

Paths and Circuits

Definitions

A **walk** in a pseudograph is an alternating sequence of vertices and edges, beginning and ending with a vertex, in which each edge is incident with every vertex immediately preceding it and the vertex immediately following it.

The **length** of a walk is the number of edges in it.

A walk is **closed** if the first vertex is the same as the last one, otherwise it is **open**.

A **trail** is a walk in which all edges are distinct.

A **path** is a walk in which all vertices are distinct.

Remark: a path is necessarily a trail.

A **circuit** is a closed trail.

A **cycle** is a circuit in which no vertex appears more than once except for the first one, which appears twice.

An **n -cycle** is a cycle with n vertices. It is **even** if n is even and **odd** if n is odd.

An **Eulerian circuit** in a pseudograph is a circuit that contains every vertex and every edge.

An **Eulerian trail** in a pseudograph is a trail that passes through every vertex and includes every edge.

A pseudograph is ***Eulerian*** iff there exists an Eulerian circuit in it.

A pseudograph is ***connected*** iff there exists a walk between any two vertices.

A ***component*** of a graph is a maximal connected subgraph, i.e., a connected subgraph that is properly contained in no other connected subgraph that has more vertices or more edges.

Remark: a graph with just one component is connected.

Theorems

A pseudograph with at least two vertices is Eulerian iff it is connected and every vertex is even.

A pseudograph G possesses an Eulerian trail between two different vertices u and v iff G is connected and all vertices except u and v are even.