Maximum, minimum, maximal, minimal, glb, lub, lattice

For elements of a poset A the following is defined:

a is ***maximum*** iff $b \preceq a \ \forall b \in A$

a is ***minimum*** iff $a \preceq b \ \forall b \in A$

a is ***maximal*** iff $\forall b \in A$, $(a \preceq b) \rightarrow (a = b)$

a is ***minimal*** iff $\forall b \in A$, $(b \preceq a) \rightarrow (b = a)$

g is ***glb*** of a, $b \in A$ (denoted $a \wedge b$, pronounced "a meet b")
iff

$g \preceq a$, $g \preceq b$, and $\forall c \in A$, $(c \preceq a) \wedge (c \preceq b) \rightarrow (c \preceq g)$

l is ***lub*** of a, $b \in A$ (denoted $a \vee b$, pronounced "a join b")
iff

$a \preceq l$, $b \preceq l$, and $\forall c \in A$, $(a \preceq c) \wedge (b \preceq c) \rightarrow (l \preceq c)$

lattice is a poset in which every two elements have a glb and a lub

Examples: $(\mathbb{R}, \leq)$, $(\mathcal{P}(S), \subseteq)$

Hasse diagram:

- There is a dot for each $a \in A$
- If $a \preceq b$, then the dot for b is positioned higher than the dot for a
- If $a \prec b$ and there is no c such that $a \prec c \prec b$, then a line is drawn from a to b (say "b covers a").