Solution to an exercise

1.2.3.b

$(p \vee r) \rightarrow [\ (q \vee \neg r) \rightarrow (\neg p \rightarrow r)\ ] \iff$
$(p \vee r) \rightarrow [\ \neg(q \vee \neg r) \vee (\neg p \rightarrow r)\ ] \iff$
$(\underline{\neg (p \vee r)}) \vee (\neg q \wedge r) \vee \underline{(p \vee r)} \iff \mathbf{1}$

Cardinality $|S|$ is the number of elements in S

$|\varnothing| = 0$

$A = B \Rightarrow |A|=|B|$

$A \subseteq B \Rightarrow |A| \leq |B|$

$A \subset B \Rightarrow |A|<|B|$

$|A \cap B| \leq |A|$

$|A \cup B| \leq |A|+|B|$

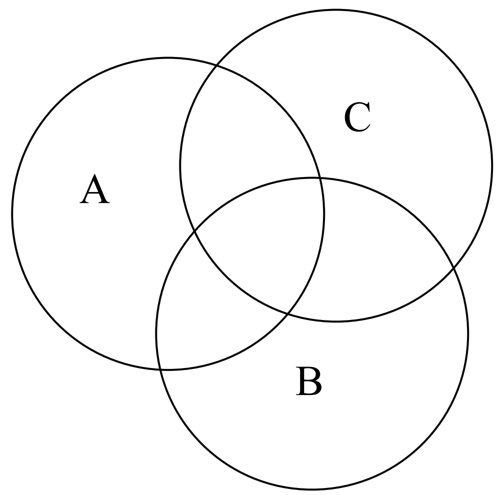
$|A \setminus B| \geq |A|-|B|$

$|\mathrm{A} \oplus \mathrm{B}| \leq |A|+|B|$

$|\wp(A)| = 2^{|A|}$

$|A \times B| = |A| \times |B|$

$|A^{\mathrm{n}}| = |A|^{\mathrm{n}}$



Binary relations

Suppose A and B are sets. A binary relation R from A to B is a subset of $A \times B$.

R on A is ***reflexive*** iff $(a,a) \in R$ for all $a \in A$.
R on A is ***symmetric*** iff $(a,b) \in R \Rightarrow (b,a) \in R$ for all $a \in A$.
antisymmetric iff $(a,b) \in R$, $(b,a) \in R \Rightarrow a=b$
transitive iff $(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R$

An ***equivalence relation*** ~ is one that is symmetric, reflexive and transitive.

An ***equivalence class*** of $a \in A$ is $\overline{a} = \{x \in A \mid x \sim a\}$
A ***quotient set*** $A / \sim$ (A *mod* ~) is the set of all equivalence classes.

A ***partition*** of A is collection of disjoint nonempty subsets of A (***called*** cells, or ***blocks***) whose union is A.

A ***partial order*** $\preceq$ is a binary relation that is reflexive, antisymmetric and transitive.
A ***partially ordered set***, or ***poset***, is a pair (A, $\preceq$). In this case a and b are ***comparable*** iff $a \preceq b$ or $b \preceq a$.
If every two elements are comparable, then $\preceq$ is a ***total order***, and (A, $\preceq$) is a ***totally ordered set***.

Maximum, minimum, maximal, minimal, glb, lub, lattice