

MATH 125-001 Spring 2007 Exam 1
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Student Name _____

Problem 1. Construct a truth table for the following expression:

$$(p \vee q) \vee ((q \vee \neg r) \wedge (p \vee r))$$

You may simplify the expression first, using equivalence relations.

Instructions

For each of the following set of problems, write an answer supported by a sound proof. Your proof may use facts and terminology of mathematical logic and set theory without references to the source. You can limit your choice of them to the textbook and/or lecture material. You can use formulas, plain English, or both. Try to be concise yet convincing.

When writing formulas, remember that symbols “T”, “F”, “ \rightarrow ” and “ \leftrightarrow ” refer to specific truth values, while “1”, “0”, “ \Rightarrow ” and “ \Leftrightarrow ” refer to entire truth tables.

In some cases a proof can be based on a specific example. Still, your responsibility is to make a connection with your general argument. Make sure you finish your proof with a sound *general* conclusion relevant to the question.

Each problem has the weight of 10 points, and the maximal total score is 50 points. *Try to do all problems.* Write your answer in the space provided or on a separate sheet of paper. Write it clearly, so that I can read it. Follow the Honor Code.

Good luck.

Problem 2. You flip 2 coins. You can either win or lose. You lose iff you get exactly one tail. In order to win, how many must be heads?

- Problem 3.** Can an implication ($p \rightarrow q$) and its inverse ($\neg p \rightarrow \neg q$) both be
- (a) true?
 - (b) false?

Problem 4. A medieval philosopher said: “If I assume that two times two equals five, then I conclude that witches fly through the chimney”. Validate the argument.

Problem 5. Simplify the expression and list all proper subsets of the set A

$$A = \emptyset \cup \{\emptyset, B\}$$

in the following three special cases:

- (a) Substitute $\{\emptyset\}$ for B
- (b) Substitute A for B
- (c) Substitute \emptyset for B (discard repetitions in a set).

Extra credit problem:

Now, if you need to convince somebody that your line of reasoning is valid, you can formally validate it using inference rules and equivalence relations, as in HW2. To show that you know how to do this, formally validate the line of reasoning in the box below using *modus tollens* (write the argument in p, q, r notations).

An argument is sound iff it is valid and its hypotheses are true. In the argument " $F \rightarrow T$ " the hypothesis is not true. Therefore, " $F \rightarrow T$ " is not a sound argument.

Solving this problem is not a requirement and will not add to your score above 50 points.