This is a two hour exam. Show all work for full credit. No electronic devices or formula sheets are allowed.

Problem 1 (10 points):

a) (5 points) Find the value of $\frac{dy}{dx}$ in terms of t for $x = 4\sin t, y = 3\cos t$.

b) (5 points) Graph $x = 4\sin t, y = 4\cos t$ for $0 \leq t \leq \pi$ and indicate the direction of motion.

Problem 2 (10 points):

Use Taylor series to evaluate $\lim_{x \to 0} \frac{e^{bx} - 1}{x}$ for a real parameter b.

Problem 3 (10 points):
Find the linear and quadratic approximations to e^x centered at $a = 1$.

Problem 4 (10 points):
Use Taylor series to find the first four nonzero terms of a series which is equal to e^3.

Problem 5 (10 points): Consider the parametric equations $y = e^{2t}, x = e^t + 2$

a) (5 points) Eliminate the parameter t to obtain an equation in x and y.

b) (5 points) Describe the curve and indicate the positive orientation.

Problem 6 (10 points):

a) (5 points) Find the function represented by the power series $\sum_{k=0}^{\infty} \left(\frac{k! + 3}{k!}\right) x^k$

b) (5 points) Find the interval of convergence for the power series $\sum_{k=0}^{\infty} \frac{(-1)^k x^{3k}}{27^k}$

Problem 7 (10 points):
Estimate the remainder for the nth order Taylor polynomial for the function $\cos x$ at a=0.

Problem 8 (10 points):
Find the Taylor series expansion for the function e^x centered at $a = \ln 3$

Problem 9 (10 points):
Find a power series representation centered at 0 for $f(x) = \dfrac{1}{(1-x)^2}$. Give the interval of convergence for the resulting series.

Problem 10 (10 points):
a) (5 points) Convert the polar equation $r = \sin\theta \sec^2\theta$ to Cartesian coordinates and graph the curve.

b) (5 points) Write the point $(x, y) = (-1, 0)$ in two different ways using polar coordinates.

Extra Credit (5 points):

Let $i = \sqrt{-1}$. Expand the complex exponential functions $e^{i\theta}$ and $e^{-i\theta}$ in Maclaurin series and show that $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$ and $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$.