CLIM 750 Geophysical Fluid Dynamics, Problem Set 1

1. In class we discussed the Stokes drift for a wave which induced a velocity $u(x, t)$ which had the shape of a square wave. If the wave has phase speed c, an amplitude (of fluid velocity) U and a wavelength λ, what is the speed of the Stokes drift?

2. For a wave given by

$$\eta = \sin(\Phi) \tag{1}$$

where

$$\Phi = ax - ay - \omega t, \tag{2}$$

(a and ω are constants) calculate the speed of the line $\Phi = 0$ (in other words, perpendicular to itself), and the speed along the x-axis.

3. Given the surface gravity wave dispersion relation,

$$\omega^2 = gK \tanh kH \tag{3}$$

show that for $\lambda \gg H$, $c_p = \sqrt{gH}$ and that for $\lambda \ll H$, $c_p = \sqrt{g\lambda/2\pi}$.

4. For a wave given by

$$\eta = \eta_0 \cos(kx - \omega t), \tag{4}$$

use the linearized equations of motion to derive expressions for u and w. In the shallow water limit, calculate the amplitudes of u and w, show that u is approximately independent of depth, and show that w goes linearly from 0 at the bottom to a maximum value at the surface.

5. Consider a gravity wave in which the sea surface height is given by

$$\eta = \eta_0 \cos(kx - \omega t). \tag{5}$$

In class, we assumed that there is no friction, but let's consider the horizontal momentum equation with an additional term on the right hand side of $+\nu u_{zz}$, where ν is the kinematic viscosity. Use scaling to derive a condition on η_0, k, ω, and ν for the conditions under which we are justified in ignoring the friction term in the momentum equation. What form does this condition take in the limit of shallow water waves and in the limit of deep water waves?