This is a two hour exam. Show all work for full credit. No electronic devices or formula sheets are allowed.

Problem 1 (10 points): Evaluate $\int \sin^2 x \cos^2 x \; dx$.

Problem 2 (10 points): Evaluate $\int \tan x \sec^4 x \; dx$.

Problem 3 (10 points):

(a) (5 points) Evaluate $\int_0^{\ln 2} xe^{2x} dx$.

(b) (5 points) Evaluate $\int e^x \sin x dx$.

Problem 4 (10 points):

Let m and n be integers with $m \neq n$. Use the fact that

$$\cos(mx)\cos(nx) = \frac{1}{2}\left[\cos((m-n)x) + \cos((m+n)x)\right]$$

to show that $\int_0^{\pi} \cos(mx)\cos(nx)dx = 0$. This relation is useful for generating Fourier series.

Problem 5(10 points): Find the partial fraction expansion of $\dfrac{z+1}{z^2(z^2+4)}$.

Problem 6 (10 points): Evaluate $\displaystyle\int_1^4 \frac{dx}{x^2 - 2x + 5}$.

Problem 7 (10 points): Evaluate $\displaystyle\int \frac{dy}{y^2\sqrt{9y^2 - 81}}$ for $y > 3$.

Problem 8(10 points): Evaluate $\int \frac{2x^3 + x^2 - 6x + 7}{x^2 + x - 6} dx$.

Problem 9 (10 points): Given a function $f(t)$, the *Laplace transform* is a new function $F(s)$ defined by

$$F(s) = \int_0^\infty e^{-st} f(t) dt$$

where we assume s is a positive real number. Find the Laplace transform of $f(t) = t$.

Problem 10 (10 points):

(a) (5 points) Determine whether $\displaystyle\int_{-\infty}^{\infty} e^{-|x|} dx$ converges or diverges.

(b) (5 points) Determine whether $\displaystyle\int_{-1}^{1} \frac{dx}{x^3}$ converges or diverges.

Extra Credit (10 points) Determine whether $\int_0^\infty \frac{dx}{\sqrt{x} + x^2}$ converges or diverges.