An Introduction to Sets 2/2/2007

A ***set*** S is a collection of distinct objects x_1, x_2, … called ***elements*** (***members***) of the set.

Notation	**Meaning**	**Name**
$S = \{x_1, x_2\}$	S is a set of 2 elements: $x1$ and $x2$	explicit definition by the list of elements
$S =$ $\{x \mid x\in \mathbb{R}, 0<x<1\}$ $S = \{x\in \mathbb{R} \mid 0<x<1\}$	S is a set of real numbers between 0 and 1	set builder
$x_1 \in S$	x_1 is a member of S x_1 belongs to S	membership
$x_1 \notin S$		negation of the above
$x = y$	two elements x and y are one and the same element	equality
$x \neq y$	x and y are two distinct elements	inequality, negation of the above
$\|S\|$	the number of elements in S	cardinality

	Examples of sets:	
$\mathbb{N}$	natural numbers	
$\mathbb{Z}$	integers	
$\mathbb{Q}$	rational numbers	
$\mathbb{R}$	real numbers	
$\mathbb{C}$	complex numbers	

Notation	**Meaning**	**Definition**
$\varnothing$	empty set	contains no elements
U	universal set	contains all elements in the given context
$A = B$	sets A and B are equal	iff A and B contain the same elements or both are empty.
$A \subseteq B$	A is a subset of B A is contained in B	iff every element of A is an element of B.
$B \supseteq A$	B is a superset of A	(same as above)
$A \not\subseteq B$		negation of the above

$A \subset B$	A is a proper subset of B	iff $A \subseteq B$ and $A \neq B$
$B \supset A$	B is a proper superset of A	same as above
$A \not\subset B$		negation of the above

The following is true for any sets A, B:

$$A \subseteq A$$

$$\varnothing \subseteq A$$

$$(A = B) \leftrightarrow (A \subseteq B) \wedge (B \subseteq A)$$

Notation	**Meaning**	**Definition**
$A \cup B$	Union	the set of all elements that appear in any of the two sets
$A \cap B$	intersection	the set of all elements that appear in both sets
$A \backslash B$	set difference	the set of those elements of A that are not in B
$\mathcal{P}(A)$	power set of A	The set of all subsets of A
$\mathrm{A} \oplus \mathrm{B}$	symmetric difference	$(A \cup B) \setminus (A \cap B)$

Notation	**Meaning**	**Definition**
$A \times B$	***Cartesian product*** (***direct product***)	the set of all ordered pairs, in each of which the first element belongs to *A* and the second element belongs to *B*
X^n	the *n*-th power of *X*	the set of all ordered *n*-tuples over *X*
$<x_1,\ldots,x_n>$	ordered n-tuple	a sequence of *n* elements
A^c	complement of A	$U \setminus A$

It follows immediately that

$(A^c)^c = A$

$U^c = \varnothing$

$\varnothing^c = U$

From the point of view of associativity and distributivity, there is a direct analogy between $\cup$ and +, $\cap$ and $\cdot$, \ and -. The analogy is even deeper between $\cup$ and $\vee$, $\cap$ and $\wedge$, \ and $\wedge\neg$: in fact, logical operations that we studied in the previous lecture can be interpreted as set operations. In this case $\varnothing$ corresponds to contradiction, and U corresponds to tautology. This observation automatically extends all familiar equivalence relations (De Morgan's laws, etc.) as well as other facts about propositions to the current topic of sets.

Set theory	**Propositional logic**	**Boolean arithmetic**
$x \in P$	T	1
$x \notin P$	F	0
$P \cap Q$	$p \wedge q$	pq
$P \cup Q$	$p \vee q$	$p+q-pq$
P^c	$\neg p$	$1-p$
$P \subseteq Q$	$p \rightarrow q$	$1-p+pq$
$P = Q$	$p \leftrightarrow q$	$1-p-q+2pq$
$P \oplus Q$	$p \veebar q$ (XOR)	$p+q-2pq$
$P \cap P=P$	$p \wedge p \Leftrightarrow p$	$p^2 \equiv p$
$P \cap P^c=\varnothing$	$p \wedge \neg p \Leftrightarrow \mathbf{0}$	$p(1-p) \equiv 0$