

Problem 1 (5 points): Find the slope of the line tangent to the curve $r = 4 \cos \theta$ at the point $\left(2, \frac{\pi}{3}\right)$.

$$\frac{dy}{dx} = \frac{f'(\theta) \sin \theta + f(\theta) \cos \theta}{f'(\theta) \cos \theta - f(\theta) \sin \theta} = \frac{-4 \sin^2 \theta + 4 \cos^2 \theta}{-4 \cos \theta \sin \theta - 4 \sin \theta \cos \theta}$$

so that the slope of the tangent line at $\left(2, \frac{\pi}{3}\right)$ is $\frac{-2}{-2\sqrt{3}} = \frac{\sqrt{3}}{3}$.

Problem 2 (5 points): Find the area of the region inside the cardioid $r = 4 + 4 \sin \theta$.

$$A = \frac{1}{2} \int_{\alpha}^{\beta} f(\theta)^2 d\theta = \frac{1}{2} \int_0^{2\pi} (4 + 4 \sin \theta)^2 d\theta = 8 \int_0^{2\pi} (1 + 2 \sin \theta + \sin^2 \theta) d\theta$$

$$= 8 \left(\theta - 2 \cos \theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right) \Big|_0^{2\pi} = 24\pi$$