UNDERSTANDING RATIONAL NUMBERS AND PROPORTIONS

Research Theme: Meaningful Mathematics Discourse in the Classroom

Broad Subject Matter Goals: Understanding Rational Numbers and Proportions

Unit Learning Objectives: Students will use real-world models to develop an understanding of fractions, decimals, unit rates, proportions, and problem solving.

Learning Objectives:
Students will

- represent parts of a whole using an area interpretation of fractions
- determine the fractional part of a whole when parts are not cut into equal-sized pieces
- develop an understanding of the quotient interpretation of fractions
- find the unit cost of items that are part of a set
- determine the relationship among parts of a whole that are unequal-sized pieces
- express fractional parts of a whole as decimal equivalents

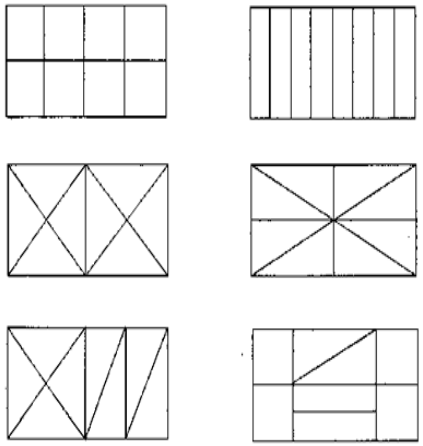
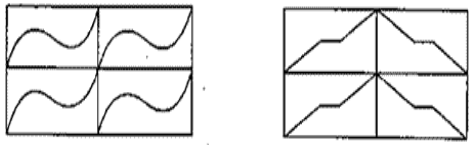
Materials:

- Making Four Pieces Overhead
- Cakes Cut Into Eighths Activity Sheet
- Cakes Cut Into Fourths Activity Sheet
- Scissors
- Calculators (optional)
- Rulers
- Grid Paper
- Grid Paper Transparency
- Overhead Projector

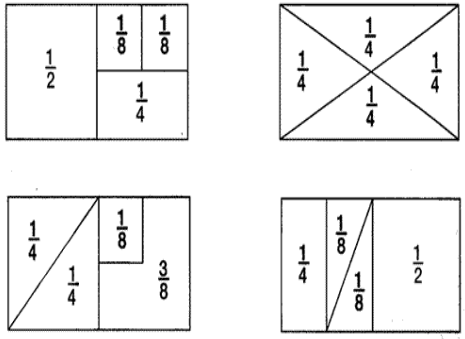
Instructional Plan

Steps	Main Learning Activities	Students' Anticipated Responses	Remarks on Teaching
Introductory Activity Evoking prior knowledge	Distribute rectangular pieces of construction paper and scissors and ask the students to cut in half. Then ask them to take the halves and cut into half again. Discuss what fraction of the original rectangle each piece represents. Ask them to cut each fourth into half again. What fraction do we now have?	Students may cut these pieces into squares, more rectangles, or even triangles.	Discuss equivalency of the different shaped pieces.
Activity 1 Posing the problem	Hand out the following problem, read with class, and allow students a couple of moments to think about it. "Customers Cut the Cake" *Each day the local baker makes several rectangular sheet cakes, which he cuts into eighths. He sells 1/8 of a sheet cake for $1.59. As part of a new promotional campaign for his store, he wants to cut his sheet cakes into eighths a different way each day. Customers who suggest a new way to cut the cakes into eighths win a free piece of cake each day for a week. What are some of the different ways to cut the cake?*	Students may need some help understanding what a sheet cake is. Students may need help connecting the fraction 1/8 to the real world representation (cake).	It would be nice to have a picture of a rectangular sheet cake as a visual.

Assessing student understanding of the problem	Some questions to ask students **before** they begin work on the problem include: ▪ What is the shape of the baker's cakes? ▪ What are the restrictions on the ways the cakes can be cut? ▪ How can we verify that pieces that are not the same shape contain the same amount of cake?	Students may have trouble with the idea that equal pieces may not have the same shape.	Answers to questions-- Rectangular cakes Pieces must contain the same amount of cake; they do not have to be the same shape. Cut the pieces into smaller parts and lay one on top of the other or we can use graph paper and count squares or we can weigh the actual cake pieces. Make graph paper available for those that may want to use. This might be helpful for those that are confused with equivalency.
Students problem solving	Hand out the Cakes Cut Into Eighths Activity Sheet and assign cooperative groups or pairs. Allow time to work.	Some students will need to cut apart and manipulate the pieces, while others will be able to visualize the equivalence on their papers. Some students may have a limited view and think that all the cuts must be parallel to one side of the rectangle.	(For differentiation, students can use rectangular pieces of paper to model the cakes, sketching the shapes or cutting the paper into eight pieces and verifying the equivalence of the pieces by cutting and overlapping.)

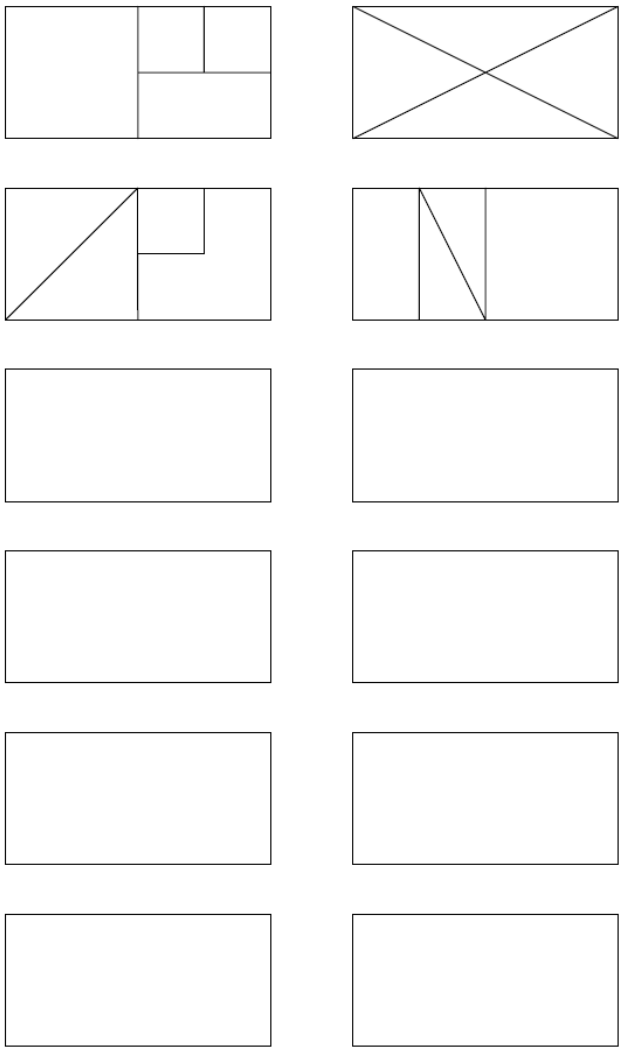
Discussion of Activity 1	Have students place their designs on the chalkboard or the overhead projector. Ask the students to decide which designs are the same and which are different. Talk about the designs and discuss whether the pieces are equivalent. What makes them equivalent?		This is an excellent place to bring in **area, size and shape**. If students are having difficulty visualizing the equivalent areas, consider using graph paper transparency to trace the shapes on, and then count the squares.
	Is it possible to create equivalent pieces using curves or combinations of straight line segments? Challenge students to do this.		The equivalence of shapes formed by cuts that are curves is difficult to determine but is a good investigation in itself. The equivalence of shapes that are formed by cuts that are combinations of straight line segments is easier to determine.
Activity 2	Hand out the following problem, read with class, and allow students a couple of moments to think about it.		

Posing the problem Assessing student understanding of problem	"You Can Eat Your Cake and Have It, Too!" *The baker is conducting a second contest, this time for his employees. As part of a new promotional campaign for his store, each day he wants to feature sheet cakes that have been cut into four pieces in a different way. The pieces do not have to be equal for this promotion. The baker has challenged his employees to suggest interesting ways to cut the cakes into four pieces. The employees must also determine the price for each piece. The bakery sells 1/8 of a sheet cake for $1.59. What are some of the different ways the cakes can be cut, and how much should each piece cost?* Some questions to ask students. ▪ What are the restrictions on the ways the cakes are cut? ▪ How can we determine the fractional parts of the pieces we cut? ▪ What will happen if we just cut four pieces at random?		Answers to questions-- Each cake must be cut into four pieces, not necessarily equal-sized pieces. We can use equivalences we know, such as 2/4 is equivalent to 1/2, to find the value of each part, if we partition by finding parts of parts. It will be difficult to determine the size and therefore the cost of each piece. This is a very important idea to stress. Students need to be able to calculate the actual fractional part of each piece. Grid paper might help with this.

Students problem solving	Hand out the Cakes Cut Into Fourths Activity Sheet and allow time to work.	Students may remember some of the ways they cut the cakes into eighths in the first activity, which may help them. Students may have difficulty determining the fractional part when shapes are not evenly partitioned. Although we have reminded them that they will have to calculate the fraction of the cake, some may still cut pieces that are not easily used with eighths.	Remind students to label the pieces with the fractional pieces and the cost. If this happens, you may have to remind them about common denominators.
Discussion of Activity 2	Have students place their designs on the chalkboard or the overhead projector. Be sure to include the fractions and cost of each piece. Be sure to have students explain their process in determining the fraction and cost of each piece.		*Size* / *Cost* 1/8 — $1.59 2/8 = 1/4 — 3.18 3/8 — 4.77 4/8 = 1/2 — 6.36 5/8 — 7.95 6/8 = ¾ — 9.54 7/8 — 11.13 8/8 = 1 — 12.72
Summary Discussion	Have the students calculate the sum of their parts and the total cost of their cake and compare with another group. Why did we all come up with the same total cost, but we may not all have the same sized pieces?	If all calculations are done correctly, they should come up with 8/8 (or an equivalent) and a total cost of $12.72.	Emphasize the concept of parts to whole and equivalency. We all had the same cake to start with, so when we add our four pieces together, we should all have the same total cost.

Making Four Pieces

NAME _____

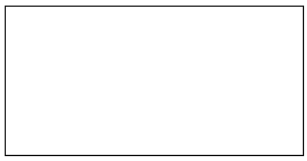
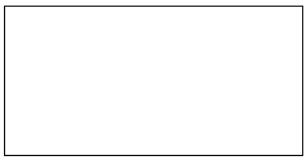


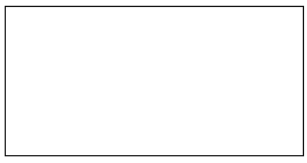
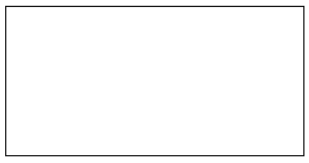
Cakes Cut Into Fourths

NAME _____

Using the rectangles below, show how you would cut each of the "cakes" into four pieces. Each piece **does not** have to have the same area as the other pieces in that cake. If you need to, you may use a piece of printer paper to practice cutting before making your drawings below.

Once you have made your cuts, find out the cost of each piece of cake, assuming the bakery sells 1/8 of a sheet cake for $1.59.

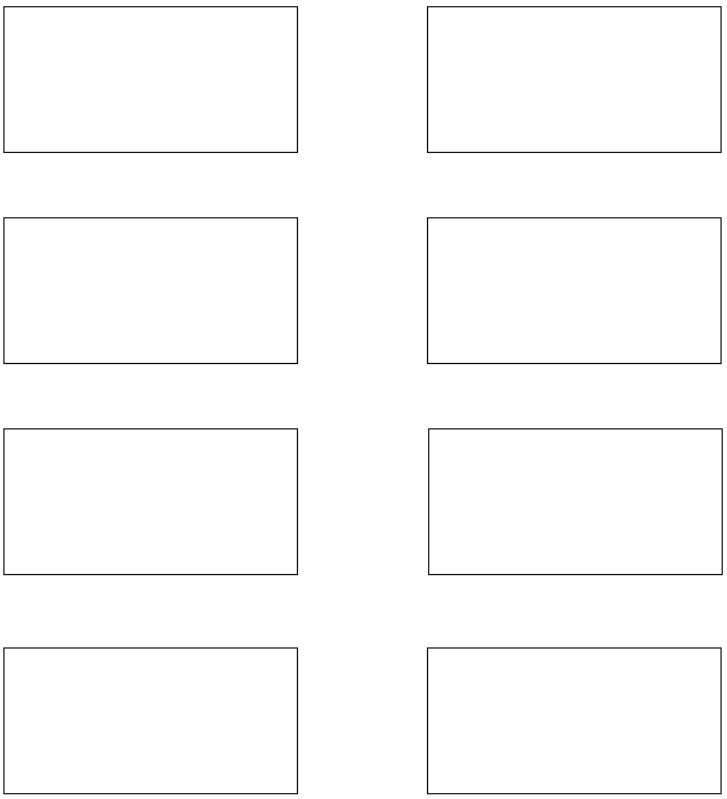



Cakes Cut Into Eighths

NAME _____

Using the rectangles below, show how you would cut each of the rectangular "cakes" into eight pieces. All pieces should have the same area. If you need to, you may use a blank piece of paper to practice cutting before making your drawings below.



Lesson Study Observation Guidelines
Lesson #3 on Understanding Rational Numbers and Proportions
(This page is for use by those who observe the selected lesson as it is taught. The focus of this observation is on the student learning, not an evaluation of the teacher.)
As you observe, keep the following questions in mind and make notes about what occurs. Please be strictly an observer and do not interact with the students during the lesson.
Our Research Goal: *Meaningful mathematics discourse (math talk) in the classroom*
Content goal:
Students will use real-world models to develop and deepen an understanding of the "parts to whole" relationship.

Effectiveness of the Lesson:

A. As you observe, note examples of how the lesson contributes toward the following goals:
   1. Mathematical ideas are communicated clearly and effectively?

   2. Students are developing into flexible problem solvers?

   3. Students are given the opportunity to describe, analyze, and make generalizations about the "parts to whole" relationship?

   4. Students have the opportunity to describe, analyze, and make generalizations about equivalent fractions?

B. What instructional strategies/activities were most effective and why?

Student engagement:

1. To what extent do students appear to be engaged in the lesson? How are they engaged? (raising hands, answering questions, explaining their thinking, etc.)

2. Are students' comments only given in response to the teacher's questions and comments, or do students respond and elaborate upon other students' comments as well? Give examples.

General comments

1. What would you have done differently if you were to teach this lesson?

2. How does this lesson compare to others you have taught or observed on the topic of equivalent fractions and the “part to whole” relationship?

Good night math friends, it's time to go
Good night math friends, it's time to go,
We hate to leave you but it's time to say
fractions are the way to go.