Math 114: Practice Exam 4
This is a 50 minute exam.

1. For \( f(x) = 2^x \):
   (a) Complete the following table and use it to find a formula for \( f^{(k)}(0) \). Recall \( \frac{d}{dx}(2^x) = 2^x \ln(2) \).

<table>
<thead>
<tr>
<th>( k )</th>
<th>( f^{(k)}(x) )</th>
<th>( f^{(k)}(0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
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<tr>
<td>1</td>
<td></td>
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<tr>
<td>2</td>
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<tr>
<td>3</td>
<td></td>
<td></td>
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</tbody>
</table>

   (b) What is the Taylor series representation for \( f(x) \) centered at 0?
   (c) Find the second degree Taylor polynomial for \( f(x) \) centered at 0, and use it to approximate \( \sqrt{2} \). Use \( \ln(2) \approx \ln(e) = 1 \) in your approximation.
   (d) How large could the error be in the above approximation? Again, use the fact that \( \ln(2) < \ln(e) = 1 \).
   (e) What is the smallest degree Taylor polynomial necessary to guarantee accuracy within \( 10^{-2} \)?

2. Find the power series representation centered at 0 for \( \frac{-2x}{(1 - x^2)^2} \) and find its interval and radius of convergence. Hint: First find the power series representation centered at 0 for \( \frac{1}{1 - x^2} \).

3. Use Taylor series to evaluate \( \lim_{x \to 0} \frac{x^2/2 - 1 + \cos(x)}{x^4} \).

4. Eliminate the parameter in \( x(t) = te^t + 1 \) and \( y(t) = t^2e^{2t} + 4te^t \) for \( 0 \leq t \leq 1 \) to obtain an equation in \( x \) and \( y \).

5. Graph the function from 4 and include orientation.
6. Find the tangent line to the above graph at $t = 0$.

7. Convert $\frac{x^2}{4} + \frac{y^2}{9} = 1$ to polar coordinates.

8. Convert $r = \tan \theta$ to Cartesian coordinates.