Probabilities

Definitions:

A ***sample space*** S is the set of all possible specific outcomes of a given experiment. An ***event*** A is a subset of S: $A \subseteq S$.

If all outcomes in a finite sample space S are equally likely, then the ***probability*** of an event A is

$$P(A) = \frac{|A|}{|S|}.$$

Theorem: Let S be a sample space. Then for any events A, B the following is true:

(1) $0 \le P(A) \le 1$, $P(\varnothing) = 0$, $P(S) = 1$.

(2) $P(A^c) = 1 - P(A)$.

(3) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Theorem: Let S be a sample space and $\{A_i\}$ events, then

$$P\left(\bigcup_i A_i\right) = \sum_i P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - \ldots + (-1)^{n+1} P\left(\bigcap_i A_i\right)$$

Proof: by the Principle of Inclusion-Exclusion.

Definitions:

Events A, B are ***mutually exclusive***, iff $A \cap B = \varnothing$.

Events $\{A_i\}$ are ***pairwise mutually exclusive***, iff

$$i \neq j \rightarrow A_i \cap A_j = \varnothing.$$

Corollary: Given a set of mutually exclusive events $\{ A_i \}$,

$$P\left(\bigcup_i A_i\right) = \sum_i P(A_i)$$