This is a two hour exam. Show all work for full credit. No electronic devices or formula sheets are allowed.

Problem 1 (10 points):
(a) (5 points) Find the volume of the solid generated by revolving the region bounded by the curves and lines $y = x, y = \sqrt{x}$ about the y-axis using the washer method.

(b) (5 points) Find the volume of the solid generated by revolving the region bounded by the curves and lines $y = x, y = \sqrt{x}$ about the y-axis using the shell method.

Problem 2 (10 points): Find the position and velocity of an object moving along a straight line with the given acceleration, initial velocity, and initial position. Assume units of meters and seconds.
$a(t) = e^{-2t}, v(0) = 10, s(0) = 20$

Problem 3 (10 points): (a) (5 points) Find the area of the region bounded by $y = \sqrt{x}$ and $y = x^3$ using an integral with respect to x.

(b) (5 points) Find the area of the region bounded by $y = \sqrt{x}$ and $y = x^3$ using an integral with respect to y.

Problem 4 (10 points): A quantity increases exponentially in time according to the function $y(t) = y_0 e^{kt}$. Find the time required for the quantity to increase p-fold.

Problem 5 (15 points):
(a) (5 points) Evaluate $\dfrac{d}{dx}(x^x)$

(b) (5 points) Evaluate $\int 3^{3x} dx$

(c) (5 points) Use the definition of the natural logarithm to prove that $\ln(xy) = \ln x + \ln y$

Problem 6 (10 points):
(a) (5 points) Find the mass of a thin bar with density $|x - 1| \ \frac{kg}{m}$ for $0 \leq x \leq 2$.

(b) (5 points) A linear spring can be stretched and held 0.5 m from its equilibrium position with a force of 20 N. Find the work done in compressing the spring 0.5 m from its equilibrium position.

Problem 7 (5 pts): Find a curve which passes through the point $\left(1, \frac{1}{5}\right)$ and has an arc length on the interval [3,6] given by $\int_3^6 \sqrt{1+x^8}dx$?

Problem 8 (10 points): Suppose the acceleration of an object moving along a line is given by $a(t) = kv(t)\ \frac{m}{s^2}$, where k is a positive constant and v is the object's velocity. Assume that the initial position and velocity are given by $s(0) = 0\ m$ and $v(0) = 20\ \frac{m}{s}$

(a) (5 points) Use $a(t) = v'(t)$ to find the velocity of the object as a function of time.

(b) (5 points) Use $v(t) = s'(t)$ to find the position of the object as a function of time.

Problem 9 (10 points): Find the length of the curve $y = \frac{x^3}{6} + \frac{1}{2x}$ on the interval [1,2]

Problem 10 (10 points): Let R be the region bounded by the curves $y = x^2$ and $y = 2x$. Find the volume of the solid generated when R is revolved around the x-axis.