_0 e^{i\Phi} \tag{1a}$$

$$v = v_0 e^{i\Phi} \tag{1b}$$

$$\eta = \eta_0 e^{i\Phi} \tag{1c}$$

where $\Phi = kx + ly - \omega t$, and u_0, v_0, and η_0 are constants. It may disturb you that the solution is complex, but since the equations are linear we can simply take the real part and ignore the imaginary part.

(a) Insert (1) into the linearized, shallow-water equations to produce a system of three equations that are linear with respect to u_0, v_0, and η_0.

(b) The equations you found in (a) can be written in the form $\mathrm{A}\mathbf{X} = \mathbf{Z}$, where X is the column vector with elements (u_0, v_0, η_0), Z is the column vector with elements $(0, 0, 0)$, and A is a matrix whose elements are functions of f, ω, k, and the other parameters. This equation only has a solution if

$$\det A = 0. \tag{2}$$

Use this expression to find the dispersion relation

$$\omega^2 = f^2 + gH_0(k^2 + l^2). \tag{3}$$

2. (a) Given the dispersion relation for Poincaré waves (see question 1), write expressions for how c_p and c_g depend on ω, and sketch $c_p(\omega)$ and $c_g(\omega)$ on the same graph.

(b) Consider a Poincaré wave that starts out at a northern-hemisphere latitude θ_1 where $f = f_1$, and which has $f_1 < \omega < f_N$, where f_N is the value of f at the northern boundary of the basin. Assuming the wave propagates northward, write expressions for K/K_1 and c_G/c_{G1} as a function of ω, f, and f_1, where the subscript "1" refers to values at latitude θ_1. What happens to K and c_G before the wave can reach the northern boundary?

(c) In general, given a dispersion relation which can be written as $\omega = \omega(K)$, where $K^2 = k^2 + l^2$, show that the group velocity $\vec{c}_G$ must be in the same direction as the phase velocity $\vec{c}_p$.

4. In class, I derived the existence of coastal Kelvin waves, and noted that there is also a similar equatorial Kelvin wave. Following my coastal Kelvin wave derivation, derive the equatorial case. The derivation will be somewhat easier if you assume that the meridional extent of the Kelvin wave is small compared to the radius of the Earth.