Answers to supplemental problems:

1) 7.67/**7.84**/7.10.8

A one-directional alternative obviously makes sense here. Why? Because the physiologist is interested in the levels of HBE, which is secreted during stress. Runners who have been jogging regularly would be expected to have relatively normal levels of HBE when resting. Runners who had just started an exercise program might be expected to have higher levels of HBE since they'd be more stressed out. They certainly wouldn't be expected to have lower levers of HBE. So a one sided test makes sense here.

Write out hypotheses:

H_0: There is no difference in HBE levels between regular joggers and new joggers (when resting).
H_1: New joggers will have higher HBE levels when resting than regular joggers.

Re-do in symbols:

H_0: $\mu_1 = \mu_2$
H_1: $\mu_1 < \mu_2$

(Note: since we're using the Mann-Whitney U-test we need to assume the shape of the two distributions is equal - you should always state that assumption).

Pick α = 0.10 (book's suggestion)

Now calculate U^*:

K1	Y1	Y2	K2
		9	0
1	13		
		18	1
2	19		
		23	2
		27	2
4	28		
		31	3
5	32		
5	32		
		33	5
6.5	37	37	5.5
7	39		
7	40		
8	41	41	8.5
		41	8.5
		42	9
		47	9
		49	9
12	52		
		54	10
		59	10
14	60		
		70	11
71.5			**93.5**

Now check: 71.5 + 93.5 = 165 = 11 x 15, so we probably did things correctly

Also check: $K_2 > K_1$, so this is consistent with the alternative hypothesis (otherwise we would stop here).

So $U^* = \max(K_1, K_2) = 93.5$.

Now compare U^* to U_{table}:

$n = 15$, $n' = 11$, and we want a one tailed test at $\alpha = 0.10$:

$U_{table} = 108$. Because 108 > 93.5, we fail to reject the H_0.

Write out conclusion:

Because we fail to reject H_0, we have no evidence to show that HBE levels vary between regular runners and new runners (we might even go so far as to say that we have no evidence that physical condition affects HBE levels).

2) 9.2/**9.2**/<u>*8.2.2*</u>:

It ought to be obvious that this is a paired design.

(a) The standard error of the mean difference should be really easy, since we just have to worry about d (the difference):

Remember that:

$$SE_{\bar{d}} = \frac{s_d}{\sqrt{(n)}} \quad \textit{which implies that} \quad SE_{\bar{d}} = \frac{59.3}{\sqrt{9}} = 19.77$$

(b) We don't know anything about the diets, so a non-directional alternative (like the book suggests) makes sense here:

Preliminaries:

Hypotheses are:

H_0: there is no difference in average steer weight gain with diet.
H_1: there is a difference in average steer weight gain with diet.

In symbols:

H_0: $\mu_1 = \mu_2$
H_1: $\mu_1 \neq \mu_2$

We pick $\alpha = 0.1$ (as suggested in the text)

Now we finally calculate t^*:

$$t^* = \frac{22.9}{19.77} = 1.158$$

Now we get t_{table}:

$$t_{table} = t_{8,0.10} = 1.860$$

And because $t^* < t_{\text{table}}$, we fail to reject H_0.

Our conclusion is that we have no evidence that diet affects the average weight gain of our steers.

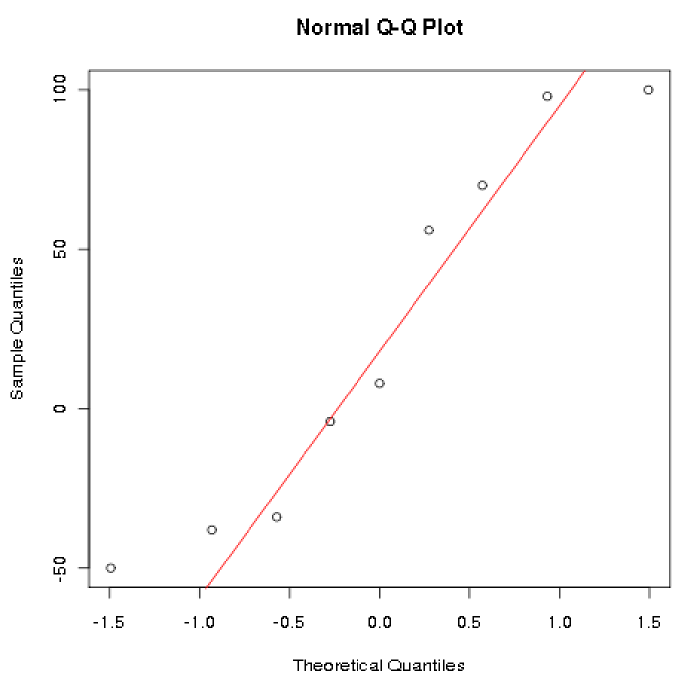
(c) This is easy:

22.9 +/- 1.860(19.77) gives us {-13.87, 59.67}

(we just pull in the SE for $\bar{d}$ from above, and use the t_{table} from above)

(d) So what does the CI mean? Simply that we are 90% confident that the true mean lies between -13.34 and 59.14. ***Because this interval includes 0***, we see that there is quite possibly no difference in the diets (0 implies there is no difference).

(e) Simply putting this information (the differences, not the original samples) into R should yield a Q-Q plot similar to the following (it shows things approximately normal):



(Note - an “S” is not so bad since it indicates short tails).

3) This is very straight forward, but before you do anything, you need to know if you're doing a one or two sided test (that's what the question is asking).

For a two sided test and $\alpha = .05$, with $d.f. = 8$, t_{table} gives: 2.306
For a one sided test and $\alpha = .05$, with $d.f. = 8$, t_{table} gives: 1.860

Please make sure you ask questions if you don't understand this!!

4) 7.68/**7.85**/_7.S.1_

I'm only giving the details for part (a) (for all four parts), and then the answers for (b) and (c). You should be able to figure everything out from what is presented below:

Part (a)

1) Assume unequal variances, and two sided test.

First, set up our hypothesis:

H_0: $\mu_1 = \mu_2$ $\quad\quad$ H_1: $\mu_1 \neq \mu_2$

(note: normally you'd want to write out your hypotheses in words first, but since this is just a bunch of numbers we're given, we'll stick with the shortcut formulas).

Second, select α:

Use $\alpha = 0.05$.

Third, calculate t^* (and v (=$d.f.$)):

$$SE_{\bar{y}_1-\bar{y}_2} = \sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = \sqrt{\frac{9.6^2}{12}+\frac{10.2^2}{13}} = 3.96$$

and so:

$$t* = \frac{42-47}{3.96} = -1.26$$

v is a bit of a pain (as usual, when we assume unequal variances):

first, let's get the SE's of the averages:

$$SE_{\bar{y}_1} = \frac{9.6}{\sqrt{12}} = 2.77 \quad and \quad SE_{\bar{y}_2} = \frac{10.2}{\sqrt{13}} = 2.83$$

so v becomes:

$$\nu = \frac{(SE_1^2+SE_2^2)}{\dfrac{SE_1^4}{n_1-1}+\dfrac{SE_2^4}{n_2-1}} = \frac{(2.77^2+2.83^2)^2}{\dfrac{2.77^4}{11}+\dfrac{2.83^4}{12}} = \frac{245.92}{5.35+5.35} = 22.99$$

(Yes, very slight rounding would get us to 23. In fact, it's so close, most people wouldn't say anything, but let's stick with rounding down to 22).

Fourth, we can finally do our comparison:

$|t^*| = |-1.26| < t_{\text{table } 22,0.05} = 2.074$ so we fail to reject H_0 and conclude that we have no evidence to show a difference in the means.

And let's not forget our assumptions:

Normally distributed data for both samples.
Unequal variances.
Random samples both within each sample and between the samples.

A lot of work to get this far. Part (2) will still be a little messy, but 3 & 4 will be very easy.

2) Assume equal variances, and two sided test:

The first two parts are identical to above:

First, set up our hypothesis:

H_0: $\mu_1 = \mu_2$ H_1: $\mu_1 \neq \mu_2$

(note: normally you'd want to write out your hypotheses in words first, but since this is just a bunch of numbers we're given, we'll stick with the shortcut formulas).

Second, select *α*:

Use $\alpha = 0.05$.

Third, calculate t* (and ν (d.f.)):

This is now different:

$$SE_{\bar{y}1-\bar{y}2} = SE_{pooled} = \sqrt{s^2_{pooled}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} \quad where \quad s^2_{pooled} = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{(n_1+n_2-2)}$$

so

$$s^2_{pooled} = \frac{11(9.6^2)+12(10.2^2)}{12+13-2} = \frac{116.04}{23} = 98.36$$

and

$$SE_{pooled} = \sqrt{98.36\left(\frac{1}{12}+\frac{1}{13}\right)} = 3.97$$

so we get t^*:

$$t^* = \frac{42-47}{3.97} = \frac{-5}{3.97} = -1.26$$

and finally, v (which is really easy this time):

$v = 12 + 13 - 2 = 23$

Fourth, we do our comparison:

$t^* = 1.26 < t_{\text{table } 23,0.05} = 2.069$ so again we fail to reject H_0 and conclude that we can't tell if the means are different

Comment: notice we did a lot of work and wound up with almost the same numbers as with the unpooled test. This shouldn't be a surprise - the n_i's are almost the same (12 and 13), and the s_i's are also very close (9.6 and 10.2). In this situation the two procedures will be very similar. For the other examples (parts (b) and (c) of this exercise), the two procedures do come out with more of a difference.

Oh - and our **assumptions** are the same as in part (1), ***except*** that we're assuming equal variances.

3) Assume unequal variances, and one sided test:

First, set up our hypothesis:

(this is a bit different now, since it's one sided):

H_0: $\mu_1 = \mu_2$ H_1: $\mu_1 < \mu_2$

(note: normally you'd want to write out your hypotheses in words first, but since this is just a bunch of numbers we're given, we'll stick with the shortcut formulas).

Second, select α:

Use $\alpha = 0.05$.

Third, verify that $\bar{y}_1 < \bar{y}_2$ (i.e., our data matches our H_1):

$\bar{y}_1 = 42$, $\bar{y}_2 = 47$, so yes, this matches our alternative hypothesis

Fourth, calculate t^* (and v (*d.f.*)):

This is now identical to what we did under part (1):

$|t^*| = |-1.26| = 1.26$ and $v = 22.99$, rounding down to 22

Fifth, we do our comparison:

We compare our t^* with t_{table} ,making sure we use the top row now instead of the bottom.

$t^* = 1.26 < t_{\text{table } 22,0.5} = 1.717$ again we fail to reject H_0 and conclude that we have no evidence to show the μ_1 is less than μ_2.

Assumptions are as in part (1)

4) Assume equal variances, and one sided test:

(again, this is one sided):

H_0: $\mu_1 = \mu_2$ H_1: $\mu_1 > \mu_2$

(note: normally you'd want to write out your hypotheses in words first, but since this is just a bunch of numbers we're given, we'll stick with the shortcut formulas).

Second, select α:

Use $\alpha = 0.05$.

Third, verify that $\bar{y}_1 > \bar{y}_2$ (i.e., our data matches our H_1)**:**

$\bar{y}_1 = 42$, $\bar{y}_2 = 47$, so no, this DOES NOT match our alternative hypothesis.

We **<u>stop</u>** here since we would automatically fail to reject. No need to calculate anything!!

Comment: if $\bar{y}_1 > \bar{y}_2$ had been true, you could have continued - in this case you'd use the pooled variance and pooled *d.f.* from above, making sure you use the top row in the *t*-tables (all your answers would have been to same as (2), except you'd need a different t_{table}).

Assumptions are as in part (2).

Part (b)

I'm just giving you the answers - using α = 0.05, you can follow the outline above for part (a):

$SE_1 = 0.58$ $SE_2 = 0.44$ $SE_{(\text{ybar1 - ybar2})} = 0.72$ $t^* = -19.38$ $\nu = 37$ (use 30)

$s^2_{\text{pooled}} = 5.59$ $SE_{\text{pooled}} = 0.74$ $t^*_{\text{pooled}} = -18.90$ $\nu_{\text{pooled}} = 39$ (use 30)

t_{table} two sided unpooled = 2.042 t_{table} two sided pooled = 2.042

t_{table} one sided unpooled = 1.697 t_{table} one sided pooled = 1.697

(Note: you should reject the H_0 in all four cases; also note that for the case of H_1: $\mu_1 > \mu_2$, you can stop and not do any more math)

Part (c)

Again, I'm just giving you the answers at $\alpha = 0.05$, you can follow the outline above for part (a):

$SE1 = 0.54$ $SE2 = 0.53$ $SE_{(\text{ybar1 - ybar2})} = 0.75$ $t^* = -2.65$ $v = 9$

$s^2_{\text{pooled}} = 1.75$ $SE_{\text{pooled}} = 0.78$ $t^*_{\text{pooled}} = -2.58$ $v_{\text{pooled}} = 10$

t_{table} two sided unpooled = 2.262 t_{table} two sided pooled = 2.228

t_{table} one sided unpooled = 1.833 t_{table} one sided pooled = 1.812

(Note: again, you should reject the H_0 in all four cases. Also, note that again for the case of H_1: $\mu_1 > \mu_2$, you can stop and not do any more math).