Graphs (continued)

Definition

The ***degree*** of a vertex *v*, written *deg v*, is the number of edges (excluding loops) that are incident with *v*, plus twice the number of loops that are incident with *v*.

Proposition (***Euler formula***): in a pseudograph $G = (V, E)$,

$$\sum_{v \in V} \deg v = 2|E|$$

Definitions

Odd an ***even vertices***: those with odd and even degrees, resp.

Measure of connectivity: ***beta index*** = e/v, where e is the number of edges and v is the number of vertices.

Recall that a ***graph*** G is a pair of sets: $G = (V, E)$, called vertices (V) and edges (E), where V is nonempty, and each element of E is a set of two elements of V.

A graph $G_1 = (V_1, E_1)$ is a ***subgraph*** of another graph $G = (V, E)$ iff $V_1 \subseteq V$ and $E_1 \subseteq E$.

Deletion of an edge or a vertex: given a graph $G = (V, E)$,

$G\backslash\{e\}$ is a subgraph whose edge set is $E\backslash\{e\}$

$G\backslash\{v\}$ is a subgraph whose vertex set is $V\backslash\{v\}$

The ***degree sequence*** of G is the sequence of degrees of all vertices of G listed in a descending order.

Isomorphism: graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are ***isomorphic***, written $G_1 \cong G_2$, iff there is a bijection $\phi: V_1 \to V_2$ that preserves edges (i.e., $\phi: E_1 \to E_2$ is also a bijection).

Isomorphism is an equivalence relation, since it is

- reflexive
- symmetric
- transitive

Therefore, all graphs are divided into ***isomorphism classes***

A graph is ***connected*** if there is a sequence of adjacent edges between any two vertices.

An ***n-cycle*** is a graph isomorphic to a polygon with n vertices. A 3-cycle is called a ***triangle***.

A ***triangulation*** of a surface is a partition of it by triangles (or, more generally, by n-cycles). Each part of the surface surrounded by a triangle (more generally, by an n-cycle) is called a ***facet***.

Euler's formula for Euler's characteristic χ of a surface

Given a triangulation of an ***orientable*** surface of ***genus***[1] g, that has c connected components and b ***boundaries*** (an example of a boundary is the edge of the circle), the Euler's characteristic is

$$\chi = f - e + v = 2c - 2g - b, \qquad (*)$$

where f is the number of facets, e is the number of edges, and v is the number of vertices in the triangulation.

[1] Genus g is actually defined by (*) and can be intuitively understood as the number of "handles" by which you can grab the surface without cutting your hands. For example, a torus has genus 1; a sphere, a circle and an annulus each have genus 0. See also http://en.wikipedia.org/wiki/Genus_%28mathematics%29