Algorithms

Definition:

Algorithm is a systematic procedure that produces – in a finite number of steps – the answer to a question or the solution of a problem.

Example: Horner's algorithm for polynomial evaluation

$$S = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$$

Step 1. Set $S = a_n$;
Step 2. For $i = 1$ to n, replace S with $a_{n-i} + S\,x$;
Step 3. Output S.

Definitions:

Given two functions $f\colon \mathbb{N} \to \mathbb{R}$, $g\colon \mathbb{N} \to \mathbb{R}$, f is ***big Oh*** of g iff

$$f = O(g) \leftrightarrow \exists\ c > 0, n_0 \in Z,\ \ni\ |f(n)| \le c|g(n)|\ \forall n \ge n_0$$

f has ***smaller order*** than g, iff

$$f \prec g \leftrightarrow f = O(g) \wedge g \ne O(f)$$

f and g have ***same order***, iff

$$f \sim g \leftrightarrow f = O(g) \wedge g = O(f)$$

Propositions:

~ is an equivalence relation (reflexive, symmetric, transitive)

Let f, g, f_1, g_1 be functions $\mathbb{N} \to \mathbb{R}$, then

$$f = O(g) \to f + g = O(g)$$

$$f = O(f_1),\ g = O(g_1) \to f g = O(f_1\, g_1)$$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 \to f \prec g$$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty \to g \prec f$$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = L,\ L \in \mathrm{R},\ L \neq 0 \to f \sim g$$

A polynomial has the same order as its highest power:

$$S = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n \sim x^n$$

$$a, b \in \mathbb{R}, a < b \ \to \ n^a \prec n^b$$

$$\log_b n \prec n \ \forall b > 1$$