MATH 125-001 Spring 2007 Exam 3
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Student Name___________________________________

Clearly state your answers and provide sufficient explanation of your reasoning to them relevant to the questions. If you need more space, write on the back. Follow the Honor Code. Good luck.

Problem 1 (9 points). From a group of 200 students, 80 take ART 101, 60 take BIOL 101, and 125 students take at least one of the two courses. How many students take

(a) exactly one of the two courses?

(b) both, ART 101 and BIOL 101?

(c) neither of the two courses?

Problem 2 (10 points). Prove that of any ten points chosen within an equilateral triangle whose sides have length 1, there are at least two points whose distance apart is at most 1/3. *Hint:* use the Pigeonhole Principle and Figure 1.

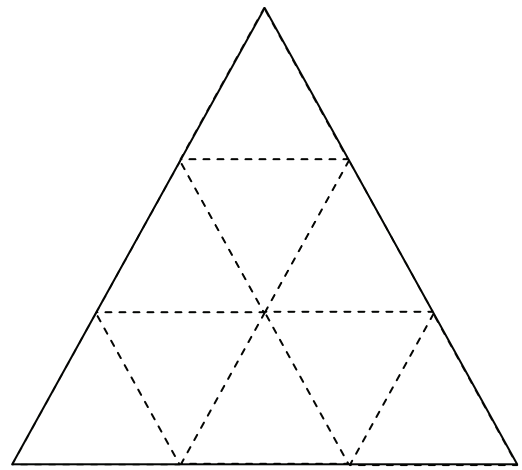

Figure 1.

Problem 3 (10 points). Consider all possible binary sequences (sequences of zeros and ones) of length 10.

(a) What is the total number of these sequences?

(b) How many of them include three or more zeros?

Problem 4 (20 points). Five marbles are hidden in four identical bins A, B, C, D. Given no further information, assume that all possible allocations of marbles are equally likely.

(a) What is the a priori probability to find bin A empty?

(b) You opened bin B and found it empty. What is now the probability to find bin A empty?

Problem 5 (16 points).

(a) Construct an example of a complexity function $f(n)$ that satisfies the relations

$$n \log(\log n) \prec f(n) \prec n \log n$$

(b) Given two complexity functions, $f(n)$ and $h(n)$, $f(n) \prec h(n)$, construct a function $g(n)$ that satisfies the relations $f(n) \prec g(n) \prec h(n)$.

Problem 6 (25 points). (a) Describe an algorithm of solving the Traveling Salesman Problem by enumerating permutations. (b) Evaluate its complexity as a function of the number of cities *n*. (c) Is this problem NP-complete?

Extra credit problem (20 points).

(solving this problem is not a requirement and will not add to your score above the maximum of 90 points)

Consider a paintball variation tournament arranged between three individual players who do not know each other. The fight is arranged in a limited space, where everybody can continuously see everybody, cannot hide and cannot run. Everyone has a loaded gun with one "deadly" shot and is a good shooter. Guns are non-transferable. At the beginning all guns are ready to use and must be pointed down, all players are positioned at equal distances from each other. Everyone can shoot at any time after a whistle. The game ends when each participant either has fired his/her shot or is "dead". Rewards are given for "survival", plus a relatively small bonus is given for eliminating an opponent. Assuming that shots cannot occur simultaneously by chance, that each shot is made in full awareness of the current situation, that each player knows the above rules and is a rational decision maker, one can find a winning strategy for this game.

(a) In a winning strategy, what is your immediate action (if any) after the whistle? What are the conditions for this action?

(b) What would be your chances of "survival" with the winning strategy in the case when the only strategy of your opponents is to shoot a randomly chosen "alive" target?