

$$\sin^2(x) + \cos^2(x) = 1$$

$$\tan^2(x) + 1 = \sec^2(x)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

$$\text{Area of a trapezoid: } \frac{1}{2}b(h_1 + h_2)$$

$$\text{Cross-sectional area of a disk: } \pi r^2$$

$$\text{Cross-sectional area of a washer: } \pi(R^2 - r^2)$$

Volume by disk/washer: Integral of the cross-sectional area

$$\text{Volume by shells: } 2\pi \int_a^b x(\text{"top" - "bottom"})dx \text{ or } 2\pi \int_c^d y(\text{"right" - "left"})dy$$

$$\text{Arc length: } \int_a^b \sqrt{1 + (f'(x))^2} dx$$

$$\int \tan(x) dx = -\ln|\cos(x)| + C$$

$$\int \sec(x) dx = \ln|\sec(x) + \tan(x)| + C$$

$$\int \sec^3(x) dx = \frac{1}{2}(\sec(x) \tan(x) + \ln|\sec(x) + \tan(x)|) + C$$

$$\int \cot(x) dx = -\ln|\sin(x)| + C$$

$$\int \csc(x) dx = -\ln|\csc(x) - \cot(x)| + C$$

$$\text{Integration by parts: } uv - \int v du$$

$$\text{Midpoint/Rectangle approximation error bound: } E_M \leq \frac{k(b-a)^3}{24n^2}$$

$$|f''(x)| \leq k \text{ on } [a, b]$$

$$\text{Trapezoid approximation error bound: } E_T \leq \frac{k(b-a)^3}{12n^2}$$

Taylor polynomial (degree  $n$ ) centered at  $a$  approximate error bound:

$$R_n(x_0) \leq M \frac{|x_0 - a|^{n+1}}{(n+1)!}, \text{ where } |f^{(n+1)}(x)| \leq M \text{ between } x_0 \text{ and } a$$

Alternating series approximation error bound:  $|R_n| \leq a_{n+1}$

Integral test approximation error bound:  $R_n \leq \int_n^\infty f(x)dx$

Linear function:  $y - y_0 = m(x - x_0)$  or  $y = mx + b$

Parametric derivative:  $\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$

Polar/Cartesian relationships:  $x^2 + y^2 = r^2$ ,  $x = r \cos \theta$ , and  $y = r \sin \theta$

**Table 9.5**

$$\begin{aligned}\frac{1}{1-x} &= 1 + x + x^2 + \cdots + x^k + \cdots = \sum_{k=0}^{\infty} x^k, \quad \text{for } |x| < 1 \\ \frac{1}{1+x} &= 1 - x + x^2 - \cdots + (-1)^k x^k + \cdots = \sum_{k=0}^{\infty} (-1)^k x^k, \quad \text{for } |x| < 1 \\ e^x &= 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^k}{k!} + \cdots = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad \text{for } |x| < \infty \\ \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{(-1)^k x^{2k+1}}{(2k+1)!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \quad \text{for } |x| < \infty \\ \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{(-1)^k x^{2k}}{(2k)!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}, \quad \text{for } |x| < \infty \\ \ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + \frac{(-1)^{k+1} x^k}{k} + \cdots = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k}, \quad \text{for } -1 < x \leq 1 \\ -\ln(1-x) &= x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^k}{k} + \cdots = \sum_{k=1}^{\infty} \frac{x^k}{k}, \quad \text{for } -1 \leq x < 1 \\ \tan^{-1} x &= x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + \frac{(-1)^k x^{2k+1}}{2k+1} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}, \quad \text{for } |x| \leq 1 \\ (1+x)^p &= \sum_{k=0}^{\infty} \binom{p}{k} x^k, \quad \text{for } |x| < 1 \quad \text{and} \quad \binom{p}{k} = \frac{p(p-1)(p-2)\cdots(p-k+1)}{k!}, \binom{p}{0} = 1\end{aligned}$$

**Table 7.3**

**The Integral Contains...**

**Corresponding Substitution**

**Useful Identity**

$$a^2 - x^2 \quad x = a \sin \theta, -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \quad a^2 - a^2 \sin^2 \theta = a^2 \cos^2 \theta$$

$$a^2 + x^2 \quad x = a \tan \theta, -\frac{\pi}{2} < \theta < \frac{\pi}{2} \quad a^2 + a^2 \tan^2 \theta = a^2 \sec^2 \theta$$

$$x^2 - a^2 \quad x = a \sec \theta, \begin{cases} 0 \leq \theta < \frac{\pi}{2} & \text{for } x \geq a \\ \frac{\pi}{2} < \theta \leq \pi & \text{for } x \leq -a \end{cases} \quad a^2 \sec^2 \theta - a^2 = a^2 \tan^2 \theta$$

**Table 8.4 Special Series and Convergence Tests**

Series or test	Form of series	Condition for convergence	Condition for divergence	Comments
Geometric series	$\sum_{k=0}^{\infty} ar^k$	$ r  < 1$	$ r  \geq 1$	If $ r  < 1$ , then $\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$
Divergence Test	$\sum_{k=1}^{\infty} a_k$	Does not apply	$\lim_{k \rightarrow \infty} a_k \neq 0$	Cannot be used to prove convergence
Integral Test	$\sum_{k=1}^{\infty} a_k$ , where $a_k = f(k)$ and $f$ is continuous, positive, and decreasing	$\int_1^{\infty} f(x) dx < \infty$	$\int_1^{\infty} f(x) dx$ does not exist.	The value of the integral is not the value of the series
$p$ -series	$\sum_{k=1}^{\infty} \frac{1}{k^p}$	$p > 1$	$p \leq 1$	Useful for comparison tests
Ratio Test	$\sum_{k=1}^{\infty} a_k$ , where $a_k > 0$	$\lim_{k \rightarrow \infty} \frac{a_{k+1}}{a_k} < 1$	$\lim_{k \rightarrow \infty} \frac{a_{k+1}}{a_k} > 1$	Inconclusive if $\lim_{k \rightarrow \infty} \frac{a_{k+1}}{a_k} = 1$
Root Test	$\sum_{k=1}^{\infty} a_k$ , where $a_k \geq 0$	$\lim_{k \rightarrow \infty} \sqrt[k]{a_k} < 1$	$\lim_{k \rightarrow \infty} \sqrt[k]{a_k} > 1$	Inconclusive if $\lim_{k \rightarrow \infty} \sqrt[k]{a_k} = 1$
Comparison Test	$\sum_{k=1}^{\infty} a_k$ , where $a_k > 0$	$0 < a_k \leq b_k$ and $\sum_{k=1}^{\infty} b_k$ converges	$0 < b_k \leq a_k$ and $\sum_{k=1}^{\infty} b_k$ diverges	$\sum_{k=1}^{\infty} a_k$ is given; you supply $\sum_{k=1}^{\infty} b_k$
Limit Comparison Test	$\sum_{k=1}^{\infty} a_k$ , where $a_k > 0, b_k > 0$	$0 \leq \lim_{k \rightarrow \infty} \frac{a_k}{b_k} < \infty$ and $\sum_{k=1}^{\infty} b_k$ converges.	$\lim_{k \rightarrow \infty} \frac{a_k}{b_k} > 0$ and $\sum_{k=1}^{\infty} b_k$ diverges.	$\sum_{k=1}^{\infty} a_k$ is given; you supply $\sum_{k=1}^{\infty} b_k$
Alternating Series Test	$\sum_{k=1}^{\infty} (-1)^k a_k$ , where $a_k > 0, 0 < a_{k+1} \leq a_k$	$\lim_{k \rightarrow \infty} a_k = 0$	$\lim_{k \rightarrow \infty} a_k \neq 0$	Remainder $R_n$ satisfies $R_n <$