

One-sided tests

I. So what is a one sided test?

1) in our original set-up, we formulated a hypothesis such as:

$$H_0: \mu_1 = \mu_2$$

our alternative was:

$$H_1: \mu_1 \neq \mu_2$$

we also discussed, briefly, that one could have:

$$H_1: \mu_1 < \mu_2 \quad \text{or} \quad H_1: \mu_1 > \mu_2$$

2) The last two alternatives give rise to what is known as a one-sided test.

a) not all tests that you will see have one-sided alternatives (an example is ANOVA).

b) So what happens in a one sided test?

i) we REJECT if we conclude $\mu_1 < \mu_2$ (H_2) or, of course $\mu_1 > \mu_2$ (H_3).

ii) we do *not* test both alternatives at once (i.e., pick H_2 or H_3 , not both).

3) How do you decide if you need to do a one sided test?

- **based on your subject matter knowledge!** Do you know something about the situation that would lead you to believe that a one sided test would do better?

- for example - you test a new medication to lower blood pressure.

- what outcome are you interested in?

- that the medicine LOWERS blood pressure. If $\mu_1 =$ "average blood pressure before medication", then you're interested in $H_1: \mu_1 > \mu_2$.

- you want to show that the medication lowers blood pressure; you're not interested if it somehow raises blood pressure.

- you make your decision based on what you know about the problem.

- NEVER, NEVER decide to do a one sided test by comparing \bar{y}_1 with \bar{y}_2 . That's cheating.

4) Here's an outline:

a) Set up our hypotheses:

i) $H_0: \mu_1 = \mu_2$

ii) $H_1: \mu_1 < \mu_2$ (or $H_1: \mu_1 > \mu_2$)

b) decide on α .

c) verify that your results agree with your alternative hypothesis

- if your alternative hypothesis is $H_1: \mu_1 < \mu_2$, make sure $\bar{y}_1 \leq \bar{y}_2$ (or vice-versa)

- if this is not true, STOP - don't do anything else. Don't calculate anything else. *Your results contradict your alternative hypothesis!*

d) calculate t^* in the usual way (remember, this depends on pooled vs. unpooled variances (std. deviations), also don't forget your assumptions, or even if things are paired).

make your comparison in the usual way:

if $|t^*| \geq t_{\text{table}}$ then reject (otherwise “fail to reject”).

make sure that you now use the top row of your t-tables to select α (up until now we've only used the bottom row).

or (as usual):

if $p\text{-value} \leq \alpha$ then reject (otherwise “fail to reject”)

e) note that this only works if you do part (c) first; otherwise this can lead you to make a mistake (the more traditional way of doing things is outlined below).

5) The above is not the traditional approach to one sided tests. The traditional way of doing things is given at the end if you're interested (it *is* a better way of doing things, and if you truly understand it, it's a lot better).

6) An example:

a) exercise 7.42 on p. 269 [7.52, p. 265] ~~{7.5.7, p. 259}~~.

i) lettuce seedlings were grown on either standard soil, or soil with extra nitrogen.

ii) what are you interested in?

- standard soil is better?? Not likely.

- nitrogen soil is better?? This makes sense.

b) So form your hypotheses:

i) $H_0: \mu_1 = \mu_2$

ii) $H_1: \mu_1 < \mu_2$

- why? because μ_1 represents plants grown in standard soil.

- also note: we could pick $H_0: \mu_1 \geq \mu_2$ (some statisticians prefer this). We still do not, however, "accept" H_0 .

c) pick $\alpha = .10$ (book picks α for us).

d) verify that our results agree with the alternative hypothesis;

- yes, $\bar{y}_1 < \bar{y}_2$

d) calculate t^* . Let's assume unpooled variances:

$$t^* = \frac{3.62 - 4.17}{\sqrt{\frac{0.54^2}{5} + \frac{0.67^2}{5}}} = -1.43$$

e) so now figure out our d.f. (same as previously):

$$\begin{aligned} v &= \frac{(SE_1^2 + SE_2^2)^2}{\frac{SE_1^4}{n_1 - 1} + \frac{SE_2^4}{n_2 - 1}} \\ &= \frac{(.2415^2 + .2996^2)^2}{\frac{.2415^4}{4} + \frac{.2996^4}{4}} \\ &= 7.67 \end{aligned}$$

(so we use 7 (we round down))

f) now we look up t with 7 d.f. and $\alpha = .10$ and get 1.415.

g) Our comparison:

$|t^*| = 1.43$ is less than or equal to 1.415,

so we reject H_0 .

h) We conclude that we do have evidence to show that adding nitrogen to soil affects the weight of lettuce seedlings.

7) Some comments:

a) note that your p -values are now half of what they were before. Why? because you don't have to put half of your probability in the other tail!! This is good, because you have a much better chance of rejecting H_0 (you get more power).

b) remember: if you look at your data after performing the test and find that $\bar{y}_1 \gg \bar{y}_2$, you just can't go back and test for $\mu_1 > \mu_2$. You'd be cheating!!

c) if you decide to do a one-tailed test, you must decide to do so before looking (snooping) through your data. *You should have a reason for choosing a one-tailed test before you even perform the experiment.*

II. The one-sided Mann-Whitney U-test.

1) No long discussion. Just a brief outline - you should be able to figure this out yourself.

2) Outline:

a) Develop your hypotheses (just as for the one-tailed t -test, except remember you're testing distributions, not means)

b) decide on α .

c) Calculate U^* . But note (like in the t -test) that if your alternative is that distribution 1 > distribution 2 (or $\mu_1 > \mu_2$), and $K_1 < K_2$, then you can stop since you're p -value is over .5.

d) Look up your U^* in table 6, but this is a little easier because the table keeps track of everything for you. Your comparison is the same as for the two-tailed test (if $U^* \geq U_{table}$, reject), except that you need to use the appropriate column for a one-tailed test. No need to worry about negative values, etc.

e) make your decision to reject H_0 or fail to reject H_0 .

3) You'll have one of these for homework.

III) ***** Optional (we won't cover this in class) *****

1) The more traditional approach (and probably better) to one sided tests (or: “more than you ever wanted to know about one sided tests”):

a) even though you calculate t^* the same way as always, you should make sure you're consistent. In other words, always subtract the same way in the numerator:

$$\bar{y}_1 - \bar{y}_2$$

- until now, this didn't make any difference to our results because we used $|t^*|$, but now we have to pay attention to the sign of t^* .

b) Note that if:

$$\bar{y}_1 - \bar{y}_2 > 0$$

this implies that $\bar{y}_1 > \bar{y}_2$, and this in turn implies that your p-value will be greater than 1/2. In this case you can stop and not bother doing the rest of the test.

(Remember, our alternative hypothesis: $H_1: \mu_1 < \mu_2$; this hypothesis would obviously be very unlikely if our results actually indicated the “opposite”.)

c) look up t_{table} . This will be different, and depend on the direction of your test. If you get confused, it's probably best to draw a couple of sketches to help keep straight what you're looking up.

i) Since we're interested in $H_1: \mu_1 < \mu_2$, what we're doing is rejecting for small values of t .

we don't mean small in “absolute value”; instead we mean negative numbers are small.

- Why? If $\bar{y}_1 < \bar{y}_2$, we will get a negative quantity in the numerator of t^* .

If this is “very” negative, then you would think our alternative is more likely to be true. So the smaller our t^* is, the more likely we are to reject.

ii) So what about our t_{table} ? We look up the upper tailed probability, and then use the negative of this quantity. There is no need to worry about dividing anything in half (i.e., we're using the top row of our table).

{*** here is some stuff on which row to use in your t-tables if you didn't modify the t-tables as described in class ***}

- since all our probability is now in one tail (we're not rejecting for large values of t^* in this example), we can read our critical t -value right off the table. For instance, if $\alpha = .05$, we just use that column, cross-index with our $d.f.$, and then use the negative of the critical value we find.

- if this sounds confusing, remember that we modified our t -tables so that we could read two sided values of α right off the bottom row and didn't have to worry about this.

- REMEMBER, for a two tailed test, we need to make sure that half of α is in each tail, so we divided our α by two, and then used the column for $\alpha/2$ (or, if you modified your table, you could just read across the bottom of your t -tables

{*** end of stuff on reading t -tables the old fashioned way ***}

iii) So our comparison is:

if $t^* \leq$ (tabulated t) reject and conclude H_1 is true.

if $t^* >$ (tabulated t) "fail to reject" and conclude we have no evidence against H_0 .

iv) We did not use absolute values here, we stuck with our "original" t^* .

d) if our alternative hypothesis had been:

$H_1 = H_1: \mu_1 > \mu_2$, our outline would have been identical to above, except that we would reverse all our inequalities. Now we would reject for "large" values of t^* .

g) It is VERY easy to confuse yourself. Go slow, if need be, draw a picture or two, and convince yourself that you are doing the right thing.