Proposition ***(the Pigeonhole principle).*** Suppose that n objects are put into m boxes. Then there is a box that contains at least $\left\lceil \frac{n}{m} \right\rceil$ objects.

Proof: by the contrapositive. Suppose that there is no box that contains at least $\left\lceil \frac{n}{m} \right\rceil$ objects. Then each of the m boxes contains at most $\left\lceil \frac{n}{m} \right\rceil - 1$ objects, which is less than $\frac{n}{m}$, and for the total number of objects n we have

$$n \le m\left(\left\lceil \frac{n}{m} \right\rceil - 1\right) < m\frac{n}{m} = n.$$

Combinatorics

Definition: the ***Pohgammer symbol*** $P(n, r)$ is

$$P(n,r) = \frac{n!}{(n-r)!} = n(n-1)(n-2)\ldots(n-r+1)$$

Definition: a ***permutation*** of a set is a linear arrangement of elements of the set in some order.

Definition: an ***r-permutation*** of a set of n symbols, $r \leq n$, is a linear arrangement of r elements of the set in some order.

Proposition: the number of permutations of a set of n elements is equal to $n!$. The number of r-permutations of n symbols is $P(n, r)$.

Proof: using the multiplication rule and the principle of mathematical induction.

Definition: the ***binomial coefficient*** is

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Definition: a subset of r objects is called an ***r-combination***.

Proposition: the number of r-combinations of a set of n objects, $0 \le r \le n$, is equal to $\binom{n}{r}$.

Proof: by counting r-permutations.

Example: the number of ways to put 3 identical marbles into 10 boxes (each marble into a separate box) is

$$\binom{10}{3} = \frac{10!}{3!(10-3)!} = \frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3} = 120.$$