Connecting Whole Number Arithmetic to Algebra:
Hands-On/Number Sense Activities Creating a Seamless Path

Margo Lynn Mankus
George Mason University
Fairfax, VA
mmankus@gmu.edu
http://mason.gmu.edu/~mmankus/

Description:

When students "hit" algebra, they are usually presented with concepts such as "balancing" equations, factoring and expanding polynomials, and linear equations. What groundwork can be set by the K-6 curriculum to lead students to see algebra as an extension of their arithmetic skills? Can the algebra curriculum bounce off students' previous knowledge of arithmetic or must the algebra curriculum start from its own frame of reference? Activities looking at the operations on whole numbers, using manipulatives and calculators, will be investigated and then extended to the "rules" of algebra. Share your thoughts on the question, "Can there be a seamless journey from arithmetic to algebra?"

Goals:

- Present a connection between the thought processes and algorithms developed while studying arithmetic and the thought processes and algorithms shown while studying algebra.
- Use word problems, investigations with manipulatives and calculators, and finally the written mathematics to present a connected route from arithmetic to algebra.
- Share thoughts on the question, "Can there be a seamless journey from arithmetic to algebra?"

In this Handout:

Part A: Word Problems: Natural Problem Solving and Inverse Operations
vs. Balancing Equations

Part B: Whole Number Algorithms and a Bit of Algebra!
Use Virtual Base 10 Blocks!
http://mason.gmu.edu/~mmankus/whole/base10/asmdb10.htm

Part C: Multiplication Tables to Linear Equations

Part D: Using the TI-73 and the TI-Ranger to learn about lines!

Part A: Word Problems:
Balancing Equations vs. Natural Problem Solving and Inverse Operations

Addition and Subtraction:

1. Maggie had 5 cookies. Jamal gave her 3 more cookies. How many cookies does Maggie have altogether?

2. Maggie has 5 cookies. How many more cookies does she need to have 8 cookies altogether?

3. Maggie had some cookies. Jamal gave her 5 more cookies. Now she has 8 cookies. How many cookies did Maggie have to start with?

4. Maggie had 8 cookies. She gave some to Jamal. Now she has 5 cookies left. How many cookies did Maggie give to Jamal?

Multiplication and Division:

1. If three children have two cookies each, how many cookies are there altogether?

2. If two children have three cookies each, how many cookies are there altogether?

3. If six cookies are shared among three children, how many would each child get?

4. If there are six cookies and each child must get three cookies, how many children can you serve?

Discussion Question:
Which should we teach? How do we link the processes?

Balancing Equations	Natural Problem Solving and Inverse Operations
$x + 5 = 8$ $\underline{\quad - 5 = -5}$ $x \quad = 3$	$x + 5 = 8 \Leftrightarrow 8 - 5 = x$ $3 = x$ or $x = 3$
$3x = 6$ $\dfrac{3x}{3} = \dfrac{6}{3}$ $x = 2$	$3x = 6 \Leftrightarrow 6 \div 3 = 2$

Modified and extended from work by:
Using Children's Mathematical Knowledge in Instruction
American Education Research Journal, Fall 93, Vol. 30, #3, pp. 555-583
Fennema et. al.
and
Teaching Mathematics in Grades K-8: Research Based Methods
Anghileri and Johnson (1988)

Part B: Whole Number Algorithms and a Bit of Algebra!
Using Base Ten Blocks to "See" Algorithms

This activity can be found at:
http://mason.gmu.edu/~mmankus/whole/base10/asmdb10.htm
You can use Virtual Base 10 Blocks from this page!

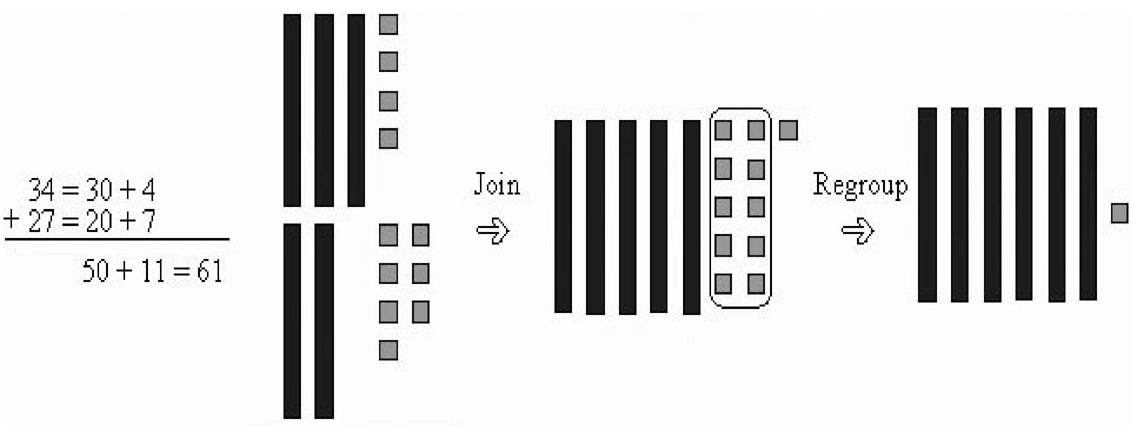
Objective: To look at addition, subtraction, multiplication and division of whole numbers from a geometric, "hands-on" perspective and an algorithmic perspective.

Audience: This activity is intended for teachers. The activity is designed to make connections between the use of manipulatives and the development of algorithms. Parents and students are welcomed!

Part 1: Addition - Focus on trading and regrouping.
Part 2: Subtraction - Focus on trading and regrouping.
Part 3: Multiplication - Focus on the distributive property and area models.
Part 4: Division - Focus on the scaffold method and area models.
Part 5: A Bit of Algebra - Focus on the distributive property and area models.

Part 1: Addition

1. **One Type of Addition Algorithm**

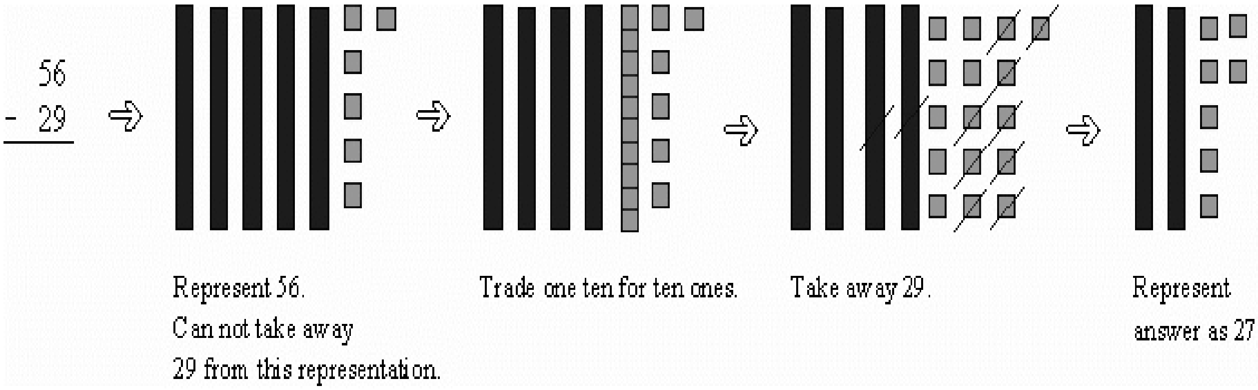


2. Try these problems using Base 10 Blocks and the algorithm. Write and draw your work.

$$\begin{array}{r} 38 \\ +\ 13 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 126 \\ +\ 45 \\ \hline \end{array}$$

Part 2: Subtraction

1. **One Type of Subtraction Algorithm**



$$\begin{array}{r} 56 \\ -\ 29 \\ \hline \end{array} \Rightarrow \begin{array}{r} (50+6) \\ -(20+9) \\ \hline \end{array} \Rightarrow \begin{array}{r} (40+16) \\ -(20+\ \ 9) \\ \hline (20+\ \ 7) = 27 \end{array}$$

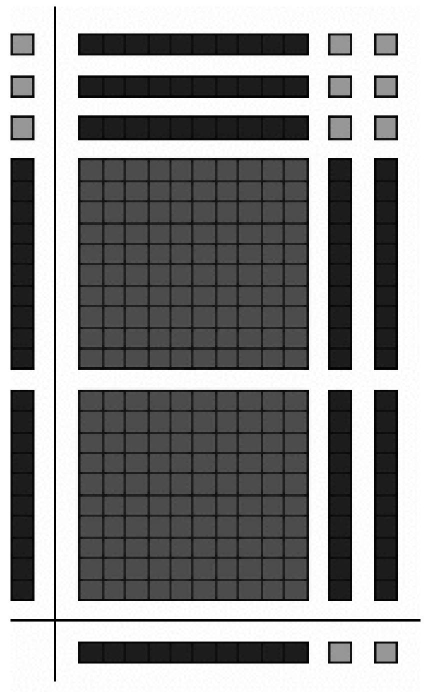
2. Try these problems using Base 10 Blocks and the algorithm. Write and draw your work.

$$\begin{array}{r} 63 \\ -\ 25 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 50 \\ -\ 23 \\ \hline \end{array}$$

Part 3: Multiplication

1. **One Type of Multiplication Algorithm -** If you have 23 students in your class and they each need 12 straws for a craft project, how many straws do you need to supply? We write this as 23 groups of 12 or 23 × 12. Write out how you would solve the problem.

2. Notice two applications of the **Distributive Property** gives us the "standard" pieces. Here you see this in both vertical and horizontal formats. Find the pieces from the computations below on the area model. Notice that we are actually finding 12, 23's.



Vertical:

```
      2  3
x     1  2
---------
         6   = 2 x 3
      4  0   = 2 x 20
      3  0   = 10 x 3
2     0  0   = 10 x 20
-------------
2     7  6
```

OR

Horizontal:

$$\begin{aligned} 12 \times 23 &= (10+2)\times(20+3) \\ &= (10\times 20)+(10\times 3)+(2\times 20)+(2\times 3) \\ &= 200+30+40+6 \\ &= 276 \end{aligned}$$

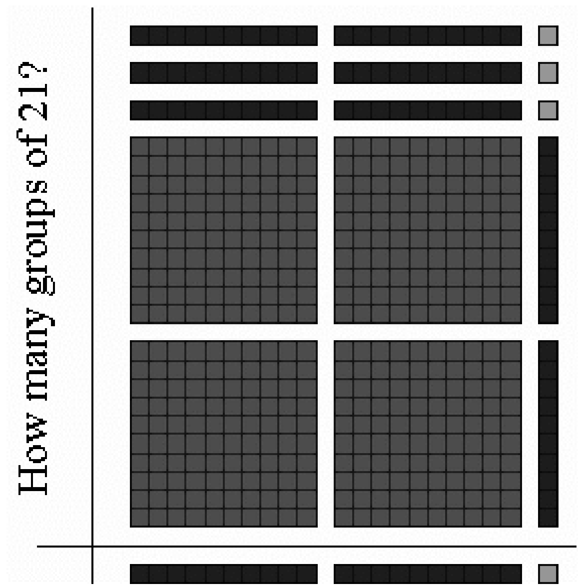
3. Try these problems using Base 10 Blocks. Draw or printout the area model you construct. Write out the details of the algorithm and find the products on your area model. Notice that the second problem is multi step. (Why?)

$$\begin{array}{r} 14 \\ \underline{\times\ \ 12} \end{array} \qquad\qquad \begin{array}{r} 24 \\ \underline{\times\ \ 13} \end{array}$$

Part 4: Division

1. **One Type of Division Algorithm -** You have 483 sea shells for a class art project. Each student needs 21 shells. How many students will be able to make the project? How many groups of 21 shells can you form out of 483 objects? We write 483 ÷ 21. Write out how you would solve this problem.

2. Find the number of groups of 21 on the Area Model. Draw in the left most column with the appropriate "Base 10 blocks."



3. Next, look at the scaffold method below. (Is there a correlation to the scaffold "good guess" method and the Area Model? Does there have to be a relationship?)

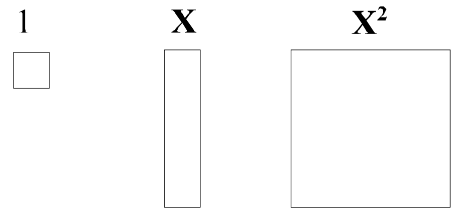
21) 483	
− 420	20 groups of 21
63	
− 63	+ 3 groups of 21
0	23 groups of 21

4. Now, you have 483 sea shells for a class art project. There are 21 students in your class. If you give each student the same number of shells, how many shells will each student have? Use the blocks to model this problem. Is it still written 483 ÷ 21? Discuss.

5. Caution: When you pick problems for illustration with Base 10 blocks, make sure you check them out first using the blocks! Sometimes a problem requires that you break up a FLAT in order to fill in the area model. This type of problem is not the best for a first use of the blocks.
Try these by drawing Base 10 Blocks. Solve also using the scaffold method.

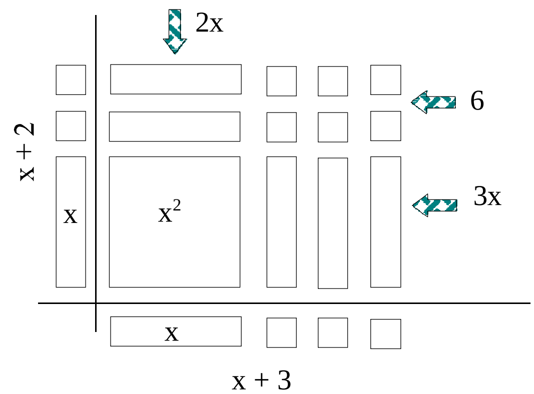
13) 299   14) 308

Part 5: Moving to Algebra!



1. We can use the Base 10 blocks idea to create Base x blocks or Algebra Tiles. x is unknown!

2. Look at the figure to the right. This is an area representation of (x + 2)(x + 3). Compare this to 23 ×12. Do you see a similarity?



3. Expand:
(x + 2)(x + 3) = ______________________

Find the pieces on the picture to the right.

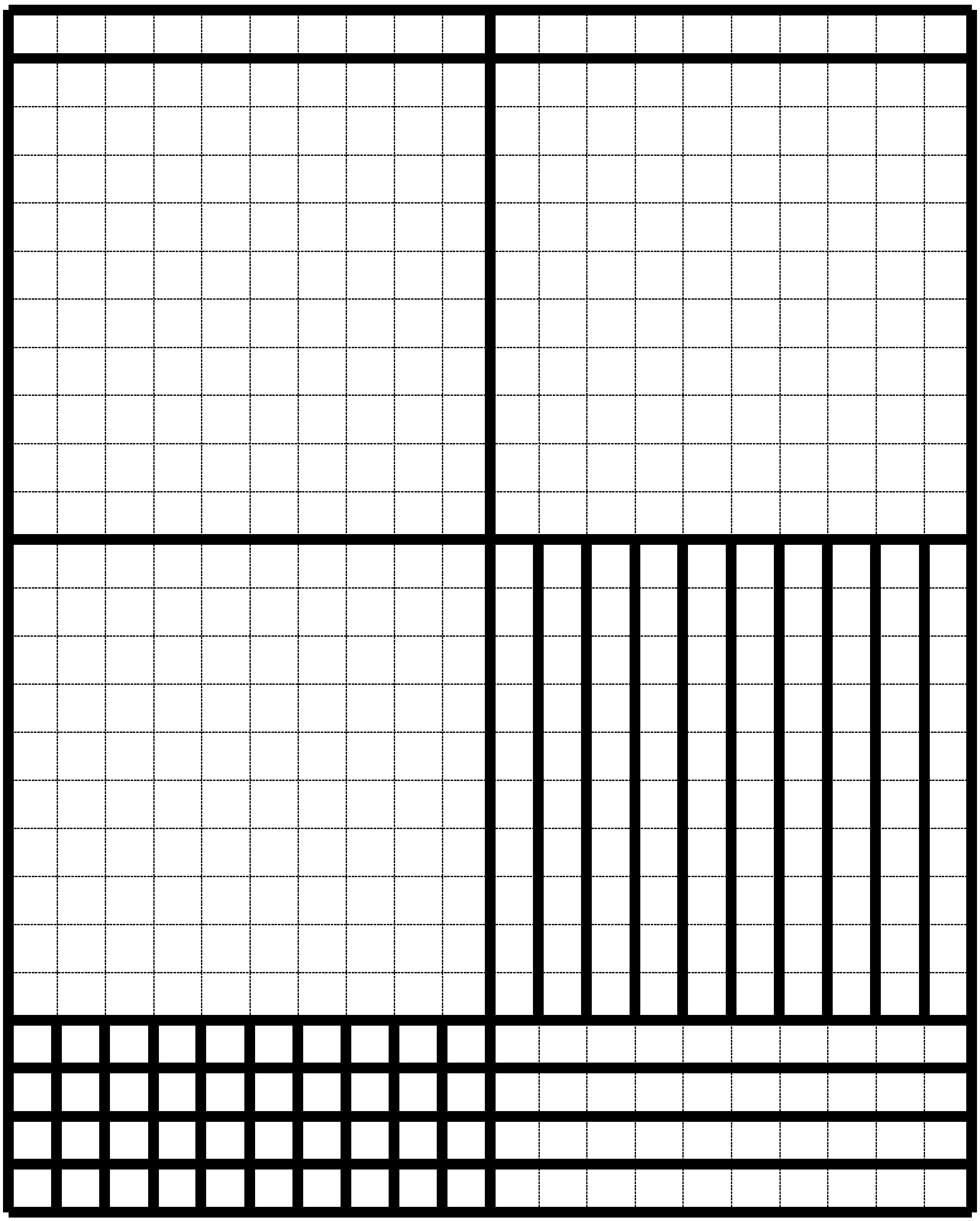
4. Try these! Draw them out and then expand. Can you see the pieces?

   a. x(x+4)

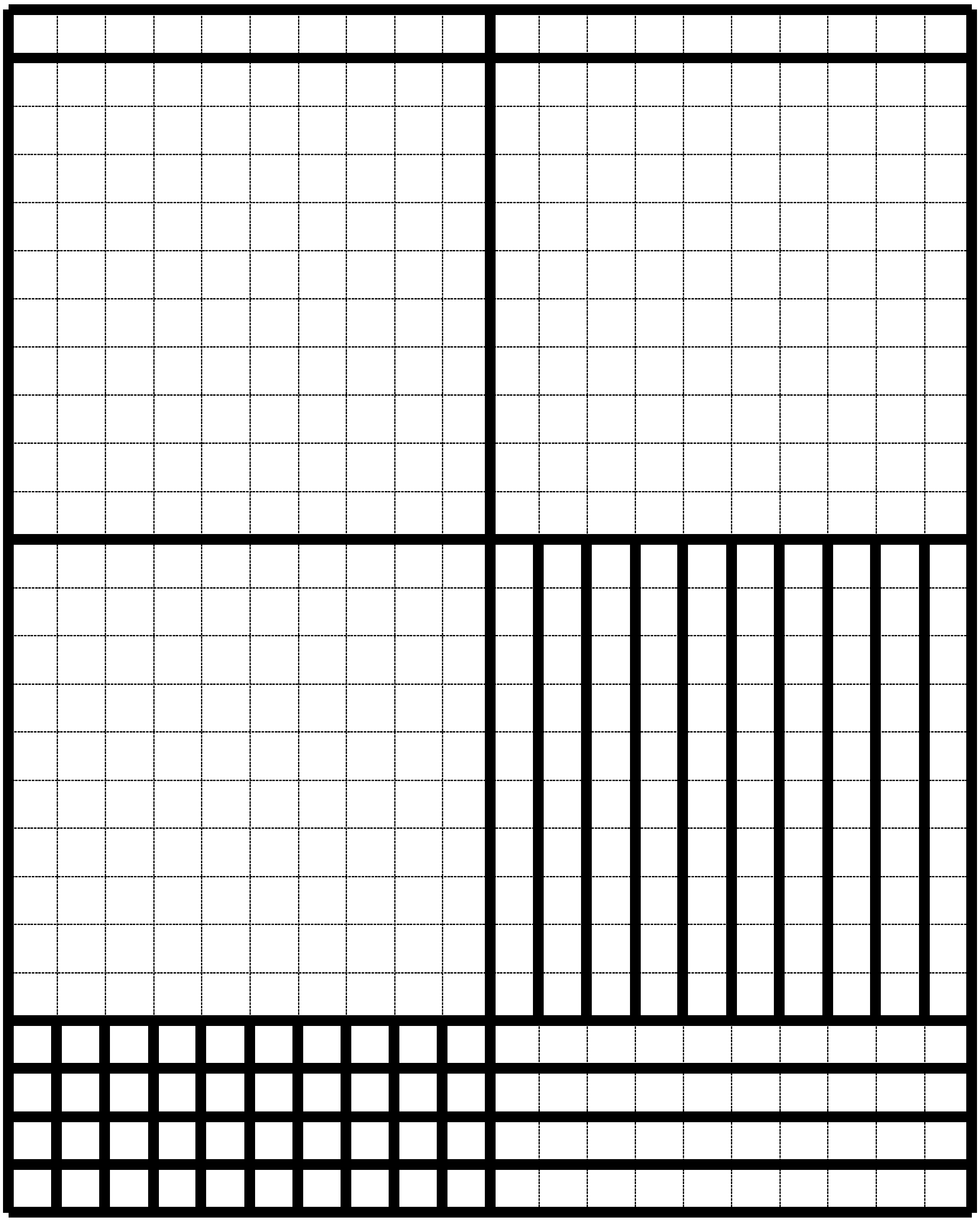
   b. (x+1)(2x+3)

   c. **Extension:** Draw and expand (x + 2y + 3)(2x + y).
   Hint: You will need y and y^2 blocks.

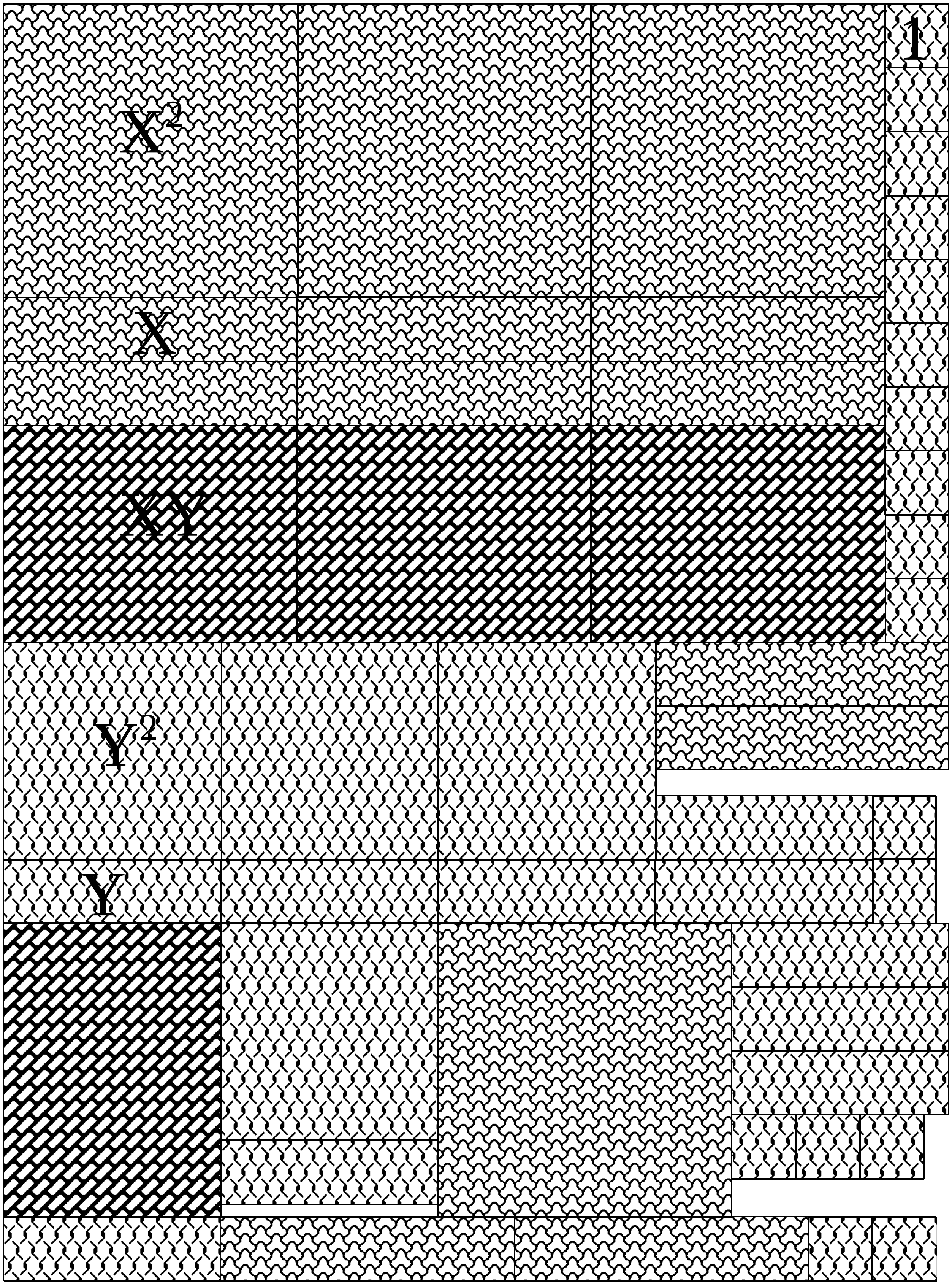
Base Ten Block Pattern



Base Ten Block Pattern



X Y Blocks

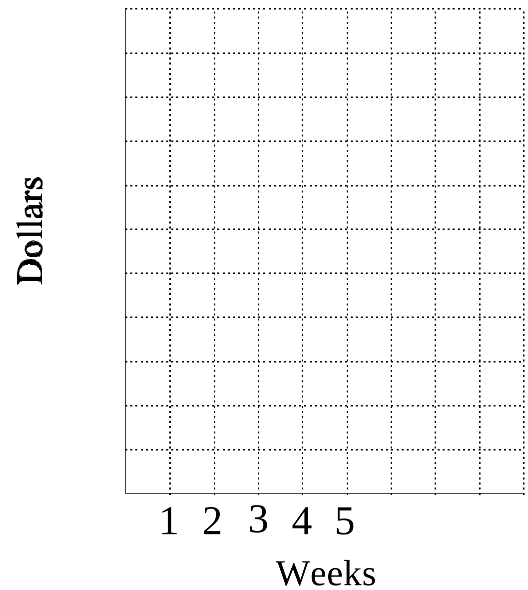


Part C: Multiplication Tables to Linear Equations

You have no money now. You find out that you are going to get an allowance of $2 each week. You would like to buy a ______________ (Calculator) which costs ____________ ($20). You will save your money until you have enough! When will you have enough? How is your money growing?

1. With your group, decide how much money you will have week to week. Use the table and the graph below to organize your work.

Weeks	**D**ollars
0	
1	
2	
3	
4	
⋮	
W	



2. If you receive $2 each week you can write that in math symbols as ____________________

3. Write the sequence of numbers that describes your money supply week to week.
   ____

4. From your graph, write a word that describes the shape of the graph. _________________

5. Write a sentence to explain when you will have enough money to make your purchase. Write how you found this answer.

6. Use your work to predict the future! If you don't spend any money for 14 Weeks, how many Dollars will you have? Write an explanation of how you found your answer. Use words such as add, subtract, multiply, and divide.

Teacher's Sheet
How Does Your Money Grow?

Classroom Organization:

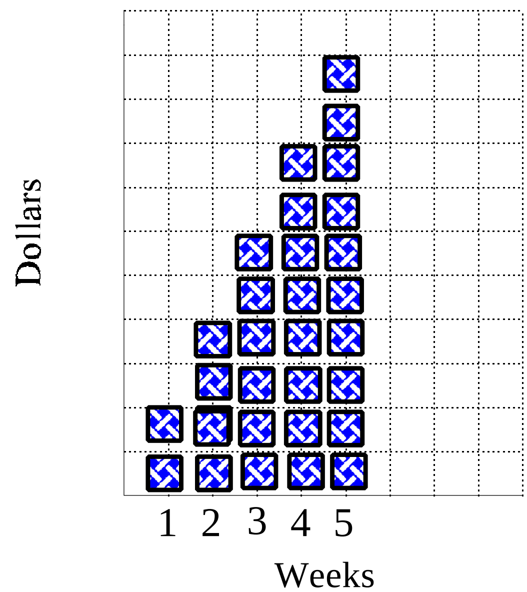
Groups work and present their solution.

(Time saver - use overheads or small white boards for each table or have board space for each group as they work.)

Discussions of Mathematics used in the solutions needs to occur.

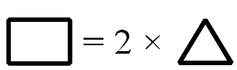
Give the suggestion of the use of Tables of Numbers and Graphs as tools.

1. Reinforce multiplication as repeated addition.



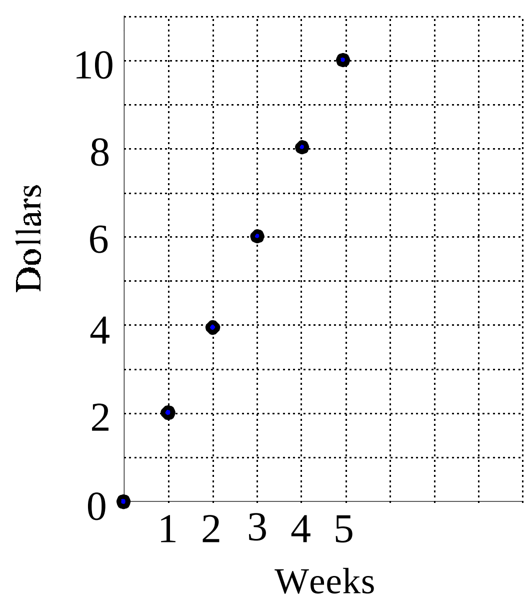
Weeks	Dollars
0	0
1	2
2	2+2 = 2×2
3	2+2+2 = 3×2
4	2+2+2+2 = 4×2
⋮	
W	2*W

Dollars = 2 × Weeks



D = 2 × W

FYI - can you see y = 2 x?



2. If you receive $2 each week you can write that in math symbols as
<u> + 2 , a constant rate of change </u>

3. Write the sequence of numbers that describes your money supply week to week.
<u>   2, 4, 6, 8, 10, ···   </u> This is an **Arithmetic Sequence** of numbers.

4. From your graph, write a word that describes the shape of the graph. <u> a line </u>

5. Write a sentence to explain when you will have enough money to make your purchase. Write how you found this answer.
Encourage students to see the relationship Dollars = 2 × Weeks This is developed using Inductive Reasoning.
Use of Division as an Inverse Operation: $\boxed{20} = 2 \times \triangle$

$\triangle = \boxed{20} \div 2$

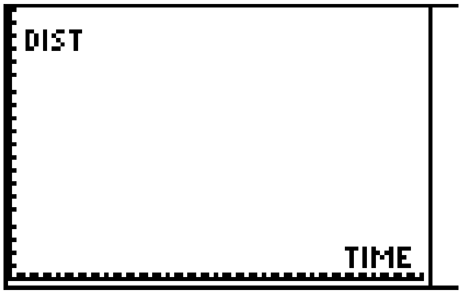
This sets the foundation of a function and an inverse function.
Students will also be able to read the answer from the table and the graph.

6. Use your work to predict the future! If you don't spend any money for 14 Weeks, how many Dollars will you have? Write an explanation of how you found your answer. Use words such as add, subtract, multiply, and divide.
Students can use the table, graph, the calculator to find the solution. Encourage the formal writing of Dollars = 2 × 14 = $28. Notice that if students use the table or the graph they will have to predict a pattern to use.

Added discussion: Commutative Property of ×
Does it matter?
Dollars = 2 × Weeks
or
Dollars = Weeks × 2

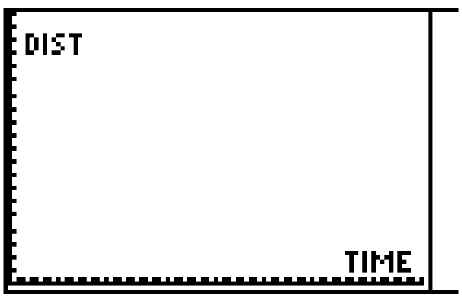
Part D: Using the TI-73 and the TI-Ranger to Learn about Lines!

Part I How Fast Do You Need To Go?

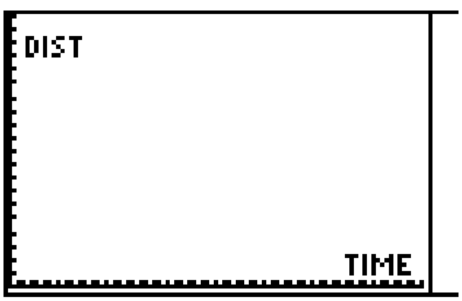


1. Use the Ranger program on your TI-73 to collect data. A distance-time graph of *constant speed* forms a *straight line*. Draw your walk on the graph at the left. Write a description of how you walked a constant speed. ______________________________
____

2. How can you make your line steeper? ______________________________
____



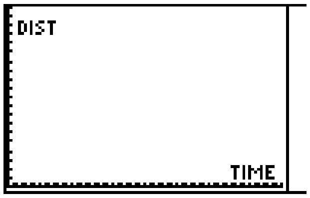
3. Repeat the exploration above and walk a distance-time graph which is steeper than your first walk. Draw your walk on the graph.



4. Repeat the exploration above and walk a steeper line. Draw your graph. What did you have to do to get a steeper line? ____________
____

5. Can you walk a line that is vertical (straight up and down)? Why or why not?
____
____

Part II How Fast Were You Going? Let's Look at Your Data!



1. Use the Ranger program on your TI-73 to gather data. Walk a distance-time graph of constant speed. Draw your walk on the graph.

2. L_1 is the time in seconds and L_2 contains your position from Ranger. Fill in the following table with the data.

L_1 - seconds	L_2 - feet
1.1	
1.6	
2.1	
2.6	
3.1	

3. The average velocity you walked can be calculated by

$$average\ velocity = \frac{change\ in\ position}{change\ in\ time}$$

Use the information in the table to answer the following questions.

4. How far did you walk from 1.1 sec to 1.6 sec? ____________________
This is your **change in position**.

5. How many seconds did you walk during 1.1 sec to 1.6 sec? ____________
This is your **change in time**.

6. Your average velocity during 1.1 sec to 1.6 sec. is found by

$$average\ velocity = \frac{change\ in\ position}{change\ in\ time} = \underline{\qquad\qquad\qquad}$$

7. Find your average velocity during 1.6 sec to 2.1 sec.

$$average\ velocity = \frac{change\ in\ position}{change\ in\ time} = \underline{\qquad\qquad\qquad}$$

8. Find your average velocity during 2.1 sec to 2.6 sec.

$$average\ velocity = \frac{change\ in\ position}{change\ in\ time} = \underline{\qquad\qquad\qquad}$$

9. Find your average velocity during 2.6 sec to 3.1 sec.

$$\text{average velocity} = \frac{\text{change in position}}{\text{change in time}} = \underline{\hspace{6cm}}$$

10. Did you come close to walking a constant speed? Explain your answer. ______________
____

11. How is the average velocity related to the **slope** of the distance-time line?
____

12. What is the **intercept** of your walk? How do you know? ___________________________

13. **Extension:** Use Manual-Fit on the TI-73 to find the equation that describes your walk. Compare it to the linear regression line found by using LinReg on the TI-73!