

**This is a two hour exam. Show all work for full credit. State which theorems or tests you are using. No electronic devices or formula sheets are allowed.**

Problem 1 (10 points): Evaluate the series  $\sum_{k=1}^{\infty} \frac{2^k}{e^k}$  or state that it diverges.

Problem 2 (10 points): Determine whether  $\sum_{k=1}^{\infty} \frac{\pi^k}{k!}$  converges or diverges.

Problem 3 (10 points): Evaluate the series  $\sum_{k=1}^{\infty} \left( \frac{1}{k} - \frac{1}{k+1} \right)$  or state that it diverges.

Problem 4 (10 points): Find the limit of the following sequences or determine that the limits do not exist.

(a) (5 points)  $a_n = \sin\left(\frac{n\pi}{2}\right)$

(b) (5 points)  $a_n = \left(1 + \frac{3}{n}\right)^n$

Problem 5 (10 points):

(a) (5 points) Determine whether  $\sum_{k=1}^{\infty} ke^{-k^2}$  converges or diverges.

(b) (5 points) How many terms of the series  $\sum_{k=1}^{\infty} \frac{3}{k^4}$  must be summed in order to obtain an approximation that is within  $10^{-3}$  of the exact value of the series?

Problem 6 (10 points): Determine whether  $\sum_{k=1}^{\infty} \left(1 + \frac{2}{k}\right)^{k^2}$  converges or diverges.

Problem 7 (10 points): Determine whether  $\sum_{k=1}^{\infty} \frac{k^2 + 3k + 5}{k^3 + 2k + 1}$  converges or diverges.

Problem 8 (10 points):

(a) (5 points) Determine whether  $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt[3]{k}}$  converges absolutely, converges conditionally, or diverges.

(b) (5 points) Find an upper bound on the remainder when the series  $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^3}$  is approximated by its first 3 terms.

Problem 9 (10 points):

(a) (5 points) Determine whether  $\sum_{k=1}^{\infty} \frac{k}{k+1}$  converges or diverges.

(b) (5 points) Determine whether  $\sum_{k=1}^{\infty} \sin k$  converges or diverges.

Problem 10 (10 points): Determine all values of  $x \geq 0$  for which  $\sum_{k=1}^{\infty} \frac{x^{2k}}{k^2}$  converges.

Extra Credit (5 points) Given that  $\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$ , determine the value of  $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}$ .