

Graphs

Definitions

A **graph** G is a pair of sets: $G = (V, E)$, called vertices (V) and edges (E), where V is nonempty, and each element of E is a set of two elements of V .

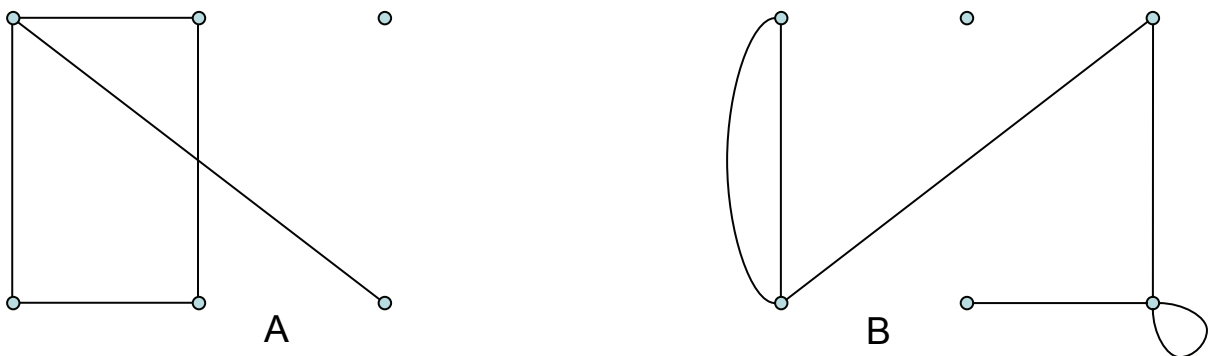
A **pseudograph** G is a pair (V, E) , where V is a nonempty set, and E is a multiset, each element of which is a pair of elements of V (here a pair could be composed of one and the same element of V).

$$G = (V, E), \quad V = \{v\} \neq \emptyset, \quad E = \{(e_i, m_i) \mid e_i = uv, u, v \in V, m_i \in \mathbb{N}\}$$

[Wikipedia]: A **multiset** (sometimes also called a **bag**) differs from a set in that each member has a **multiplicity**, which is a natural number.

For example, $\{a, a, b, b, b, c\}$ is a multiset, in which the multiplicities of the members a, b, c are respectively 2, 3, and 1. The order of listing is ignored.

Therefore: (i) a pseudograph may have multiple edges and loops, while a graph cannot. (ii) Every graph is a pseudograph; however, the converse is false. Examples: a graph (A), a pseudograph (B).



Definitions

Given a (pseudo)graph $G = (V, E)$, the following definitions hold. For a given edge

$$e = \{v, w\} \stackrel{\text{notation}}{=} vw \in E,$$

the two vertices $v, w \in V$ are called the **end vertices**, or **ends** of e , and each of them is **incident** with e .

The edge e **joins** the vertices v and w and is **incident** with v and w .

Two vertices are **adjacent** if they are joined by an edge, two edges are **adjacent** if they have a common vertex.

The **degree** of a vertex v , written $\deg v$, is the number of edges (excluding loops) that are incident with v , plus twice the number of loops that are incident with v .

The **complete graph** K_n of n vertices is the graph of n vertices in which every two vertices are adjacent.

A **bipartite graph** is one whose vertices can be partitioned into two disjoint sets V_1 and V_2 , called **bipartition sets**. In particular, there are no edges that have both ends within V_1 or within V_2 .

A **complete bipartite graph** $K_{n,m}$ on bipartition sets V_1 of n vertices and V_2 of m vertices is a bipartite graph in which every vertex in V_1 is adjacent to every vertex in V_2 .

Proposition (Euler formula): in a pseudograph $G = (V, E)$,

$$\sum_{v \in V} \deg v = 2|E|$$