Patterns, Functions, and Algebra For Elementary School Teachers

A Professional Development Training Program to Implement the 2001 Virginia Standards of Learning

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Introduction

The *Patterns, Functions and Algebra for Elementary School Teachers* is a staff development training program designed to assist teachers in implementing the Virginia Standards of Learning for mathematics. This staff development program provides a sample of meaningful and engaging activities correlated to the Patterns, Functions and Algebra strand of the grades K-5 Mathematics *Standards of Learning*.

The purpose of the staff development program is to enhance teachers' content knowledge and their use of instructional strategies for teaching in the patterns, functions and algebra strand of the K-5 Mathematics *Standards of Learning*. Teachers will receive intensive training in ways to develop student understanding of patterning, functional relationships and the foundations of algebraic thinking. Through explorations, problem solving, and hands-on experiences, teachers will engage in discussions and strategies to guide instruction and classroom assessment. Elementary teachers will work to develop techniques to help students:

- recognize, construct, extend, create, analyze, generalize, and describe patterns;
- use pattern-based thinking to understand and represent mathematical and real-world phenomena;
- develop categorization and classification skills;
- determine mathematical rules and develop an understanding of functional relationships;
- use tables, rules, variables, open sentences, and graphs to describe patterns and other relationships;
- model real world situations by representing data in tables, pictures, graphs, open sentences, equations or inequalities, rules, and functions;
- develop strategies for evaluating expressions and finding the solution to equations and inequalities; and
- form and verify generalizations based on observations of patterns and relationships.

Through these activities, it is anticipated that teachers will develop new techniques that are sure to enhance student achievement in their classroom.

Designed to be presented by teacher trainers, this staff development program includes directions for the trainer, as well as the black line masters for overhead transparencies and handouts. In some instances, related student activities are included. Trainers should adapt the materials to best fit the needs of their audience, adding materials that may be more appropriate for their audience and eliminating materials that have been used in previous training sessions. Trainers are encouraged to use technology, as appropriate. All materials in this document may be duplicated and distributed as desired for use in Virginia.

The training programs are organized into five three-hour modules that may be offered by school divisions for recertification points or for a one-credit graduate course, when university credit can be arranged.

The *Patterns, Functions and Algebra for Elementary School Teachers* training program is being provided to school divisions through an appropriation from the General Assembly and in accordance with the Virginia Department of Education's responsibility to develop and pilot model teacher, principal, and superintendent training activities geared to the Standards of Learning content and assessments, and to technology applications.

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GLOSSARY

Additive Inverse	A number's opposite. When the number and it's opposite are added together the sum is zero. Example: 2 + (-2) = 0.
Arithmetic sequence	A sequence where the difference between consecutive terms is always the same. Example: 3, 6, 9, ...
Common factor	If a number is a factor of two or more numbers, it is a common factor of that set of numbers.
Common multiple	A number that is a multiple of each of two or more given numbers. Example: 24 is a common multiple of 4 and 3.
Composite number	A whole number greater than 1 that has more than two factors.
Constant	A quantity whose value does not change.
Cube (in numeration)	The third power of a number. Example: 8^3 is read "Eight to the third power."
Equation	A mathematical statement in which two expressions are equal. Example: x - 10 = 6
Exponent	A number telling how many times the base is used as a factor. Example: $8^3 = 8 \times 8 \times 8$, where 3 is the exponent and 8 is the base.
Expression	A mathematical phrase made up of variables and/or numbers and symbols. Example: 3x + 4
Factor	A whole number that divides another whole number without leaving a remainder. Example: 8 is a factor of 48.
Formula	A rule showing relationships among quantities. Example: A=bh
Function	A rule that matches two sets of numbers such that for each first number there is only one possible second number according to the rule.
Geometric Sequence	A sequence where the ratio between consecutive terms is always the same. Example: 3, 6, 12, ...
Greatest Common Factor (GCF)	The largest factor two numbers have in common. Example: 6 is the GCF of 24 and 18.
Inequality	A statement that two expressions are not equal. Example: $x + 2 \geq 6$

Inverse operations	Operations that “undo” each other, such as addition and subtraction.
Least Common Denominator (LCD)	The least common multiple (LCM) of two or more denominators.
Least Common Multiple (LCM)	The smallest common multiple of two numbers. Example: 56 is the LCM of 8 and 14.
Multiple	The product of a given number and another whole number. Example: 21 is a multiple of 3 (and 7) because 3 x 7 = 21.
Order of operations	Rules describing the sequence to use in computation: 1) compute within grouping symbols; 2) compute exponents and/or roots; 3) multiply and divide from left to right; 4) add and subtract from left to right.
Perfect square	The square of a whole number.
Positive numbers	Numbers greater than zero.
Power	An exponent.
Prime factorization	Writing a number as a product of prime numbers. Example: 30 = 2 x 3 x 5.
Prime number	A whole number greater than 1 whose only factors are 1 and itself. The first five primes are 2, 3, 5, 7, and 11.
Proportion	A statement showing two equal ratios.
Radical sign	$\sqrt{\ }$ used to represent a square root.
Reciprocals	Two numbers whose product is 1. Example: 5/7 and 7/5 are reciprocals.
Solutions of an equation or inequality	Values of a variable that make an equation or inequality true.
Solve	To find the solutions of an equation or inequality.
Square root	The length of the side of a square with an area equal to a given number. Example: 6 is the square root of 36 because 6 x 6 = 36.
Substitute	To replace a variable with a known value.
Term	One number in a sequence.
Variable	A quantity whose values may vary.
Whole number	A number in the set (0, 1, 2, 3,…)

Algebraic Thinking and Classification
Session 1

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Algebraic Thinking	Creating and Identifying Patterns (Warm-Up)	3	K.17, K.18, 1.20, 1.21, 2.25, 2.26, 3.24, 3.25, 4.21, 4.22, 5.20, 5.21, 5.22		
	The Big Ideas of Algebra	4	K.17, K.18, 1.20, 1.21, 2.25, 2.26, 3.24, 3.25, 4.21, 4.22, 5.20, 5.21, 5.22	The Big Ideas of Algebraic Thinking, Algebraic Thinking: Making the Connection, Patterns, Functions, and Algebra SOL	
	Why is Algebraic Thinking Important?	9	K.17, K.18, 1.20, 1.21, 2.25, 2.26, 3.24, 3.25, 4.21, 4.22, 5.20, 5.21, 5.22	Patterns, Functions, and Algebra SOL, Why is Algebraic Thinking Important in the K-5 Curriculum?, Information Sheet on Patterns, Functions, and Algebra from NCTM, Algebraic Thinking in the NCTM Standards Summary Sheet	
Classification	What's In the Box? Tibby Warm-Up)	31	K.17, K.18, 1.20, 1.21, 2.25, 3.24, 4.21, 5.20		32 piece set of attribute materials
	Play!	32	K.17, K.18, 1.20, 1.21, 2.25, 3.24, 4.21, , 5.20		32 piece set of attribute materials
	Missing Pieces	33	K.17, 1.20, 2.25, 3.24, 4.21, 5.20		32 piece set of attribute materials
	20 Questions Game	34	4.21, 5.20		32 piece set of attribute materials
	Who Am I? Game	36	K.17, K.18, 1.20, 1.21, 2.25, 3.24, 3.25, 4.21, 5.20,		32 piece set of attribute materials. Sample clue cards

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
	Differences - Train and Games	39	K.17, K.18, 1.20, 1.21, 2.25, 3.24, 4.21, 5.20,	Differences	32 piece set of attribute materials
Classification (continued)	Hidden Number Patterns	42	4.21, 5.20	Hidden Number Pattern Sheet	32 piece set of attribute materials
	Attribute Networks	45	4.21	Attribute Networks	32 piece set of attribute materials
	Two-Loop Problems	47	4.21, 5.20	Attribute Cards	32 piece set of attribute materials

Activity: Creating and Identifying Patterns (Warm-Up)

Format: Small Group

Objectives: Participants will develop a pattern in small groups and present it to the whole group. This activity is designed to allow participants to introduce themselves and for the whole group to determine the pattern shown by each small group.

Related SOL: Patterns, Functions, and Algebra Standards of Learning

Materials: None needed

Time Required: 15 minutes

Directions:

1. Divide participants into small groups of four.
2. Participants should introduce themselves in the small groups and discuss a possible pattern that exists in their group. Examples of possible patterns could be (brown hair, blonde hair, brown hair, blonde hair) or (glasses, no glasses, glasses, no glasses).
3. Ask each small group to stand up so that the pattern that they have selected is visible to the rest of the participants (e.g., brown hair, blonde hair, brown hair, blonde hair). The rest of the participants should make conjectures about what the pattern could be. The small group will then confirm and/or explain their pattern. They should then introduce themselves to the whole group (name, school division, grade level).

Activity: **The Big Ideas of Algebra**

Format: Small Group

Objectives: Participants will discuss the meaning of algebraic thinking and identify big mathematical ideas that constitute algebraic thinking in the K-5 curriculum.

Related SOL: Patterns, Functions, and Algebra Standards of Learning.

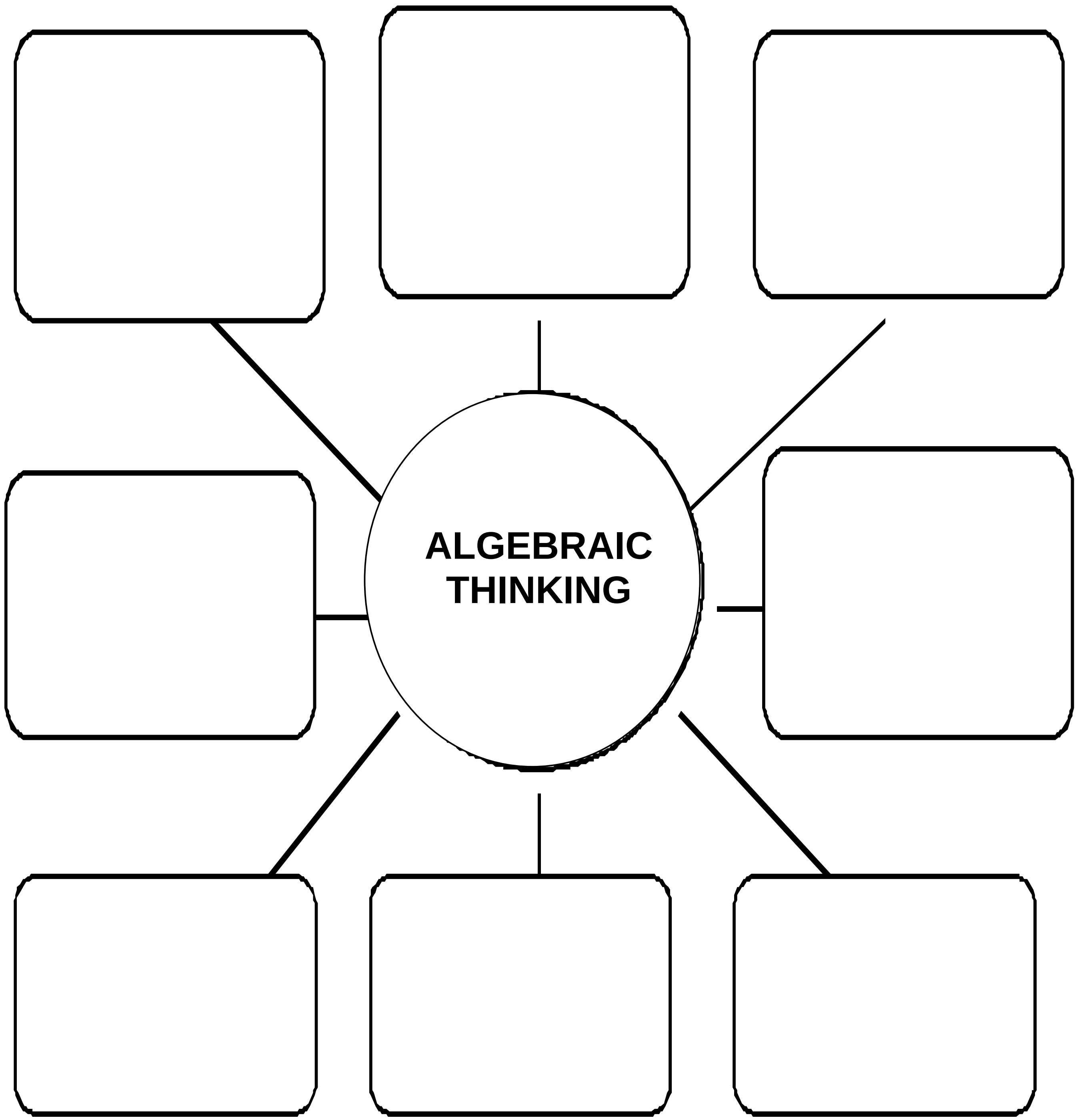
Materials: The Big Ideas of Algebraic Thinking and Algebraic Thinking: Making the Connections Activity Sheets; Standards of Learning Handout

Time Required: 15 minutes

Directions:

1. Divide participants into small groups of four (preferably with different grade levels represented).
2. Pose the question: "When you think of the term algebraic thinking, what do you think of? What are some of the key words in the Standards of Learning that constitute algebraic thinking?" Have participants refer to the Standards of Learning Sheet to look for key words and ideas. Assign a group recorder the task of writing the words the group associates with algebraic thinking on the handout. Give groups about 5 minutes to discuss.
3. Have groups contribute ideas. Record ideas in a webbing fashion to show how ideas connect on the transparency of "The Big Ideas of Algebraic Thinking". Discuss each briefly and give examples.
4. The big ideas should include such things as: patterns, functions, equations, variables, sorting and classifying, proportional reasoning, number relationships, expressions, and graphing.
5. Use the Algebraic Thinking: Making the Connections Activity Sheet to discuss how algebraic thinking is explicitly written in the SOL.

The Big Ideas of Algebraic Thinking



Virginia Standards of Learning

Patterns, Functions and Algebra Strand K-5

K.17 The student will sort and classify objects according to similar attributes (size, shape, and color).

K.18 The student will identify, describe, and extend a repeating relationship (pattern) found in common objects, sounds, and movements.

1.20 The student will sort and classify concrete objects according to one or more attributes, including color, size, shape, and thickness.

1.21 The student will recognize, describe, extend, and create a wide variety of patterns, including rhythmic, color, shape, and numeric. Patterns will include both growing and repeating patterns. Concrete materials and calculators will be used by students.

2.25 The student will identify, create, and extend a wide variety of patterns, using numbers, concrete objects, and pictures.

2.26 The student will solve problems by completing a numerical sentence involving the basic facts for addition and subtraction. Examples include: 3 + __ = 7, or 9 - __ = 2. Students will create story problems using the numerical sentences.

3.24 The student will recognize and describe a variety of patterns formed using concrete objects, numbers, tables, and pictures, and extend the pattern, using the same or different forms (concrete objects, numbers, tables, and pictures).

3.25 The student will a) investigate and create patterns involving numbers, operations (addition and multiplication), and relations that model the identity and commutative properties for addition and multiplication; and b) demonstrate an understanding of equality by recognizing that the equal sign (=) links equivalent quantities, such as $4 \cdot 3 = 2 \cdot 6$.

4.21 The student will recognize, create, and extend numerical and geometric patterns, using concrete materials, number lines, symbols, tables, and words.

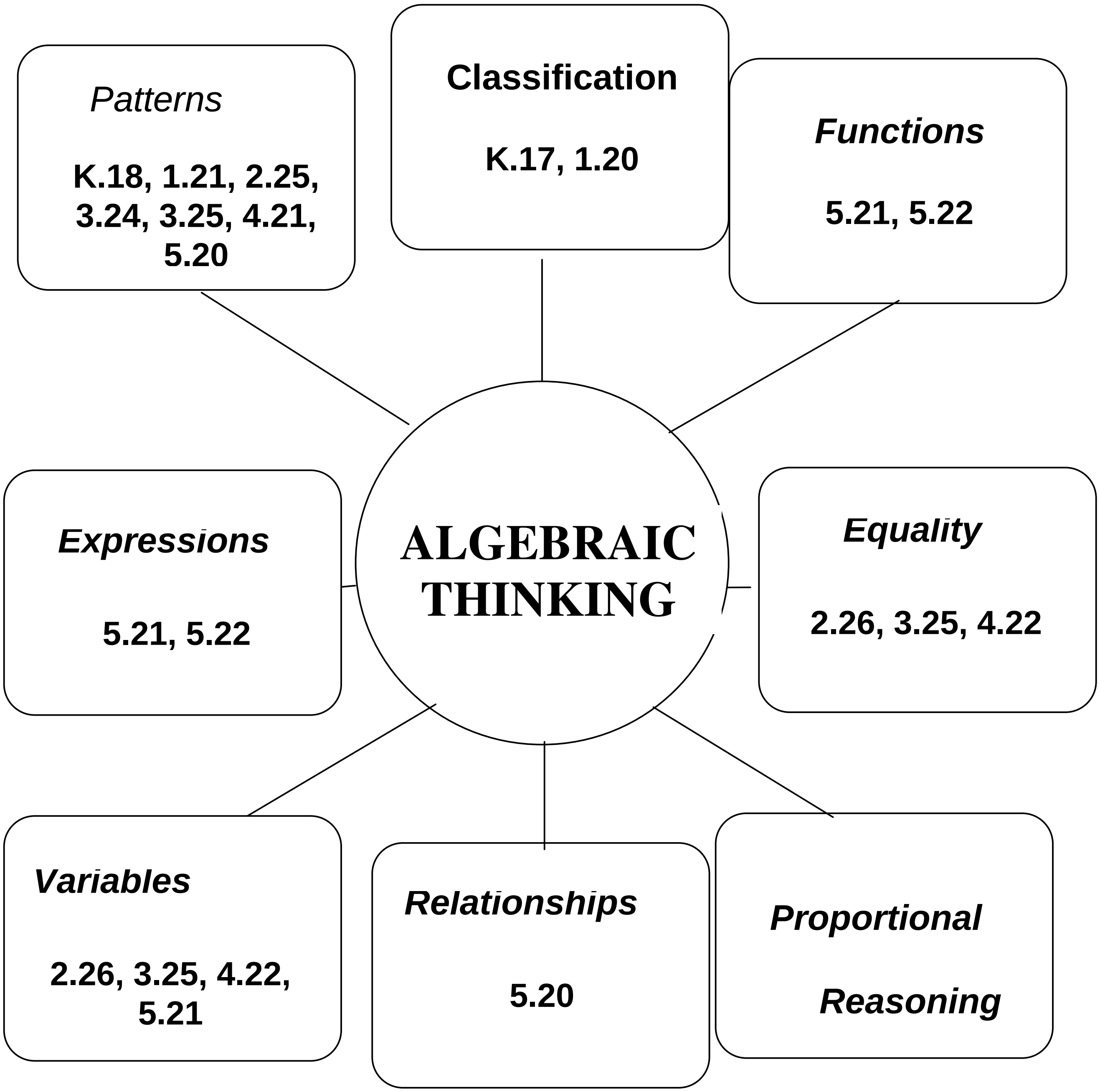
4.22 The student will recognize and demonstrate the meaning of equality, using symbols representing numbers, operations, and relations [e.g., 3 + 5 = 5 + 3 and 15 + (35 + 16) = (15 + 35) + 16].

5.20 The student will analyze the structure of numerical and geometric patterns (how they change or grow) and express the relationship, using words, tables, graphs, or a mathematical sentence. Concrete materials and calculators will be used.

5.21 The student will
- a) investigate and describe the concept of variable;
- b) use a variable expression to represent a given verbal quantitative expression, involving one operation; and
- c) write an open sentence to represent a given mathematical relationship, using a variable.

5.22 The student will create a problem situation based on a given open sentence using a single variable.

Algebraic Thinking: Making the Connections



Activity: **Why is Algebraic Thinking Important?**

Format: Whole group, Mini-Lecture

Objectives: Participants will understand the rationale for including algebraic topics in the K-5 curriculum by discussing how the big ideas of algebraic thinking are connected with the NCTM *Principles and Standards for School Mathematics.*

Related SOL: Patterns, Functions, and Algebra Standards of Learning

Materials: Why is Algebraic Thinking Important in the K-5 Curriculum? Activity Sheet, Algebraic Thinking in the NCTM *Principles and Standards for School Mathematics* Activity Sheet, copies for each participant of NCTM resource for discussion, "Why is Algebraic Thinking Important in the K-5 Curriculum?"

Time Required: 15 minutes

Directions:

1. Distribute copies of the NCTM resource for discussion, "Why is Algebraic Thinking Important in the K-5 Curriculum? ". Ask participants to review the information. Use the transparency of "Why is Algebraic Thinking Important in the K-5 Curriculum?" to guide the discussion. Discuss each of the bullets on the transparency.
2. Discuss the National Council of Teachers of Mathematics organization and the 2000 Principles and Standards for School Mathematics document. Discuss how algebraic thinking is emphasized in the Standards and how the Virginia SOL reflect the recommendations made in the document. Use the transparencies of "Algebraic Thinking in the NCTM Principles and Standards for School Mathematics".

Resource for Discussion from the NCTM *Principles and Standards for School Mathematics* on "Why is Algebraic Thinking Important in the K-5 Curriculum?"

NCTM Standard 2: Patterns, Functions, and Algebra
Instructional programs from prekindergarten through grade 12 should enable all students to—

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

Elaboration: Grades Pre-K-2
Algebraic concepts can evolve and continue to develop during prekindergarten through grade 2. They will be manifested through work with classification, patterns and relations, operations with whole numbers, explorations of function, and step-by-step processes. Although the concepts discussed in this Standard are algebraic, this does not mean that students in the early grades are going to deal with the symbolism often taught in a traditional high school algebra course.

Even before formal schooling, children develop beginning concepts related to patterns, functions, and algebra. They learn repetitive songs, rhythmic chants, and predictive poems that are based on repeating and growing patterns. The recognition, comparison, and analysis of patterns are important components of a student's intellectual development. When students notice that operations seem to have particular properties, they are beginning to think algebraically. For example, they realize that changing the order in which two numbers are added does not change the result or that adding zero to a number leaves that number unchanged. Students' observations and discussions of how quantities relate to one another lead to initial experiences with function relationships, and their representations of mathematical situations using concrete objects, pictures, and symbols are the beginnings of mathematical modeling. Many of the step-by-step processes that students use form the basis of understanding iteration and recursion.

Standard 2 Focus Areas for Grades Pre-K-2

Understand patterns, relations, and functions
In grades preK-2, all students should:

- sort, classify, and order objects by size, number, and other properties;
- recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another;

- analyze how both repeating and growing patterns are generated.

Represent and analyze mathematical situations and structures using algebraic symbols
In grades preK-2, all students should

- illustrate general principles and properties of operations, such as commutativity, using specific numbers;
- use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations.

Use mathematical models to represent and understand quantitative relationships
In grades preK-2, all students should

- model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols

Analyze change in various contexts
In grades preK-2, all students should

- describe qualitative change, such as a student's growing taller;
- describe quantitative change, such as a student's growing two inches in one year.

Discussion
Algebraic concepts can evolve and continue to develop during prekindergarten through grade 2. They will be manifested through work with classification, patterns and relations, operations with whole numbers, explorations of function, and step-by-step processes. Although the concepts discussed in this Standard are algebraic, this does not mean that students in the early grades are going to deal with the symbolism often taught in a traditional high school algebra course.

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Understand patterns, relations, and functions
Sorting, classifying, and ordering facilitate work with patterns, geometric shapes, and data. Given a package of assorted stickers, children quickly notice many differences among the items. They can sort the stickers into groups having similar traits such as color, size, or design and order them from smallest to largest. Caregivers and teachers should elicit from children the criteria they are using as they sort and group objects. Patterns are a way for young students to recognize order and to organize their world and are important in all aspects of mathematics at this level. Preschoolers recognize patterns in their environment and, through experiences in school, should become more skilled in noticing patterns in arrangements of objects, shapes, and numbers and in using patterns to predict what comes next in an arrangement. Students know, for example, that "first comes breakfast, then school," and "Monday we go to art, Tuesday we go to music." Students who see the digits "0, 1, 2, 3, 4, 5, 6, 7, 8, 9" repeated over and over will see a pattern that helps them learn to count to 100—a formidable task for students who do not recognize the pattern.

Teachers should help students develop the ability to form generalizations by asking such questions as "How could you describe this pattern?" or "How can it be repeated or extended?" or "How are these patterns alike?" For example, students should recognize that the color pattern "blue, blue, red, blue, blue, red" is the same in form as "clap, clap, step, clap, clap, step." This recognition lays the foundation for the idea that two very different situations can have the same mathematical » features and thus are the same in some important ways. Knowing that each pattern above could be described as having the form AABAAB is for students an early introduction to the power of algebra.

By encouraging students to explore and model relationships using language and notation that is meaningful for them, teachers can help students see different relationships and make conjectures and generalizations from their experiences with numbers. Teachers can, for instance, deepen students' understanding of numbers by asking them to model the same quantity in many ways—for example, eighteen is nine groups of two, 1 ten and 8 ones, three groups of six, or six groups of three. Pairing counting numbers with a repeating pattern of objects can create a function (see fig. 4.7) that teachers can explore with students: What is the second shape? To continue the pattern, what shape comes next? What number comes next when you are counting? What do you notice about the numbers that are beneath the triangles? What shape would 14 be?

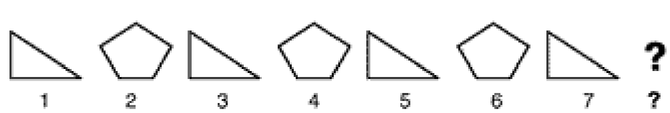


Fig. **4.7.** Pairing counting numbers with a repeating pattern

Students should learn to solve problems by identifying specific processes. For example, when students are skip-counting three, six, nine, twelve, ..., one way to obtain the next term is to add three to the previous number. Students can use a similar process to compute how much to pay for seven balloons if one balloon costs 20¢. If they recognize the sequence 20, 40, 60, ... and continue to add 20, they can find the cost for seven balloons. Alternatively, students can realize that the total amount to be paid is determined by the number of balloons bought and find a way to compute the total directly. Teachers in grades 1 and 2 should provide experiences for students to learn to use charts and tables for recording and organizing information in varying formats (see figs. 4.8 and 4.9). They also should discuss the different notations for showing amounts of money. (One balloon costs 20¢, or $0.20, and seven balloons cost $1.40.)

Cost of Balloons

Number of Balloons	Cost of Balloons in Cents
1	20
2	40
3	60
4	80
5	?
6	?
7	?

Fig. **4.8.** A vertical chart for recording and organizing information

Cost of Balloons

Number of balloons	1	2	3	4	5	6	7
Cost of balloons in cents	20	40	60	80	?	?	?

Fig. **4.9.** A horizontal chart for recording and organizing information

Skip-counting by different numbers can create a variety of patterns on a hundred chart that students can easily recognize and describe (see fig. 4.10). Teachers can simultaneously use hundred charts to help students learn about number patterns and to assess students' understanding of counting patterns. By asking questions such as "If you count by tens beginning at 36, what number would you color next?" and "If you continued counting by tens, would you color 87?" teachers can observe whether students understand the correspondence between the visual pattern formed by the shaded numbers and the counting pattern. Using a calculator and a hundred chart enables the students to see the same pattern in two different formats.

1	2	**3**	4	5	**6**	7	8	**9**	10
11	**12**	13	14	**15**	16	17	**18**	19	20
21	22	23	**24**	25	26	**27**	28	29	**30**
31	32	**33**	34	35	**36**	37	38	**39**	40
41	**42**	43	44	**45**	46	47	**48**	49	50
51	52	53	**54**	55	56	**57**	58	59	**60**
61	62	**63**	64	65	**66**	67	68	**69**	70
71	**72**	73	74	**75**	76	77	**78**	79	80
81	82	83	**84**	85	86	**87**	88	89	**90**
91	92	**93**	94	95	**96**	97	98	**99**	100

Counting by threes

1	2	3	4	5	**6**	7	8	9	10
11	**12**	13	14	15	16	17	**18**	19	20
21	22	23	**24**	25	26	27	28	29	**30**
31	32	33	34	35	**36**	37	38	39	40
41	**42**	43	44	45	46	47	**48**	49	50
51	52	53	**54**	55	56	57	58	59	**60**
61	62	63	64	65	**66**	67	68	69	70
71	**72**	73	74	75	76	77	**78**	79	80
81	82	83	**84**	85	86	87	88	89	**90**
91	92	93	94	95	**96**	97	98	99	100

Counting by sixes

Fig. **4.10.** Skip-counting on a hundred chart

Represent and analyze mathematical situations and structures using algebraic symbols
Two central themes of algebraic thinking are appropriate for young students. The first involves making generalizations and using symbols to represent mathematical ideas, and the second is representing and solving problems (Carpenter and Levi 1999). For example, adding pairs of numbers in different orders such as 3 + 5 and 5 + 3 can lead students to infer that when two numbers are added, the order does not matter. As students generalize from observations about number and operations, they are forming the basis of algebraic thinking.

Similarly, when students decompose numbers in order to compute, they often use the associative property for the computation. For instance, they may compute 8 + 5, saying, "8 + 2 is 10, and 3 more is 13." Students often discover and make generalizations about other properties. Although it is not necessary to introduce vocabulary such as *commutativity* or *associativity,* teachers must be aware of the algebraic properties used by students at this age. They should build students' understanding of the importance of their observations about mathematical situations and challenge them to investigate whether specific observations and conjectures hold for all cases.

Teachers should take advantage of their observations of students, as illustrated in this story drawn from an experience in a kindergarten class.

> The teacher had prepared two groups of cards for her students. In the first group, the number on the front and back of each card differed by 1. In the second group, these numbers differed by 2.
>
> The teacher showed the students a card with 12 written on it and explained, "On the back of this card, I've written another number." She turned the card over to show the number 13. Then she showed the students a second card with 15 on the front and 16 on the back. » As she continued showing the students the cards, each time she asked the students, "What do you think will be on the back?" Soon the students figured out that she was adding 1 to the number on the front to get the number on the back of the card.
>
> Then the teacher brought out a second set of cards. These were also numbered front and back, but the numbers differed by 2, for example, 33 and 35, 46 and 48, 22 and 24. Again, the teacher showed the students a sample card and continued with other cards, encouraging them to predict what number was on the back of each card. Soon the students figured out that the numbers on the backs of the cards were 2 more than the numbers on the fronts.
>
> When the set of cards was exhausted, the students wanted to play again. "But," said the teacher, "we can't do that until I make another set of cards." One student spoke up, "You don't have to do that, we can just flip the cards over. The cards will all be minus 2."

As a follow-up to the discussion, this teacher could have described what was on each group of cards in a more algebraic manner. The numbers on the backs of the cards in the first group could be named as "front number plus 1" and the second as "front number plus 2." Following the student's suggestion, if the cards in the second group were flipped over, the numbers on the backs could then be described as "front number minus 2." Such activities, together with the discussions and analysis that follow them, build a foundation for understanding the inverse relationship.

Through classroom discussions of different representations during the pre-K–2 years, students should develop an increased ability to use symbols as a means of recording their thinking. In the earliest years, teachers may provide scaffolding for students by writing for them until they have the ability to record their ideas. Original representations remain important throughout the students' mathematical study and should be encouraged. Symbolic representation and manipulation should be embedded in instructional experiences as another vehicle for understanding and making sense of mathematics.

Equality is an important algebraic concept that students must encounter and begin to understand in the lower grades. A common explanation of the equals sign given by students is that "the answer is coming," but they need to recognize that the equals sign indicates a relationship—that the quantities on each side are equivalent, for example, 10 = 4 + 6 or 4 + 6 = 5 + 5. In the later years of this grade band, teachers should provide opportunities for students to make connections from symbolic notation to the representation of the equation. For example, if a student records the addition of four 7s as shown on the left in figure 4.11, the teacher could show a series of additions correctly, as shown on the right, and use a balance and cubes to demonstrate the equalities.

$$7+7=14+7=21+7=28 \qquad \begin{aligned} 7+7&=14\\ 14+7&=21\\ 21+7&=28 \end{aligned}$$

Fig. **4.11.** A student's representation of adding four 7s (left) and a teacher's correct representation of the same addition

Use mathematical models to represent and understand quantitative relationships
Students should learn to make models to represent and solve problems. For example, a teacher may pose the following problem:

> There are six chairs and stools. The chairs have four legs and the stools have three legs. All together, there are 20 legs. How many chairs and how many stools are there?

One student may represent the situation by drawing six circles and then putting tallies inside to represent the number of legs. Another student may represent the situation by using symbols, making a first guess that the number of stools and chairs is the same and adding 3 + 3 + 3 + 4 + 4 + 4. Realizing that the sum is too large, the student might adjust the number of chairs and stools so that the sum of their legs is 20.

Analyze change in various contexts
Change is an important idea that students encounter early on. When students measure something over time, they can describe change both qualitatively (e.g., "Today is colder than yesterday") and quantitatively (e.g., "I am two inches taller than I was a year ago"). Some changes are predictable. For instance, students grow taller, not shorter, as they get older. The understanding that most things change over time, that many such changes can be described mathematically, and that many changes are predictable helps lay a foundation for applying mathematics to other fields and for understanding the world.

Resource for Discussion from the NCTM *Principles and Standards for School Mathematics* on "Why is Algebraic Thinking Important in the K-5 Curriculum?"

NCTM Standard 2: Patterns, Functions, and Algebra
Instructional programs from prekindergarten through grade 12 should enable all students to—

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

Elaboration: Grades 3-5
Although *algebra* is a word that has not commonly been heard in grades 3–5 classrooms, the mathematical investigations and conversations of students in these grades frequently include elements of algebraic reasoning. These experiences and conversations provide rich contexts for advancing mathematical understanding and are also an important precursor to the more formalized study of algebra in the middle and secondary grades. In grades 3–5, algebraic ideas should emerge and be investigated as students—

- identify or build numerical and geometric patterns;
- describe patterns verbally and represent them with tables or symbols;
- look for and apply relationships between varying quantities to make predictions;
- make and explain generalizations that seem to always work in particular situations;
- use graphs to describe patterns and make predictions;
- explore number properties;
- use invented notation, standard symbols, and variables to express a pattern, generalization, or situation.

Focus Areas for Grades 3-5

Understand patterns, relations, and functions
In grades 3-5, all students should

- describe, extend, and make generalizations about geometric and numeric patterns;
- represent and analyze patterns and functions, using words, tables, and graph

Represent and analyze mathematical situations and structures using algebraic symbols

In grades 3-5, all students should

- model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.
- identify such properties as commutativity, associativity, and distributivity, and use them to compute with whole numbers;
- represent the idea of a variable as an unknown quantity using a letter or a symbol;
- express mathematical relationships using equations.

Use mathematical models to represent and understand quantitative relationships

In grades 3-5, all students should-

- model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.

Analyze change in various contexts

In grades 3-5, all students should-

- investigate how a change in one variable relates to a change in a second variable;
- identify and describe situations with constant or varying rates of change and compare them.

Discussion

Understand patterns, relations, and functions

In grades 3–5, students should investigate numerical and geometric patterns and express them mathematically in words or symbols. They should analyze the structure of the pattern and how it grows or changes, organize this information systematically, and use their analysis to develop generalizations about the mathematical relationships in the pattern. For example, a teacher might ask students to describe patterns they see in the "growing squares" display (see fig. 5.3) and express the patterns in mathematical sentences. Students should be encouraged to explain these patterns verbally and to make predictions about what will happen if the sequence is continued.

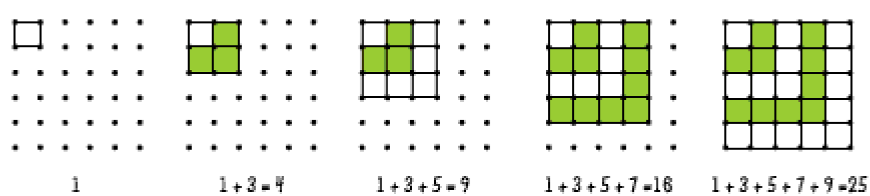


Fig. **5.3.** Expressing "growing squares" in mathematical sentences (Adapted from Burton et al. 1992, p. 6)

In this example, one student might notice that the area changes in a predictable way—it increases by the next odd number with each new square. Another student might notice that the previous square always fits into the "corner" of the next-larger square. This observation might lead to a description of the area of a square as equal to the area of the previous square plus "its two sides and one more." A student might represent his thinking as in figure 5.4.»

Area of a 5x5 square = area of a 4x4 square +4+4+1

Fig. **5.4.** A possible student observation about the area of the 5 × 5 square in the "growing squares" pattern

Examples like this one give the teacher important opportunities to engage students in thinking about how to articulate and express a generalization—"How can we talk about how this pattern works for a square of any size?" Students in grade 3 should be able to predict the next element in a sequence by examining a specific set of examples. By the end of fifth grade, students should be able to make generalizations by reasoning about the structure of the pattern. For example, a fifth-grade student might explain that "if you add the first n odd numbers, the sum is the same as n × n."

As they study ways to measure geometric objects, students will have opportunities to make generalizations based on patterns. For example, consider the problem in figure 5.5. Fourth graders might make a table (see fig. 5.6) and note the iterative nature of the pattern. That is, there is a consistent relationship between the surface area of one tower and the next-bigger tower: "You add four to the previous number." Fifth graders could be challenged to justify a general rule with reference to the geometric model, for example, "The surface area is always four times the number of cubes plus two more because there are always four square units around each cube and one extra on each end of the tower." Once a relationship is established, students should be able to use it to answer questions like, "What is the surface area of a tower with fifty cubes?" or "How many cubes would there be in a tower with a surface area of 242 square units?"

What is the surface area of each tower of cubes (include the bottom)?
As the towers get taller, how does the surface area change?

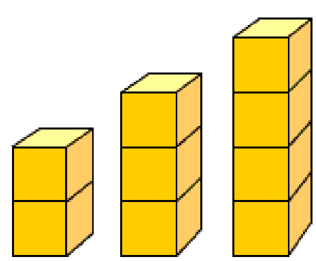


Fig. **5.5.** Finding surface areas of towers of cubes

Number of Cubes (N)	Surface Area in Square Units (S)
1	6
2	10
3	14
4	18

Fig. 5.6**.** A table used in the "tower of cubes" problem

Represent and analyze mathematical situations and structures using algebraic symbols
In grades 3–5, students can investigate properties such as commutativity, associativity, and distributivity of multiplication over addition. Is 3×5 the same as 5×3? Is 15×27 equal to 27×15? Will reversing the factors always result in the same product? What if one of the factors is a decimal number (e.g., 1.5×6)? An area model can help students see that two factors in either order have equal products, as represented by congruent rectangles with different orientations (see fig. 5.7). »

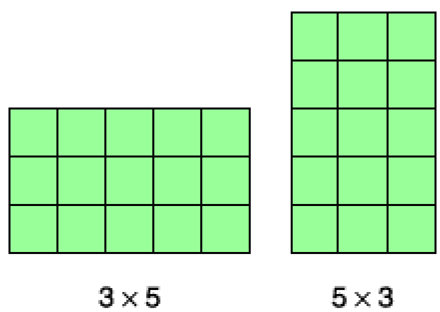


Fig. **5.7.** Area models illustrating the commutative property of multiplication

An area model can also be used to investigate the distributive property. For example, the representation in figure 5.8 shows how 8×14 can be decomposed into 8×10 and 8×4.

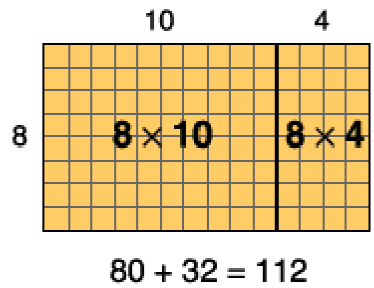


Fig. **5.8.** Area model showing the distributive property of multiplication

As students learn about the meaning of multiplication and develop strategies to solve multiplication problems, they will begin to use properties such as distributivity naturally (Schifter 1999). However, discussion about the properties themselves, as well as how they serve as tools for solving a range of problems, is important if students are to add strength to their intuitive notions and advance their understanding of multiplicative structures. For example, students might explore questions such as these: Why can't 24×32 be solved by adding the results of 20×30 and 4×2? If a number is tripled, then tripled again, what is the relationship of the result to the original number? Analyzing the properties of operations gives students opportunities to extend their thinking and to build a foundation for applying these understandings to other situations.

At this grade band the idea and usefulness of a variable (represented by a box, letter, or symbol) should also be emerging and developing more fully. As students explore patterns and note relationships, they should be encouraged to represent their thinking. In the example showing the sequence of squares that grow (fig. 5.3), students are beginning to use the idea of a variable as they think about how to describe a rule for finding the area of any square from the pattern they have observed. As students become more experienced in investigating, articulating, and justifying generalizations, they can begin to use variable notation and equations to represent their thinking. Teachers will need to model how to represent thinking in the form of equations. In this way, they can » help students connect the ways they are describing their findings to mathematical notation. For example, a student's description of the surface area of a cube tower of any size ("You get the surface area by multiplying the number of cubes by 4 and adding 2") can be recorded by the teacher as $S = 4 \times n + 2$. Students should also understand the use of a variable as a placeholder in an expression or equation. For example, they should explore the role of n in the equation $80 \times 15 = 40 \times n$ and be able to find the value of n that makes the equation true.

Use mathematical models to represent and understand quantitative relationships
Historically, much of the mathematics used today was developed to model real-world situations, with the goal of making predictions about those situations. As patterns are identified, they can be expressed numerically, graphically, or symbolically and used to

predict how the pattern will continue. Students in grades 3–5 develop the idea that a mathematical model has both descriptive and predictive power.

Students in these grades can model a variety of situations, including geometric patterns, real-world situations, and scientific experiments. Sometimes they will use their model to predict the next element in a pattern, as students did when they described the area of a square in terms of the previous smaller square (see fig. 5.3). At other times, students will be able to make a general statement about how one variable is related to another variable: If a sandwich costs $3, you can figure out how many dollars any number of sandwiches costs by multiplying that number by 3 (two sandwiches cost $6, three sandwiches cost $9, and so forth). In this case, students have developed a model of a proportional relationship: the value of one variable (total cost, C) is always three times the value of the other (number of sandwiches, S), or $C = 3 \cdot S$.

In modeling situations that involve real-world data, students need to know that their predictions will not always match observed outcomes for a variety of reasons. For example, data often contain measurement error, experiments are influenced by many factors that cause fluctuations, and some models may hold only for a certain range of values. However, predictions based on good models should be reasonably close to what actually happens.

Students in grades 3–5 should begin to understand that different models for the same situation can give the same results. For example, as a group of students investigates the relationship between the number of cubes in a tower and its surface area, several models emerge. One student thinks about each side of the tower as having the same number of units of surface area as the number of cubes (n). There are four sides and an extra unit on each end of the tower, so the surface area is four times the number of cubes plus two ($4 \cdot n + 2$). Another student thinks about how much surface area is contributed by *each* cube in the tower: each end cube contributes five units of surface area and each "middle" cube contributes four units of surface area. Algebraically, the surface area would be $2 \cdot 5 + (n - 2) \cdot 4$. For a tower of twelve cubes, the first student thinks, "4 times 12, that's 48, plus 2 is 50." The second student thinks, "The two end cubes each have 5, so that's 10. There are 10 » more cubes. They each have 4, so that's 40. 40 plus 10 is 50." Students in this grade band may not be able to show how these solutions are algebraically equivalent, but they can recognize that these different models lead to the same solution.

Analyze change in various contexts

Change is an important mathematical idea that can be studied using the tools of algebra. For example, as part of a science project, students might plant seeds and record the growth of a plant. Using the data represented in the table and graph (fig. 5.9), students can describe how the rate of growth varies over time. For example, a student might express the rate of growth in this way: "My plant didn't grow for the first four days, then it grew slowly for the next two days, then it started to grow faster, then it slowed down again." In this situation, students are focusing not simply on the height of the plant each day but on what has happened between the recorded heights. This work is a precursor to later, more focused attention on what the slope of a line represents, that is, what the steepness of the line shows about the rate of change. Students should have opportunities to study situations that display different patterns of change—change that occurs at a constant rate, such as someone walking at a constant speed, and rates of change that increase or decrease, as in the growing-plant example.

Time (days)	Height (cm)	Change (cm)
0	0	
2	0	0
4	0	0
6	1	1
8	2	1
10	4	2
12	6	2
14	7.5	1.5
16	8.5	1
18	8.5	0
20	9	0.5

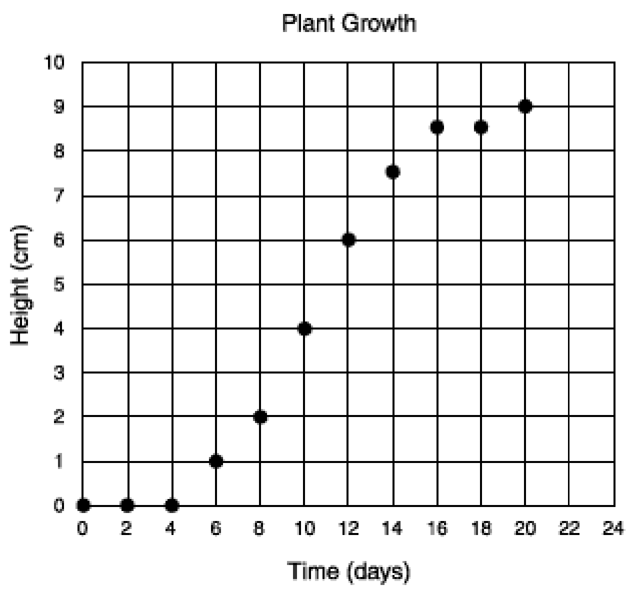


Fig. **5.9.** A table and graph showing growth of a plant

Reference: *Principles and Standards for School Mathematics* Electronic, 2000, National Council of Teachers of Mathematics

Why is Algebraic Thinking Important in the K-5 Curriculum?

1. **Algebraic thinking** provides the foundation for students to move beyond the specific computational skills to the general manipulation of symbols.
   Examples:
   $3 + \square = 7$ $\qquad$ $3 + x = 7$

   $2(3 + 5) = 2(3) + 2(5)$ $\qquad$ $2(a + b) = 2(a) +2(b)$

2. **Algebraic thinking** is looking for, expecting, and understanding patterns.
   a. Repeating and growing patterns can be found in numbers.
      Find and continue the pattern:
      1, 2, 1, 2, ___, ___, ___
      1, 3, 5, 9, ___, ___, ___
      1, 3, 6, 10, 15, 21, ___, ___, ___

   b. Patterns are a vehicle that enables children to make sense of the world around them.
      Examples:
      Rhythmic patterns, seasons, phases of the moon

   c. Using patterns to solve problems is an extremely powerful tool for making connections in mathematics.
      Examples:
      Doubles: 2 + 2 = 4
      Doubles Plus One: 2 + 3 = 5
      Skip Counting 3, 6, 9, 12, 15, 18, ...
      Function Machines

□	Δ
3	4
4	5
5	6

function: $\square + 1 = \Delta$

3. **Algebraic thinking** is fundamental to functioning in business, industry, science, technology and daily life.
   Daily Life Example:
   Determining the cost for a McDonald's fast food order
   3 Cheeseburger + 2 fries + 2 soda+ 1 shake= ?
   3 C + 2F + 2 S + 1 K = 3 (.99) + 2 (.89) + 2 (.79) + 1 (.89) = ?
   Second Example:
   Best Buys 5 for $3.00 or 3 for $2.00

Technology Example:
Computer Spreadsheets - Models use algebraic reasoning and symbolic notation

4. **Algebraic thinking** is the gatekeeper for higher mathematics and science courses.
   Example:
   - Algebra is a prerequisite to other high school mathematics courses
   - Success in Algebra is tied to gradation requirements
   - The foundation for algebraic thinking must begin in Kindergarten.

5. **Algebraic thinking** is a critical filter for employment and advanced training.
   Example:
   - More than 75% of all jobs require proficiency in fundamental algebraic concepts.

Algebraic Thinking in the NCTM *Standards*

In grades K-4, the mathematics curriculum should include the study of patterns and relationships so that students can:

- recognize, describe, extend, and create a wide variety of patterns;
- represent and describe mathematical relationships;
- explore the use of variables and open sentences to express relationships.

In grades 5-8, the mathematics curriculum should include explorations of patterns and functions so that students can:

- describe, extend, analyze, and create a wide variety of patterns;
- describe and represent relationships with tables, graphs, and rules;
- analyze functional relationships to explain how a change in one quantity results in a change in another;
- use patterns and functions to represent and solve problems.

In grades 5-8, the mathematics curriculum should include explorations of algebraic concepts and processes so that students can:

- understand the concepts of variable, expression, and equation;
- represent situations and number patterns with tables, graphs, verbal rules, and equations and explore the interrelationships of these representations;
- analyze tables and graphs to identify properties and relationships;
- develop confidence in solving linear equations using concrete, informal, and formal methods;
- investigate inequalities and nonlinear equations informally;
- apply algebraic methods to solve a variety of real-world and mathematical problems.

Overview of Classification

Key Idea: Classification Using Attribute Block Materials

Description: Participants will explore the concept of classifying which is a basic process of mathematical thinking that is essential to many concepts that are developed in the grades K - 5 mathematics curriculum. Classification involves the understanding of relationships. Classification activities, which require observing likenesses and differences can be presented through problem solving situations and provide students with the opportunity to develop logical reasoning abilities. Logical reasoning skills and especially the meaningful use of the language of logic (e.g., if-then, and, or, not, all, some, etc.) are valuable across all areas of mathematics. An understanding of classification, or the recognition of the various attributes of items, is also an essential skill to patterning (e.g., extending, exploring, and creating patterns or sequences). These classification skills can be taught through a variety of materials (e.g., collections of leaves, buttons, etc), however, attribute pieces will be the manipulative used for this session.

Attribute Materials: *Attribute materials* are sets of objects that lend themselves to being sorted and classified in different ways. Natural or *unstructured* attribute materials include such things as seashells, leaves, the children themselves, or the set of the children's shoes. The *attributes* refer to the characteristic, quality or trait of the item and are the means by which materials can be sorted. For example, hair color, height, and gender are attributes of children. Each attribute has a number of different *values*: for example, blond, brown or red (for the attribute of hair color), tall or short (for height), male or female (for gender).

A *structured* set of attribute pieces has exactly one piece for every possible combination of values for each attribute. For example, several commercial sets of plastic attribute materials have four attributes: color (red, yellow, blue), shape (circle, triangle, rectangle, square, hexagon), size (big, little), and thickness (thick, thin). In the set just described there is exactly one large, red, thin, triangle as well as one each of all other combinations. The specific values, number of values, or number of attributes that a set may have is not important.

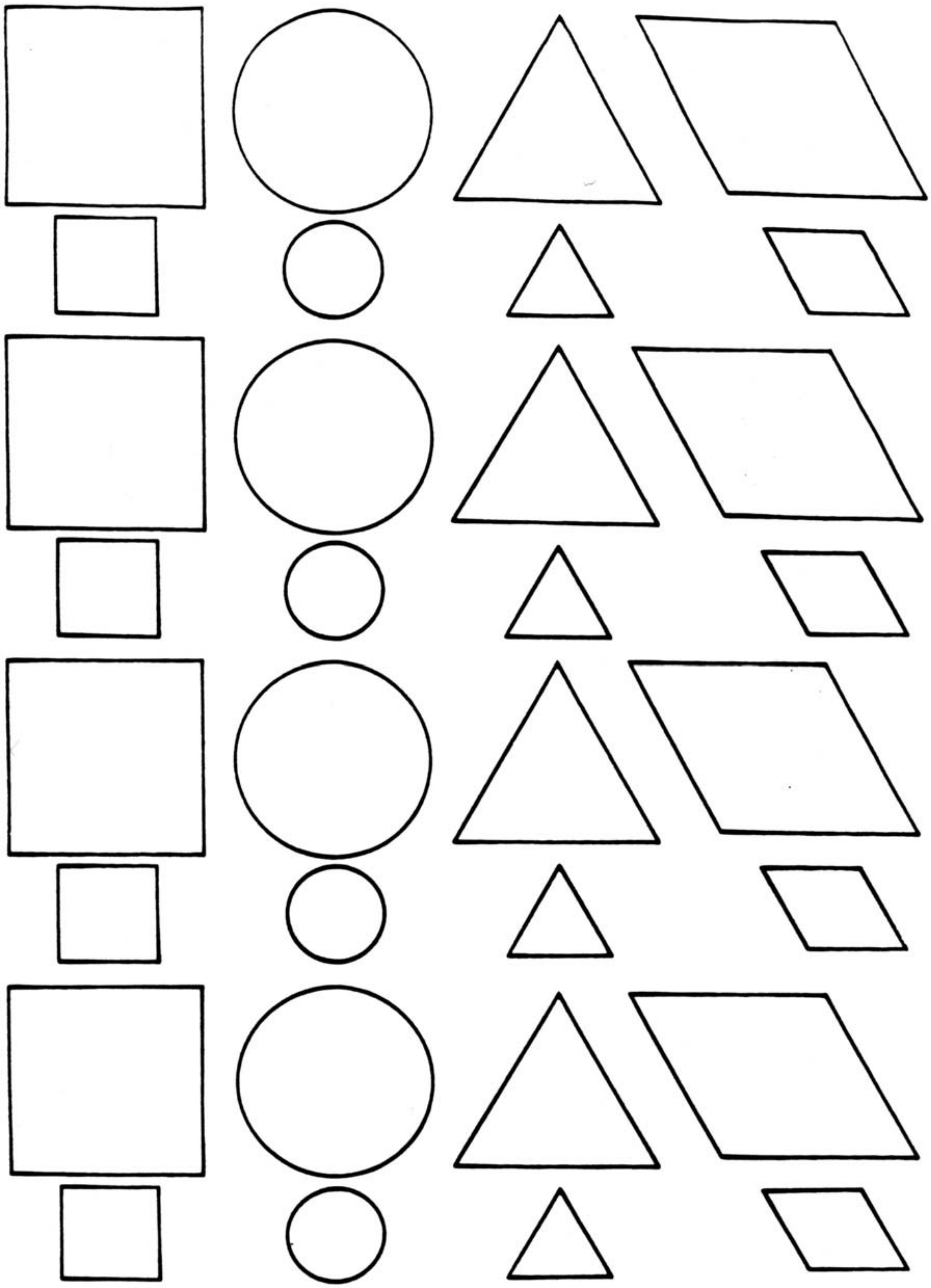
The value of using structured attribute materials (instead of unstructured materials) is that the attributes and values are very clearly identified and easily articulated to students. There is no confusion or argument concerning what values a particular piece possesses. In this way we can focus our attention in the activities on the reasoning skills that the materials and activities are meant to serve. Even though a nice set of attribute pieces may contain geometric shapes or different colors and sizes, they are not very good materials for teaching shape, color, or size. A set of attribute shapes does not provide enough variability in any of the shapes to help students develop anything but very limited geometric ideas. In fact, simple shapes, primary colors, and two sizes are usually chosen because they are most easily discriminated and identified by even the youngest of students.

Most attribute activities are best done in a teacher-directed format. Young children can sit on the floor in a large circle where all can see and have access to the materials. Older children can work in groups of four to six students, each group with its own set of materials. Older children can work with sets of attribute materials that have a greater number of pieces and attributes (i.e., the 60 piece set rather than the 32 piece set). In that format, problems can be addressed to the full class and groups can explore them independently. All activities should be conducted in an easygoing manner that encourages risks, good thinking, attentiveness and discussion of ideas. The atmosphere should be non-threatening, non-punitive, and non-evaluative.

You may wish to motivate primary students to think about attributes by reading them the book titled The Important Book by Margaret Wise Brown. As an extension, you may wish to have students develop their own "important book" about themselves as a project during your work with classification.

Constructing A Set of Attribute Blocks

To make four sets of attribute blocks, print the "Attribute Block" pattern (found on the next page) onto four different colors of heavy paper (red, yellow, blue, green). You may wish to laminate the pieces before you cut them out. Using this pattern, you will create four sets with 32 pieces: Four Colors (red, yellow, blue, green); Four Shapes (circle, triangle, rhombus [diamond], square); and Two Sizes (big, little). Make enough sets for your classroom so that two to four students share one set.



Attribute Block Pattern Warm-Up Activity: Tibby

Select an attribute, such as the color of a participant's shirt, hair, or another attribute. Do not tell the participants what has been selected. Call a participant's name and have him/her stand up. Say, "You're a Tibby" if the participant has on the color of the shirt you're thinking of (or other attribute); otherwise, say, "You're not a Tibby". Continue choosing participants that are Tibbys and not Tibbys. Have participants try to guess what makes a participant a Tibby or not a Tibby.

Variations:

- Let participants take the lead and select a characteristic and determine who is and who is not a Tibby.
- Involve two or more attributes in the determination of Tibbys.

Activity: **What's In the Box?**

Format: Whole Group

Objective: Participants will use logical reasoning to determine the number of pieces in the whole set of attribute pieces after asking questions and receiving information about a few items in the set.

Related SOL: K.17, K.18, 1.20, 1.21, 2.25, 3.24, 4.21, 5.20

Materials: Use the 32-piece set of attribute pieces. Before beginning these activities, check your set to be sure all 32 pieces are available: color (red, yellow, blue, green), shape (circle, triangle, square, rhombus or diamond), and size (big, little).

Time Required: 12 minutes

Directions:

1. Tell the participants, "This box (or bag) contains some attribute pieces which have similarities and differences." Shake it so the participants can hear. "I'd like you to ask me some questions with "yes" or "no" answers to figure out what is in the box." If the answer to the question is yes, (i.e., Do you have something red in the box?) the instructor pulls out a block from the box that has this attribute and will help the participants determine the number of items in the entire set (i.e., pull-out a red circle, then if asked again for a red item, pull out a red rhombus, to show there are other attributes beside color, etc.)
2. During the questioning process, stop at certain points and ask the participants:
   - How many pieces do you think I have left in the box?
   - How many pieces would the total set of attribute pieces contain? Why?
   - What are the characteristics (attributes) of this set of attribute pieces? (Size: large or small; Color: red, yellow, green or blue; Shape: square, rhombus, triangle and circle)
   - Is it possible to find a pair of attribute pieces that have neither size, color nor shape in common?
   - Is each of the attribute pieces in your complete set unique?
   - How can you determine the number of pieces in the set of attribute pieces? (Answer: multiply the number of each attribute: Size - 2; Color - 4; Shape - 4; so it is 2x4x4=32 pieces.)

Activity: **Play!**

Format: Whole Group; Small Group; Mini Lecture; etc.

Objectives: Participants have an opportunity for free play with the attribute pieces so that they:

- have an opportunity to informally learn about the characteristics of the pieces by actually handling them; and
- may begin to use the vocabulary of color, size, and shape.

Participants will create a design, pattern, or picture using the small attribute pieces and describe how their "creation" is mathematical.

Related SOL: K.17, K.18, 1.20, 1.21, 2.25, 3.24, 4.21, 5.20

Materials: Use the 32-piece set of attribute pieces.

Time Required: 20 minutes

Directions:

1. The instructor will ask participants to get into partner pairs. The instructor asks one partner to create a design, pattern, or picture using the small attribute pieces; the other, using the large pieces.
2. The instructor asks each partner to convince the other partner that your "creation" is "mathematical."

Activity: **Missing Pieces**

Format: Small Group

Objective: Participants have an opportunity to visualize a whole set and divide into a number of subsets based on the attributes in the set.

Related SOL: K.17, 1.20, 2.25, 3.24, 4.21, 5.20

Materials: Use the 32-piece set of attribute pieces

Time Required: 10 minutes

Directions:

1. The instructor will ask participants to get into partner pairs. The instructor tells the participants to spread the attribute pieces on the table. One partner removes a piece while the other partner looks away. The partner who hides his/her face is asked to identify the missing piece without touching any of the pieces on the table. When the missing piece has been identified, partners are asked to exchange roles and repeat this activity
2. The instructor will ask participants to describe the strategies used to organize the pieces.
3. Extension: Think of a "secret rule" to classify the set of attribute pieces into two piles. Use that rule to slowly sort the pieces as your partner observes. At any time, your partner can call "stop" and guess the rule. After the correct rule identification, players reverse roles. Each incorrect guess results in a one-point penalty. The loser is the first player to accumulate 7 points.

Activity: **20 Questions Game**

Format: Whole Group

Objective: Participants play the "20 Questions" game to develop skill in using the strategy of elimination. The objective of the game is to reduce the set of attribute blocks in the fewest number of questions in order to identify the mystery attribute piece. In this game the participants should develop the strategy of reducing the set by half each time they ask a question, as this strategy always provides the answer within 5 guesses (i.e., $32 = 2^5$).

Related SOL: 4.21, 5.20

Materials: Use the 32-piece set of attribute pieces.

Time Required: 10 minutes

Directions:

1. This is a variation of the standard "20 Questions" game. Ask one participant to think of an attribute piece from the set of attribute pieces. The participant tells the name of the attribute piece to the instructor or writes it on a piece of paper without letting the other participant see it. The other participants, in turn, ask "yes/no" questions about the mystery attribute piece. After each question is answered, the participants move to one side those attribute pieces that do not fit the clues already disclosed. A scorekeeper can count the number of questions asked. Participants try to find the mystery block using the fewest questions possible.
2. After a few games, the instructor can ask, "What is the first best question to eliminate the greatest number of blocks?" The participants may suggest, "Is the block four-sided?" This may not be the best first question, especially when the answer is no. Help the participants recognize that a strategy of eliminating the set by half is the quickest way to reduce the set and identify a specific element. In this game, the participants should learn to reduce the set by half each time, as this strategy always provides the answer within 5 guesses (i.e., $32 = 2^5$).

Game: "Twenty Questions"

Best Strategy: Eliminating Half

Question Number "n"	Eliminate	Left
1	$1/2^{1}$	*1/2*
2	$1/2^{2}$	*1/4*
3	$1/2^{3}$	*1/8*
4	$1/2^{4}$	*1/16*
5	$1/2^{15}$	*1/32*
n	$1/2^{n}$	$1/2^{n}$

When using the 32 attribute pieces, reducing the set so that it has 1/32 left, will result in finding the mystery piece. Questions that eliminate half of the set include:

- Does it have four sides?
- Is it large?
- Is it small?
- Is the color one of the colors in the colors of the U.S. Flag?

Activity: **Who Am I? Game**

Format: Individuals or Small Groups

Objective: Participants reinforce their understanding of attributes through a game where they use clues to identify a specific attribute piece. This is an opportunity to integrate simple rhymes into mathematics.

Related SOL: K.17, K.18, 1.20, 1.21, 2.25, 3.24, 4.21, 5.10

Materials: Use the 32 piece set of attribute pieces and give participants copies of the appropriate sample Clue Cards.

Time Required: 10 minutes

Directions:

1. This game can be played by individuals or by teams. The players are shown a "clue card." The first player to discover the mystery piece is the winner.
2. Extension: Participants can write their own "Who Am I?" Cards

Answers to the Clue Cards:

32 Piece Attribute Set:

1.) Large, red rhombus
2.) Small, red circle
3.) Large, green rhombus
4.) Small green circle
5.) Large, red triangle

60 piece Attribute Set:

1.) Large thick, red rectangle
2.) Small thin, red circle
3.) Large thin, blue rectangle
4.) Small thin, yellow circle
5.) Large thin, red triangle

Sample Clue Cards For The 32 Piece Attribute Set

(1) *Who am I?* I am large I am not yellow. I have four sides. I am not blue or green. I am not a rhombus. Who am I?	(2) *Who am I?* I am not large. I am green or red. I am not four sided. I have no corners. I am not green. Who am I?
(3) *Who am I?* I do not fit in a round hole. I have four corners. I am not red. I am large. I am green. I am not square. Who am I?	(4) *Who am I?* I am lost, help me find myself. When you find me, hold me in your hand. I am small. I am not blue. I am not square. I am green. I will roll off the table. Who am I?
(5) *Who am I?* I am blue or large or square. I am not green. I am small or a triangle. I am red or blue. I am not a circle. I am blue or large. I am not blue. Who am I?	(6) *Who am I?* Write your own.

Sample Clue Cards For The 60 Piece Attribute Set

(1) *Who am I?* I am large and not a square. I am not yellow. I have four sides. I am not blue or thin. I am not a rhombus. Who am I?	(2) *Who am I?* I am not large. I am yellow or red. I am not four sided. I have no corners. I am not yellow or thick. Who am I?
(3) *Who am I?* I do not fit in a round hole. I have four corners. I am not red. I am large. I am blue and thin. I am not a square. I am not a rhombus. Who am I?	(4) *Who am I?* I am lost, help me find myself. When you find me, hold me in your hand. I am small. I am not blue. I am not square or thick. I am yellow. I will roll off the table. Who am I?
(5) *Who am I?* I am blue or large or square. I am not yellow. I am small or a triangle. I am red or blue. I am not a circle. I am blue or large. I am not blue. Who am I?	(6) *Who am I?* Write your own.

Activity: **Differences - Trains and Games**

Format: Small Group

Objective: Participants reinforce their understanding of differences and similarities through some problem solving activities and games. Participants will identify the number of differences between two objects (i.e., one difference, two-differences, three-differences, etc.) as they create difference trains and play games where they must identify the number of differences.

Related SOL: K.17, K.18, 1.20, 1.21, 2.25, 3.24, 4.21, 5.20

Materials: Attribute pieces and the Differences Activity Sheet

Time Required: 20 minutes

Directions:
1. Ask participants to compare attribute pieces in terms of their differences and similarities. Hold up an attribute piece and ask the participants to hold up an attribute piece, which differs in one way. Repeat this with several attribute pieces, and then ask them to hold up an attribute piece, which differs in two ways, then in three ways. At the same time, ask the participants to hold up attribute pieces that are similar in two, one or no ways.
2. Tell the participants that "Difference Trains" have engines and cars. Place the large, red circle on the table as the engine of the train. Cars are to be sequentially attached to the train according to the given rule. Start with the rule that the car to be attached must differ from the preceding car by a single attribute - by one difference. That is, if the engine is a large, red circle, then there are a variety of possibilities that could be attached as the cars; a small, red circle; or a large yellow circle. Have participants identify all of the possibilities. Ask the participants "Why could the small, blue square not be the first car attached to the large, red circle engine? " Taking turns with their partners, have the participants build a train at least 20 cars long, verbalizing the difference as the next car is but into place.
3. Ask the participants "Could you build a train using all of the attribute pieces? Try it!
4. Two-Difference Variation: Have the participants start with the same engine. This time attach a car that differs from the car to which it is attached by two-differences. Ask the participants "Could you build a train using the 16 large pieces before using any small pieces? Try it!"
5. Three-Difference Variation: Have the participants agree on the attribute piece to be the engine. Build a train so that the adjacent cars will differ by exactly three differences.

6. "Differences" is a game for two players or two teams on a four-by-four game mat. Randomly divide the attribute pieces between the two players, so each has 16 pieces.
7. **Game One:** Alternate, taking turns to place an attribute piece on the game mat. THE ONLY RULE is that an attribute piece must differ from its horizontal and vertical neighbors in exactly two ways. The first player who cannot place a block loses.
8. **Game Two:** Alternate, taking turns to place an attribute piece on the game mat. THE ONLY RULE is that an attribute piece must differ from its neighbors horizontally, vertically, and diagonally. The first player who cannot place a block loses.

Differences

Randomly divide the attribute pieces between the two players, so each has 16 pieces.
Game One: Alternate, taking turns to place an attribute piece on the game mat. THE ONLY RULE is that an attribute piece must differ from its horizontal and vertical neighbors in exactly two ways. The first player who cannot place a block loses.
Game Two: Alternate, taking turns to place an attribute piece on the game mat. THE ONLY RULE is that an attribute piece must differ from its neighbors horizontally, vertically, and diagonally. The first player who cannot place a block loses.

Activity: **Hidden Number Patterns**

Format: Individual

Objective: Participants apply their understanding of differences and similarities by identifying number patterns related to the differences in a sequence of attribute pieces.

Related SOL: 4.21, 5.20

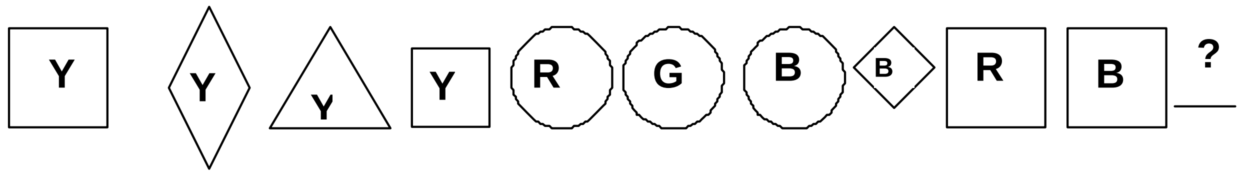
Materials: Use the 32-piece set of attribute pieces and the Hidden Number Activity Sheets.

Time Required: 10 minutes

Directions:

1. Have the participants identify the number of differences between two objects (i.e., one difference, two-differences, three-differences, etc.) in the Hidden Number Activity Sheets (see attachments).
2. Have participants create difference trains where they develop a number pattern for the number of differences in their trains.

HIDDEN NUMBER PATTERN #1



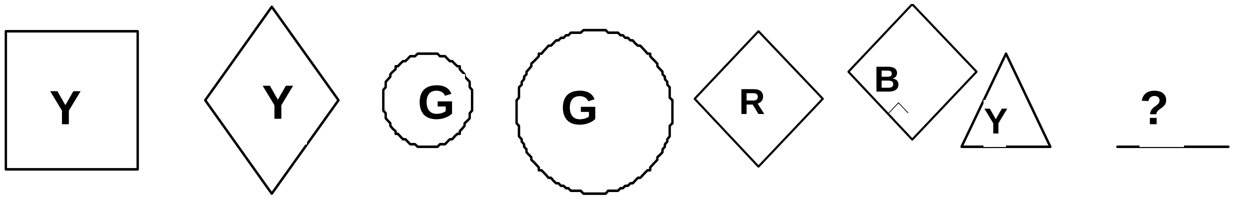
WHAT BLOCK WOULD YOU PLACE AFTER THE LARGE BLUE SQUARE?

WHAT IS THE NUMBER PATTERN (OF DIFFERENCES) ASSOCIATED WITH THIS TRAIN?

ARE THERE ANY OTHER BLOCKS THAT COULD IMMEDIATELY FOLLOW THE LARGE BLUE SQUARE?

WHAT BLOCKS COULD BE PLACED AFTER THE LARGE BLUE SQUARE?

HIDDEN NUMBER PATTERN #2:



WHAT BLOCK WOULD YOU PLACE AFTER THE LARGE YELLOW TRIANGLE?

WHY DID YOU SELECT THAT BLOCK?

ARE THERE ANY OTHER BLOCKS THAT COULD IMMEDIATELY FOLLOW THE LARGE YELLOW TRIANGLE?

WHAT NUMBER PATTERN (OF DIFFERENCES) DID YOU DISCOVER IN THIS TRAIN?

Activity: **Attribute Networks**

Format: Whole Group, Small Group; Mini Lecture; Assessment

Objective: Participants reinforce their understanding of the relationships by focusing on differences and similarities in an attribute network problem. This may be used as an assessment of a participant's understanding of differences.

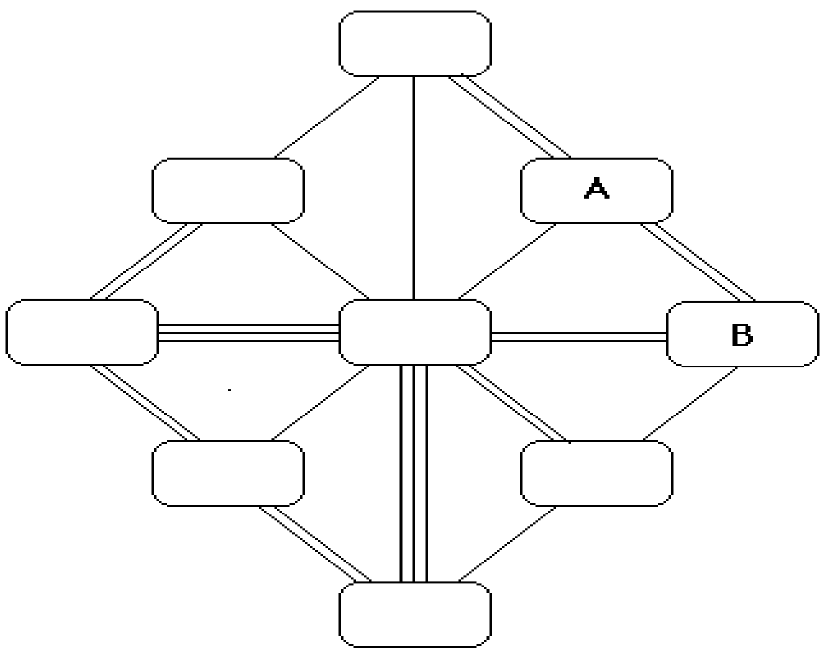
Related SOL: 4.21

Materials: Attribute pieces and a copy of the Attribute Networks Activity Sheet

Time Required: 20 minutes

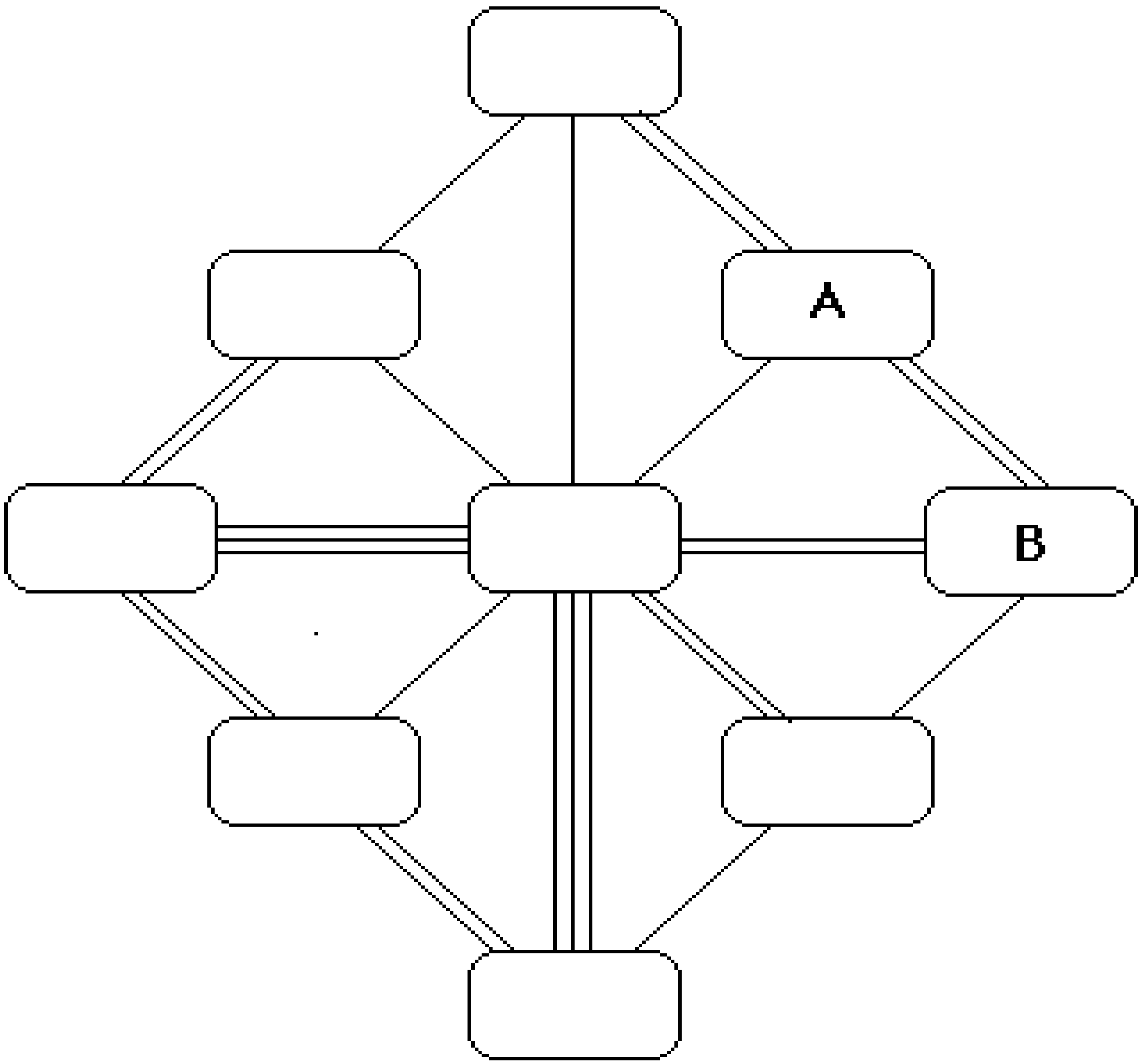
Directions:

1. Place an attribute piece on one of the regions. Place another attribute piece in an adjacent region, which differs from the first piece by as many variables (i.e., color, shape, size) as there are lines connecting the regions. For example, the piece in Region B must differ from the piece in Region A by exactly two attributes.
2. Once you have created a solution that works, write your answers in each block.



NOTE: From A to B, identify blocks that have 2-Differences since there are two lines.

Attribute Networks **Name:**______________________________



Activity: Two-Loop Problems

Format: Whole Group, Small Group; Mini Lecture

Objective: Through problem solving and games, these activities will strengthen the participant's understanding of differences and similarities, which are used in patterning, analyzing and interpreting graphs.

Related SOL: 4.21, 5.20

Materials: Use the 32-piece set of attribute pieces. Before beginning these activities, check your set to be sure all 32 pieces are available. Also have available two pieces of string, each tied in a circle to make two large loops, and a set of Attribute Cards listing one attribute of the set you are using on each card (i.e., red, yellow, square, small, etc.).

Time Required: 30 minutes

Directions:

1. Place the two loops on a table, independent of each other. Select two Attribute Cards, which describe sets with no common pieces. (For example: Squares and Triangles.) Have the participants place the pieces on the table where they belong (either in one of the loops if they are squares or triangles or outside the loops if they are not). After all of the pieces have been placed, have the participants carefully check to see that all the pieces have been correctly placed. Play again with cards such as blue/green; circle/rhombus; etc.
2. Place two overlapping loops on the table. Shuffle the Attribute Cards and place an Attribute Card on each of the loops. Define the sections of the loops. The blocks that would be placed in both loops are placed in the overlapping part of the loops because they have both attributes. For example, if the attribute cards red and triangle were used for the two circles, the intersection would contain only the large red triangle and the small red triangle. Have the participants place the attribute pieces in the loops in their correct positions. Repeat this activity with other Attribute Cards in the loops.
3. Note: The blocks that are in either one loop, the other, and in the overlapping area are in either the red set or the triangle set, or in both. The blocks which are not in the intersection may be described as "triangle but not red," or "red but not triangle."
4. Play the game "In the Loops." This game is played by two players or two teams. Shuffle the Attribute Cards and place them face down in the center of the table. Place the two loops so they overlap. One participant picks two Label Cards and looks at them without showing the other player, then places them face down, one on each loop. The second player chooses a block and asks which section of the loops it belongs or whether it belongs outside the loops. The goal of the game is to name the sets with the fewest number of pieces attempted.

5. As soon as there are sufficient “clues”, the second player tries to name the two sets. When both sets have been named, count the number of pieces the player placed in order to name a set (those both inside and outside the loops). This sum is the player’s score. The winner is the player, or team, with the lowest score after an agreed upon number of turns.
6. Have the participants set out three overlapping loops on the table. Shuffle the Attribute Cards and place an Attribute Card on each of the loops. Define the sections of the loops. The blocks that would be placed in all three loops are placed in the overlapping part of the three loops because they have both attributes. Other overlapping areas of the two loops have the attributes of the two loops. Have the participants try to place all the attribute pieces on the table within the three loops or outside the loops to further reinforce their understanding of intersections.
7. Extension: Play the game “In the Loops” with three overlapping loops.

Attribute Cards:

Print this page on card stock and then cut the pieces out to use for the loop games.

RED	NOT RED
YELLOW	NOT YELLOW
GREEN	NOT GREEN
BLUE	NOT BLUE
LARGE	NOT LARGE
SMALL	NOT SMALL
TRIANGLE	NOT TRIANGLE
SQUARE	NOT SQUARE
RHOMBUS	NOT RHOMBUS
CIRCLE	NOT CIRCLE

Patterns
Session 2

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Overview of Patterns	Overview of Patterns	52			
Exploring Various Types of Patterns	I Need A Necktie, Please!	54	1.21, 2.25, 3.24, 4.21	I Need a Necktie, Please!	Pattern blocks, adding machine tape strips, paper pattern blocks
	Up, Up and Away!	57	1.21, 2.25, 3.24, 4.21	Up, Up, and Away!	Linking cubes, graph paper, adding machine strip tape
	How High Are My Castle Walls?	59	1.21, 2.25, 3.24, 4.21	How High Are My Castle Walls?	Wooden pattern blocks, adding machine tape, paper pattern blocks
	Exactly How Many Doors Are We Talking About?	61	3.24, 4..21, 5.20	Exactly How Many Doors Are We Talking About?	Wooden pattern blocks
	Building Staircases	63	4.21, 5.20	Building Staircases	Centimeter cubes
Hidden Patterns	Tons of Tunnels	65	K.18, 1.21, 2.25, 3.24,	Tons of Tunnels	Linking cubes, cardboard tubes
	How Many Beads are Hidden Under the Clouds?	67	4.21, 5.20	How Many Beads are Hidden Under the Clouds?	Colored beads, string or yarn, needles
Patterns in Nature	The Jeweled Snake	70	K.18, 1.21, 2.25, 3.24, 4.21	The Story of The Jeweled Snake	Pattern blocks, overhead pattern blocks
	Fibonacci Numbers	74	K.18, 1.21, 2.25, 3.24, 4.21, 5.20	Patterns and Relationships	Examples or pictures of Fibonacci Numbers, tree branches with leaves, pinecones, pineapple, sunflower or daisy

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Patterns on the Hundreds Board	Grid Pictures	77	1.21, 2.25, 3.24, 4.21	Picture This, Grid Pictures	Linking cubes or small colored squares
	What Comes Next on the "Picture This" Chart	81	2.25, 3.24, 4.21	Picture This Chart, What a Cut Up!, What Comes Next?	Overhead 1-99 Chart
	Patterning on the 100 Chart	85	1.21, 2.25, 3.24, 4.21, 5.20	Patterning on the 100 Chart	Linking cubes, recording sheet
	Odd and Even	90	2.5		Color tiles
	Line Up	94	4.21	Line Up! Activity Sheet	Meter sticks, yarn, and clothes pins
	Arrow Math	96	1.21, 2.25, 3.24, 4.21, 5.20	Arrow Math	
Patterns and Literature	The King's Commissioner	99	3.24, 4.21, 5.20		*The King's Commissioners*, 100s boards, centimeter cubes, crayons or markers
	The King's Chessboard	101	3.24, 4.21, 5.20		*The King's Chessboard*, 100s boards, grains of rice

Activity: Overview of Patterns

The Virginia Standards of Learning state that students will be able to create, recognize, describe, analyze and extend a variety of patterns. This instructional module focuses on the mathematics of specific types of patterns and presents staff development activities to engage teachers in further developing their knowledge of patterns that can inform their instructional practices. Problem-solving activities are intended to engage participants in learning through the use of manipulatives, through the discovery and development of strategies, through the use of thought-provoking questions, and through activities where patterning can be found in real-world situations. Attention should be given to strategies for identifying and generalizing patterns. Patterns should be represented and modeled in a variety of ways including numerical, geometric, graphic, and algebraic.

Since the physical world is full of patterns, students enjoy finding them and understanding them. Children often have questions about patterns such as those found in nature: “Is there a pattern to the number of petals on flowers?”, “Why do all snowflakes have six sides?”, ” Why do all pineapples have the same number of clockwise and counterclockwise spirals?” As teachers, it is important to help students to learn about patterns through observation, experimentation, and discovery and help them to apply this understanding to learn mathematics. Opportunities to understand the various types of patterns and experience the wonder of patterns should be an objective of the elementary mathematics curriculum.

Repeating Patterns

The simplest types of patterns are **repeating patterns**. The patterns can be oral, such as the refrain in “Old MacDonald’s Farm” -- “e-i-e-i-o”; or physical with clapping and snapping patterns; or combinations of both with songs like the “hokey pokey”. In each case students need to identify the basic unit of the pattern and repeat it. Opportunities to create, recognize, describe and extend repeating patterns are essential to the primary school experience.

Sample Repeating Patterns (i.e., repeating the basic unit)

1. ABABABAB
2. ABCABC
3. AABBAABBAABB
4. AABAAB
5. AABCAABC
6. ABACABAC

Growing Patterns

Students find that **growing patterns** are more difficult to understand than repeating patterns, as they must not only determine what comes next, but they must also begin the process of generalization. Students need experiences with growing patterns both in arithmetic and geometric formats.

Sample Growing Patterns (i.e., where one variable changes in the basic unit)

1. ABAABAAABAAAAB
2. ABABBABBBABBBB
3. ABCAABCAAABC

Arithmetic Patterns

In **arithmetic patterns**, also called arithmetic sequences, students must determine the difference, called the "common difference", between each succeeding number in order to determine what is added to each previous number to obtain the next number.

Sample Arithmetic Patterns
6, 9, 12, 15, 18, ...
5, 7, 9, 11, 13, ...
16, 12, 8, 4, 0, -4, -8 ...

A generalization for these **arithmetic patterns** could be expressed as follows:

$\square,\ \square + \Delta,\ (\square + \Delta) + \Delta,\ (\square + \Delta + \Delta) + \Delta,\ (\square + \Delta + \Delta + \Delta) + \Delta,\ (\square + \Delta + \Delta + \Delta + \Delta) + \Delta,\ \ldots$

Geometric Patterns

In **geometric patterns**, students must determine what each number is multiplied by to obtain the next number in the geometric sequence. This multiplier is called the "common ratio".

Sample Geometric Patterns
2, 4, 8, 16, 32, ...
1, 5, 25, 125, 625, ...
80, 20, 5, 1.25, ...

A generalization for these **geometric patterns** could be expressed as follows:

$\square,\ \square * \Delta,\ (\square * \Delta) * \Delta,\ (\square * \Delta * \Delta) * \Delta,\ (\square * \Delta * \Delta * \Delta) * \Delta,\ (\square * \Delta * \Delta * \Delta * \Delta) * \Delta,\ \ldots$

Activity: I Need A Necktie, Please!

Format: Small Group

Objective: Participants will experience rhythmic patterns, describe those same patterns using wooden pattern blocks, and finally copy the patterns onto paper as they create a necktie for Father Giraffe.

Related SOL: 1.21, 2.25, 3.24, 4.21

Materials: Pattern blocks, adding machine tape strips, paper pattern blocks, I Need A Necktie, Please! Activity Sheet

Time Required: 20 minutes

Directions:
1. Engage participants in the rhythmic pattern clap, clap, pat your lap, clap, clap, pat your lap, and so on.
2. Ask participants to create a pattern with their pattern blocks that is the same as the clap, clap, pat pattern such as red, red, blue, red, red, blue and so on.
3. Repeat this activity several times with different rhythmic patterns such as snap, clap, snap, clap until all participants have created a pattern with their pattern blocks.
4. Distribute adding machine tape strips and explain to participants that they are to copy their pattern onto the strips with paper pattern blocks because they are going to make Father Giraffe a special necktie with a pattern design.
5. Remind participants that they are to repeat their pattern over and over until they have filled Father's necktie from top to bottom.
6. Instruct participants to paste their neckties on Father Giraffe.
7. When participants have completed their neckties, explain that they are now going to display their ties for all to see.
8. Discuss the different patterns and the numerous ways to sort and organize the patterns on the neckties such as ABB, AABB, AB, or even ABCD.

Patterns

Repeating Patterns (Repeating the basic unit)

1. ABABABAB
2. ABCABC
3. AABBAABBAABB
4. AABAAB
5. AABCAABC
6. ABACABAC

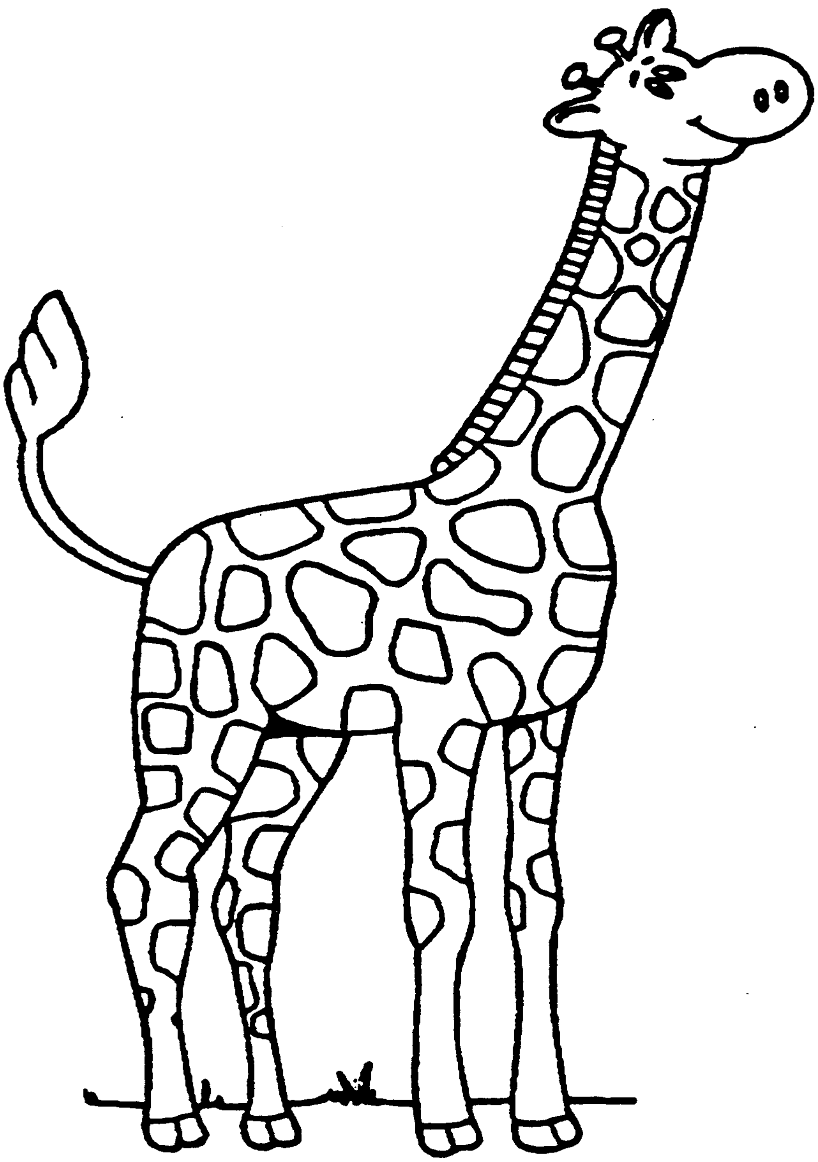
Growing Patterns

1. ABAABAAABAAAAB
2. ABABBABBBABBBB
3. ABCAABCAAABC

Other Patterns - Unexpected, Unusual

1. Seasons
2. Nature
3. Honeycombs
4. Stop Light - G,Y,R, G,Y,R
5. Class Schedules
6. Music
7. Rhythm -- clapping / soft-loud

I Need A Necktie, Please! **Name:** ________________



Activity: Up, Up and Away!

Format: Small Group

Objective: Participants will create a repeating pattern with linking cubes and interpret that same pattern with rhythmic gestures. In turn, those patterns will be identified according to a letter value such as ABAB or ABBA. Participants will then be asked to transcribe that pattern onto adding machine tape with paper pattern blocks to create the tail on a kite.

Related SOL: 1.21, 2.25, 3.24, 4.21

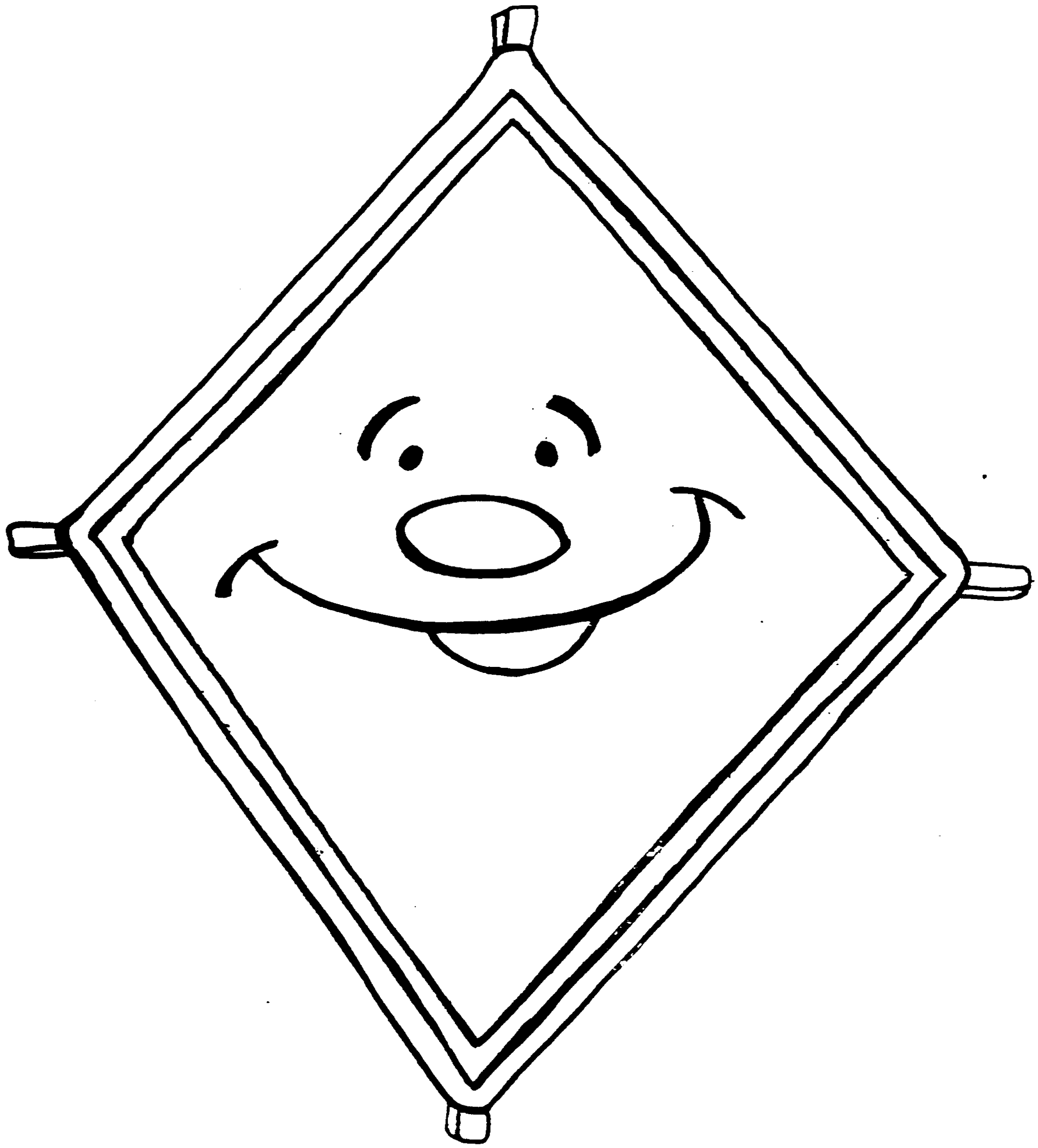
Materials: Linking cubes, graph paper, adding machine strip tape, Up, Up, and Away! Activity Sheet

Time Required: 20 minutes

Directions:

1. Ask participants to make a repeating pattern with two colors of linking cubes.
2. Explain to participants that they should make two or three repeats of the pattern with the cubes.
3. Discuss the different patterns participants have made with their linking cubes. Interpret each participant's pattern by snapping or clapping, by saying the colors aloud, or by playing touching games. The pattern ABB can be interpreted snap, clap, clap, snap, clap, clap and so on. The pattern ABAB could be interpreted red, blue, red, blue, red, blue, red, blue, and so on. An AABB pattern could be interpreted touching nose, nose, shoulders, shoulders and so on, and an ABAB could be interpreted touching nose, shoulders, nose, shoulders, nose, shoulders, and so on.
4. As each participant's pattern is interpreted, be sure to emphasize that the pattern continues on beyond the number of linking cubes.
5. Have the participants record their patterns by coloring the pattern they have created on graph paper. Participants should color two to three feet of their pattern on the graph paper and then paste their completed pattern on adding machine tape in order to create a kite tail.
6. Instruct the participants to cut out the kite and then paste on the completed patterned tail.

Up, Up and Away! **Name:** ________________



Activity: How High Are my Castle Walls?

Format: Small Group

Objective: Participants will investigate linear patterns using concrete objects as they construct castle walls to encompass a ready-made castle.

Related SOL: 1.21, 2.25, 3.24, 4.21

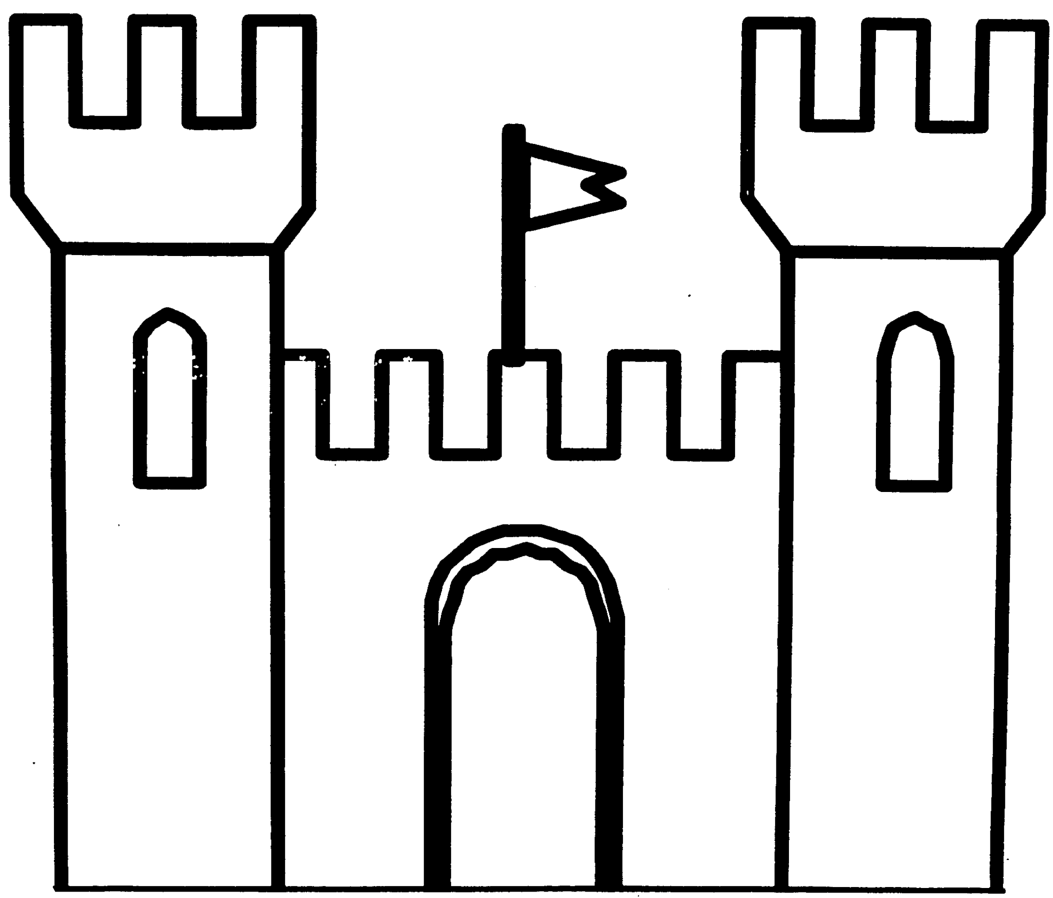
Materials: Wooden pattern blocks, adding machine tape, paper pattern blocks, How High Are My Castle Walls? Activity Sheet

Time Required: 20 minutes

Directions:

1. Have participants choose pattern blocks and create a wall with a repeating pattern around the castle.
2. Encourage participants to discuss the different pattern walls that they create and the names of the different colors or shapes that they use to create their castle wall such as square, triangle, triangle, square, triangle, triangle and so on.
3. Instruct participants to copy their pattern walls onto adding machine tape with paper pattern blocks. Remind them to repeat their pattern over and over.
4. Ask participants to tape the ends of their pattern walls together to form a circle around the castle.
5. Participants should color and cut out the castle, roll it into a cylinder and secure it in a standing position. Conclude the activity by placing participants' paper pattern block wall around their castles.

How High Are my Castle Walls? **Name:** ________________



Activity: Exactly How Many Doors Are We Talking About?

Format: Small group

Objective: Participants will use concrete objects to construct "walls" and then determine how many "doors" are located along the wall. Participants will be able to develop a number pattern and use that pattern to look for a rule to predict the number of blocks.

Related SOL: 3.24, 4.21, 5.20

Materials: Pattern blocks, Exactly How Many Doors Are We Talking About? Activity Sheet

Time Required: 20 minutes

Directions:

1. Instruct the participants to create a wall of pattern blocks. Each section of the wall will contain a sequence of one red trapezoid, one green triangle, and one red trapezoid. A wall with one green door will take three blocks to build.
2. Ask the participants to add more blocks to increase the size of this wall. The expanded wall with two green doors will take five blocks to build.
3. Explain that the wall will continue with the same pattern. Challenge the participants to compute the total number of pattern blocks in the wall when there are fifty green doors. {(50 x 2) +1 = 101}
4. Participants will not have enough blocks to actually build the entire wall so encourage them to fill out the chart in order to generalize the number pattern and the rule the wall represents. {(Doors x 2) + 1 = total blocks}
5. Have participants write their answers on the recording sheet and write about their findings.

Name: ________________

Exactly How Many Doors Are We Talking About?

Number of Doors	Total Number of Blocks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
|	
|	
|	
50	

Activity: Building Staircases

Format: Small Group

Objective: Participants will determine how many centimeter cubes will be needed to build a staircase with one hundred steps. Participants will be able to extend a given pattern using concrete manipulatives to create a staircase with one hundred steps.

Related SOL: 4.21, 5.20

Materials: Centimeter cubes, Building Staircases Activity Sheet

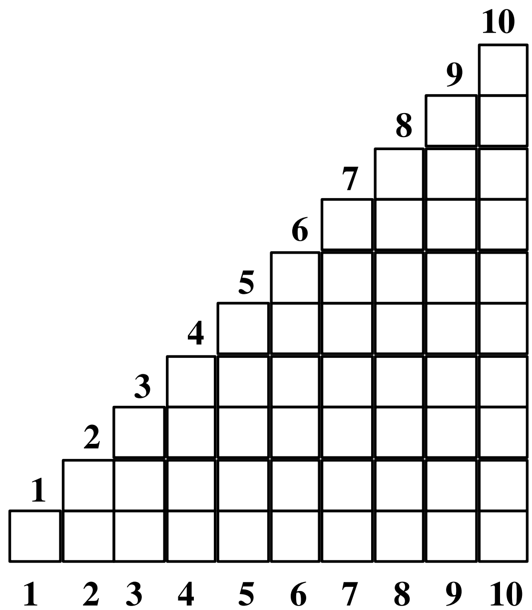
Time Required: 20 minutes

Directions:

1. On the overhead projector, the teacher should build a staircase, beginning with one step and continuing until four steps have been created.
2. As you continue to increase the staircase pattern, ask participants, "In this staircase pattern, which step is this? How many centimeter cubes does the pattern have so far? How many centimeter cubes will we need to add to make the next step?"
3. Instruct the participants to work with their partner using centimeter cubes to build staircase models.
4. Encourage participants to record the patterns they begin to see. Ask the participants: "How many centimeter cubes will you need to build a staircase with one-hundred steps?" Help participants explore different possibilities for finding the correct answer.

Building Staircases

Name: ________________



Step	Total Number of Cubes
1	
2	
3	
4	
5	
.	
.	
100	
n	

Activity: Tons of Tunnels

Format: Small Group

Objective: The participant, using concrete materials, will be able to identify the missing pattern pieces that are hidden from view.

Related SOL: K.18, 1.21, 2.25, 3.24

Materials: Linking cubes, top half of cardboard tubes from paper towels, Tons of Tunnels Activity Sheet

Time Required: 10 minutes

Directions:

1. Direct participants to make a ABA, AABB, ABAC, ABC or even a ABCA pattern using linking cubes. Encourage the participants to repeat their patterns at least three times until they have used 12 linking cubes.
2. Ask participants to call this strip of repeating pattern with the linking cubes a pattern strip.
3. Model for the class how parts of the pattern can be hidden by placing a cardboard tube over a section of the pattern strip.
4. Ask participants to work with a partner and hide a section of their pattern by covering it with the cardboard tube they are provided.
5. Instruct the participants to work with their partner and guess what is hidden under the tunnel.
6. Allow participants to check their predictions by removing the cardboard tube.
7. Reverse the process and allow the partner to build the pattern strip and hide the pattern with the cardboard tube.

Tunnels

Name: ____________________

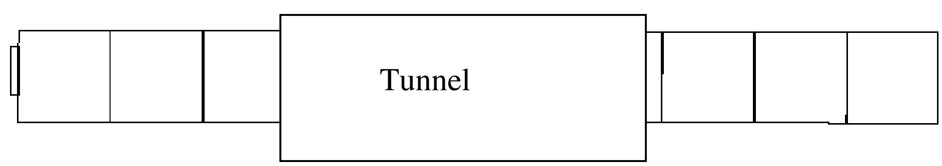
Materials Needed: Linking cubes and top half of cardboard tubes

1. Make an ABA, AAB, AABB, AABC, ABAC, ABC or an ABCA pattern using linking cubes. After you select your pattern unit, repeat it at least 3 times until you use 12 linking cubes. This is called your pattern strip.

2. Place the cardboard tube over part of your pattern strip.

3. Find a partner and ask him/her to guess what is under the tunnel. Let them check the prediction by lifting the tunnel.

4. Repeat steps 1-3 letting your partner make and hide the pattern strip.



Activity: How Many Beads are Hidden Under that Cloud?

Format: Small Group

Objective: The participant will be able to determine how many beads are hidden under the cloud.

Related SOL: 4.21. 5.20

Materials: Colored beads, string or yarn, needles, How Many Beads Are Hidden Under The Cloud? Activity Sheet

Time Required: 10 minutes

Directions:

1. Invite participants to create a growing pattern by stringing two colors of beads.
2. Encourage participants to repeat their growing patterns at least six times.
3. Such a pattern could then be identified as an ABBAABBBBAAABBBBBBBB pattern.
4. Ask for volunteers to explain what the A pattern and the B pattern are (doubling).
5. Help participants to see that the A pattern and the B pattern are growing at different rates.
6. After the participants are comfortable with the doubling effect of the A and B patterns they have created with their colored beads, direct that they work with a partner and cover a section of their beaded patterns from view.
7. Allow time for each team to reverse roles and determine the number of missing pattern pieces.
8. Invite whole group discussion and inquiry in order to determine if team understanding is adequate.
9. Display the “How Many Beads are Hidden under the Cloud?” Activity Sheet on the overhead projector.
10. Ask participants to again work with a partner to determine the number of missing beads.

ANSWER:

Using the symbols: Black Bead = B   White Bead = W

The sequence would proceed as follows:
1B, 2W, 2B, 4W, 3B, 8W, 4B, 16W, 5B, 32W, 6B, 64W, 7B. ...

What's Missing?

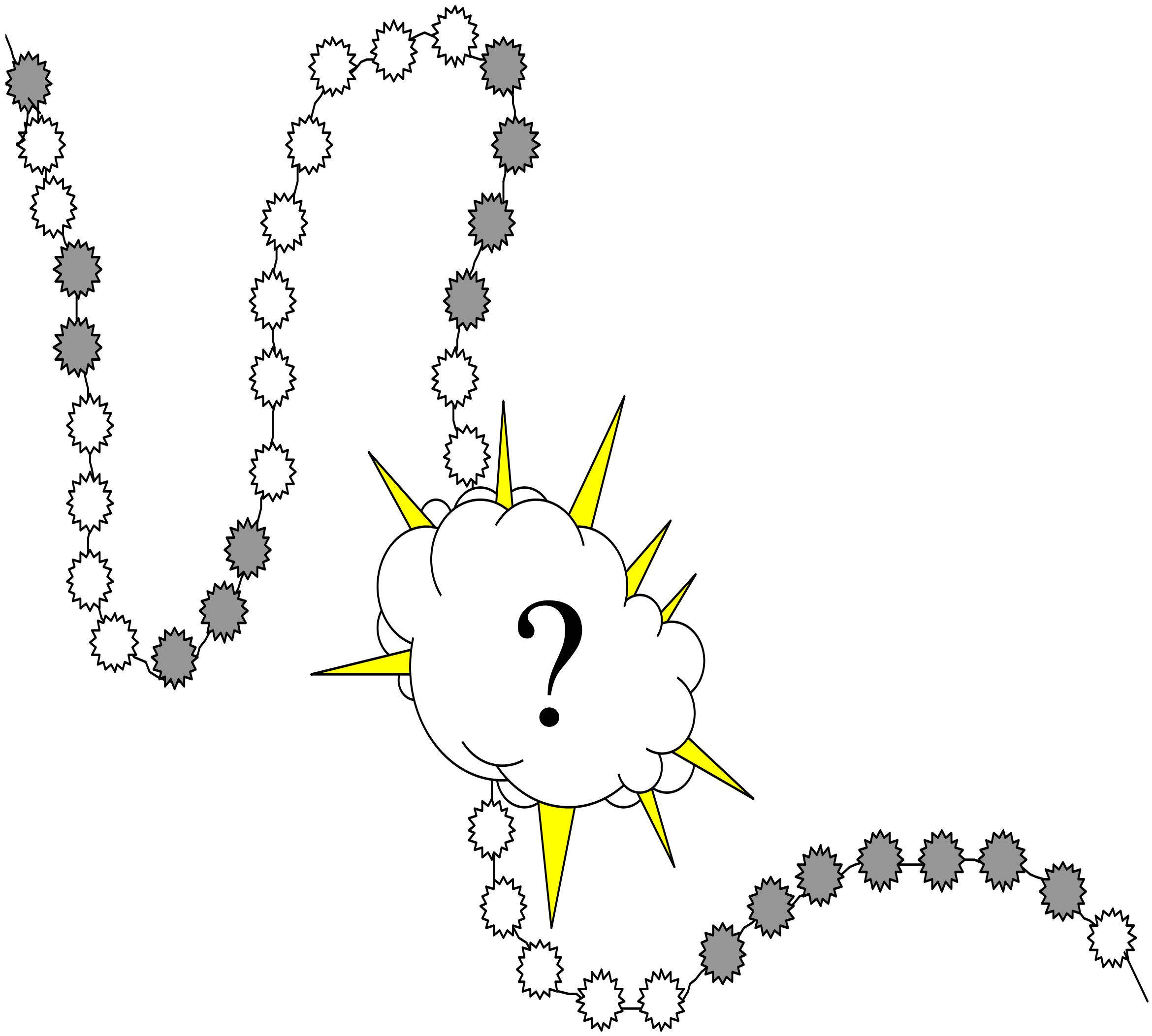
Of the 16 White Beads - 2 are showing and 14 are under the cloud
Then 5 Black Beads are under the cloud
Then 32 White Beads are under the cloud
Then 6 Black Beads are under the cloud

Of the next 64 White Beads, 5 are showing, so 59 are under the cloud

Under the Cloud:
14 + 5 + 32 + 6 + 59 = 116 are hidden under the cloud

Clouds **Name:** ________________

How Many Beads are Hidden Under the Cloud?



Activity: The Jeweled Snake

Format: Whole Group

Objective: Participants will experience an integration of literature and mathematics when they are asked to create twelve linear and geometric patterns with paper pattern blocks and actual pattern blocks after listening to the selection, "The Jeweled Snake". Participants will be able to recognize, describe, and extend a repeating pattern using pattern blocks as the manipulative.

Related SOL: K.18, 1.21, 2.25, 3.24, 4.21

Materials: Story and picture of the "Jeweled Snakes" Activity Sheet, wooden pattern blocks for participants, overhead plastic pattern blocks for instructor

Time Required: 20 minutes

Directions:

1. Identify for participants the various pattern block shapes: yellow hexagons, blue rhombuses, red trapezoids, green triangles, orange squares, and tan parallelograms.
2. Read the selection "Jeweled Snakes" to the participants and give them the handout of the story.
3. Model on the overhead projector with overhead pattern blocks the Father Snake's pattern, "Yellow hexagon and blue rhombus shapes that fit together".
4. Model on the overhead projector with overhead pattern blocks Mother Snake's pattern, "Yellow hexagon shapes with red trapezoids in between".
5. Distribute the picture of the Jeweled Snake and ask the participants to use pattern blocks to make the repeating patterns of the ten remaining snakes.
6. Have various sets of partners model for the class the different patterns they have created for each member of the snake family.

Story of the Jeweled Snakes

At the back of Farmer Max's garden was a deep, dark hole where a family of snakes lived. As snake families go, it was a medium-sized family. In all, there were 12 snakes: a father, a mother, their 6 children, a grandmother and a grandfather, and an aunt and an uncle. They all lived together in their little snake home.

There was something very special about these snakes. Their skin was covered with colorful shapes that looked like bright colored jewels running along their backs in beautiful repeating patterns. One shape that all of the snakes had in common was a hexagon.

The father snake was just about a foot long. He was covered with yellow hexagon and blue rhombus shapes that fit together in a long row. The shapes ran all the way down his back, one colorful shape after another. Now, the mother snake was covered with yellow hexagons, too. But instead of blue shapes, there were red trapezoids in between. Both snakes were beautiful.

In fact, each snake looked different:

* The grandfather snake was covered with yellow, red, and green shapes.
* The grandmother snake was covered with yellow, blue, and green shapes.
* The uncle snake was covered with yellow, orange, and tan shapes.
* The aunt snake was covered with yellow, blue, and tan shapes.
* Each of the children snakes was covered with yellow, blue, and red shapes. But they each looked different.

One day the snakes were going out looking for food, just as the father snake slithered from the hole, Max caught sight of him. "What a lovely snake," Max thought. "The pattern looks like jewels." Max decided the pattern would be perfect to put on the wood frame he was making for a picture. So he carefully studied the way that the blue shapes and yellow hexagons fit together, and he made a copy with some pattern blocks. Then he went to get his wife, Cleo.

By that time, the father snake was gone and the mother snake was just inching out of the hole. But Max didn't notice that the snake he was looking at now was a different snake. His wife, Cleo, said, "That snake is beautiful, but it doesn't look like the pattern you made." "I don't understand it," said Max. "I thought I was being so careful when I copied the pattern."

"Oh, well," said Cleo. "Let's copy the pattern and make another frame." So they did. Then they went to get their daughter, Josephine.

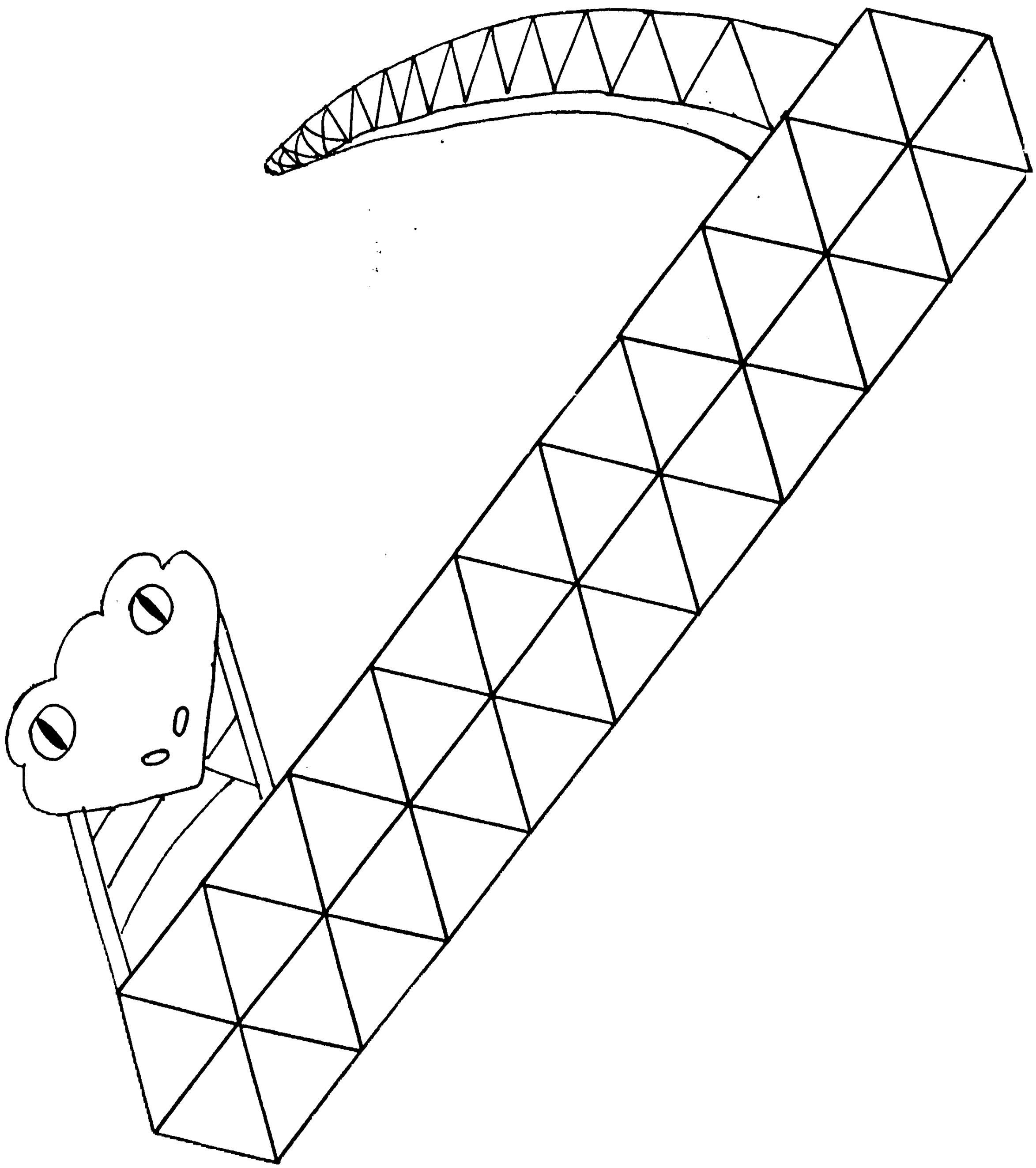
By that time, the mother snake was gone and the oldest snake child was just coming out of the hole. But Max, Cleo, and Josephine didn't know that. Josephine said, "That's a beautiful snake, but it doesn't look like either of the patterns you made."

"You're right, daughter!" replied Max. "Well, let's copy this one, too."

Max, Cleo, and Josephine copied the patterns of the 12 snakes that came out of the hole, one by one. Each time they saw a different snake. So each time the pattern was different. They never did figure it out. But you should see all the beautiful frames they made.

.

The Jeweled Snake **Name:** ________________



Activity: Fibonacci Numbers

Format: Whole Group

Objective: Participants will be able to create, recognize, describe, analyze, and extend a variety of patterns.

Related SOL: K.18, 1.21, 2.25, 3.24, 4.21, 5.20

Materials: Examples or pictures of Fibonacci Numbers, tree branches with leaves, pinecones, pineapple, sunflower or daisy, Patterns and Relationships Activity Sheet

Time Required: 30 minutes

Directions:

1. Say: *Research and our experiences as teachers continue to tell us that children who comprehend patterns in the world around them acquire and retain information much more quickly than those children who do not see or understand patterns. Those children who never seem to comprehend patterns must reinvent the wheel each and every time. For example, the child who never recognizes that a word ending in the letter "e" usually has a long vowel sound must begin the task of determining long or short vowels anew each time they are questioned because no connections have been made. Today's question is exactly how to help all children recognize and use patterns.*

   Teachers must not start from ground zero because each and every child comes to school with many years of experience in patterning. Each time the sun comes up and each time the sun sets, each time the child's father smiles when happy and frowns when upset, each time a light comes on and each time that light is turned off, and each time the child drops his spoon and it falls to the ground that child experiences patterning. Just learning to talk requires a tremendous grasp of patterning.

   Our job as teachers is to help put children's understandings into words. Our job as classroom teachers is to help students to develop their understated knowledge and name what they already know.

   Some patterns in nature have been identified as Fibonacci Numbers, named after an Italian mathematician named Leonardo de Pisa and nicknamed Fibonacci. Today Fibonacci is best remembered for a sequence of numbers that bear his name. This sequence of numbers always begins with 1,1. Each new number is then found by adding the two preceding numbers.

2. Show participants the first six Fibonacci Numbers.

   1, 1, 2, 3, 5, 8, ...

3. Invite participants to work with a partner to determine the next six numbers in the sequence of Fibonacci Numbers.

   1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

4. Say: *The Fibonacci Numbers describe a wide variety of phenomena in music, art, and nature. The numbers of leaves on many plants are Fibonacci Numbers as well as many branches on certain trees.*

   The numbers of spirals on pineapples or pinecones are Fibonacci Numbers. The center of a daisy and a sunflower has clockwise and counterclockwise spirals that are consecutive Fibonacci Numbers.

5. Encourage participants to investigate other examples of Fibonacci Numbers found in nature.
6. Conclude the concrete investigations by moving the participants to the abstract.
7. Display the Patterns and Relationships Activity Sheet on the overhead projector.
8. Assign partners the task of determining pattern commonalities and pattern differences.

Patterns and Relationships

For each pair, give at least one characteristic that the patterns have in common and at least one way that the patterns are different.

Example: 2, 4, 6, 8, 10 ..... versus 3, 5, 7, 9, 11 ....

Same: Difference of 2 between terms / Increasing

Different: One is even numbers and the other is odd

1. 2, 4, 6, 8, 10 ..... versus 2, 4, 8, 16, 32 .....
   Same: Different:

2. 3, 4, 3, 4, 3, 4 .... versus a, b, a, b, a, b .....
   Same: Different:

3. 5, 6, 7, 8, 9 ...... versus 9, 8, 7, 6, 5 .....
   Same: Different:

4. 5, 10, 15, 20, 25 .... versus 3, 8, 13, 18, 23 .....
   Same: Different:

5. 10, 20, 30, 40 ..... versus 100, 200, 300, 400 ....
   Same: Different:

Activity: Grid Pictures

Format: Small group

Objective: The participant will become familiar with the 0-99 chart and will be able to identify number patterns and relationships on the chart.

Related SOL: 1.21, 2.25, 3.24, 4.21

Materials: 0 — 99 chart for each participant, linking cubes or small colored squares of paper that will cover a number on the chart

Time Required: 10 minutes

Directions:

1. Distribute Picture This! Activity Sheet and linking cubes to participants.
2. Participants will cover the appropriate number with the indicated color linking cube as the teacher reads each clue.
3. Use the following example or make up other clues. After participants are familiar with this activity, let them create their own picture and a corresponding set of clues. Participants may exchange clue lists to try new problems.

Example 1: FLOWER

Yellow: 11,12,21,22,16,17,26,27, 51,52,61,62,56,57,66,67

Brown: 33,34,35,43,44,45

Green: 54,64,74,84,94,85,76,93,82

Example 2: UMBRELLA

Red: 4,13,15,22,23,24,25,26, 31,32,33,34,35,36,37, 40,41,42,43,44,45,46,47, 48,50,52,54,56,58

Black: 64,74,84,94,82,92,93

Grid Pictures

Example 1: FLAG

Yellow: 0

Brown: 10, 20, 30, 40, 50, 60, 70, 80, 90

Blue: 11, 12, 21, 22

Red: 13, 4, 15, 16, 17, 18, 19, 31, 32, 33, 34, 35, 36, 37, 38, 39, 51, 52, 53,54, 55, 56, 57, 58, 59

White: 23, 24, 25, 26, 27,2 8, 29, 41, 42, 43, 44, 45, 46, 47, 48, 49

Example 2: TREE

Green: 4, 5, 13, 14, 15, 16, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 51, 52, 53, 54, 55, 56, 57, 58, 61, 62, 63, 64, 65, 66, 67, 68

Brown: 74, 5, 84, 85,94, 95

Example 3: HEART (all red)

a. one more than 53	b one less than 8	c. one more than 17
d. one less than 33	e. one more than 13	f one less than 13
g. one less than 48	h. one more than 37	i one more than 20
j. one more than 24	k one less than 4	l. one more than 55
m. one less than 30	n. one less than 44	o. one more than 15
p. one less than 66		

Example 4: TURTLE (all green)

a. 5 x 7	b. 4 x 9	c. 5 x 13
d. 3 x 15	e. 2 x 23	f. 1 x 47
g. 53 x 1	h. 4 x 16	i. 11 x 5
j. 8 x 9	k. 3 x 23	l. 4 x17
m. 8 x 7	n. 1 x 57	o. 1x 79
p. 1 x 67	q. 3 x 22	r. 4 x 11
s. 2 x 29	t. 8 x 5	u. 2 x 27
v. 1 x 41	w. 2 x 25	x. 1 x 51
y. 2 x 31	z. 3 x 21	

Example 5: KITE

Red

a. 1 less than 2 dozen
B 3 x 11
c. 3 tens and 4 ones
d. all numbers between 30 and 60 with a 2 in the ones place
e. 2 less than 55
f. 1 greater than 40
g. all numbers between 42 - 46
h. 5 tens and 4 ones
i. 70-7

Yellow

j. 1 less than 75
k. 15 less than 100
l. 100 — 4

Example 6: FLOWER

Yellow

1 ten and 1 one	one more than 50	one less than 68	9 x 7
one more than 51	4 x 3	7 x 3	one less than 62
9 x 3	one more than 56	8 x 2	8 x 7
one more than 21	6 tens and 2 ones	one less than 18	one more than 65

Brown:

3 tens and 3 ones	one more than 33	9 x 5	5 x 7
4 x 11		1 x 43	

Green

9 x 6	1 more than 63	1 less than 75	1 less than 85
9 tens and 4 ones	1 more than 84	7 tens and 6 ones	9 tens and 3 ones
2 more than 80			

PICTURE THIS

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Activity: What Comes Next on the "Picture This" Chart

Format: Small groups and pairs

Objective: Participants will explore patterns on the chart. Putting together a hundred chart puzzle requires participants to consider the place value patterns of one more, one less, and ten more and ten less. Participants will be able to identify the vertical and horizontal relationships on the 0-99 chart.

Related SOL: 2.25, 3.24, 4.21

Materials: An overhead 0-99 chart or large wall chart for instructor, "Picture This" charts for participants, "What a Cut Up" cut into puzzle pieces, "What Comes Next" Activity Sheet

Time Required: 15 minutes

Directions:

1. Give each pair of participants a chart such as the Picture This! Activity Sheet. Have participants identify patterns that they see on the chart. They will probably identify the place value patterns dealing with the tens and ones places.
2. Pick a row. Add the digits of consecutive numbers, for example: 30, 31, 32, 33, 34. What pattern do you see? Does it hold in every row of the chart?
3. Now that participants have some familiarity with the chart, give out a chart that has been cut into puzzle pieces (use What A Cut Up Activity Sheet). Before allowing participants to reassemble the hundred chart have them first place the pieces on their desk. What clues do they see that indicate which pieces go together?

   Note: Participants who need assistance with this task may be given an uncut 0-99 chart to use as a base on which to place the cut out pieces.
4. Participants assemble the chart and discuss with their partner the different techniques that could be used to reassemble the chart.
5. Have participants find the piece that has the number 65 on it. Let participants reassemble the 0-99 chart beginning with the piece that has 65 on it. Did they use the same strategies?
6. The What Comes Next? Activity Sheet shows pieces of a 0-99 chart. Have participants fill in the missing numbers and follow with a discussion about the student's reasoning as they filled in numbers.

PICTURE THIS

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

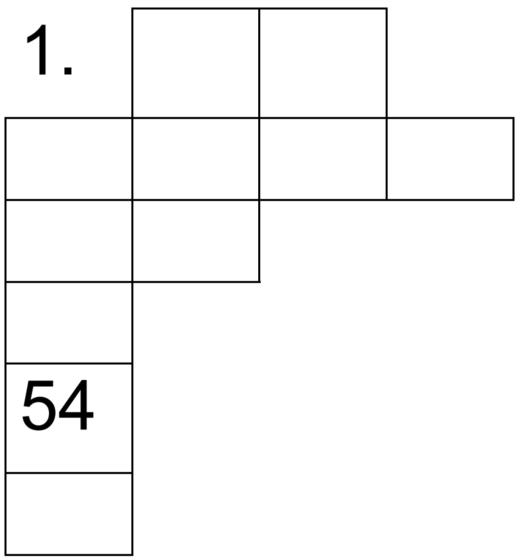
WHAT A CUT UP!
Cut on the dark lines to create puzzle pieces

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

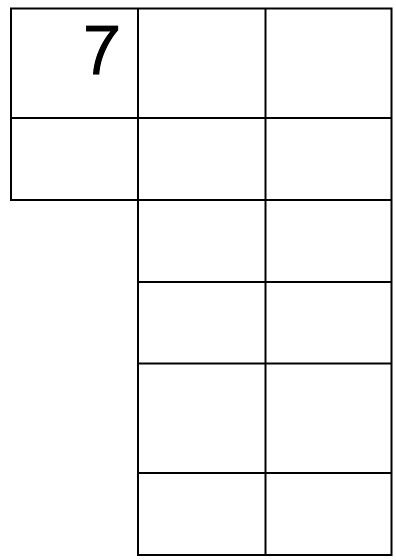
WHAT COMES NEXT?

Here are sections of a 0-99 chart. As you can see, many of the numbers are missing. Your job is to fill in the numbers to complete the section.

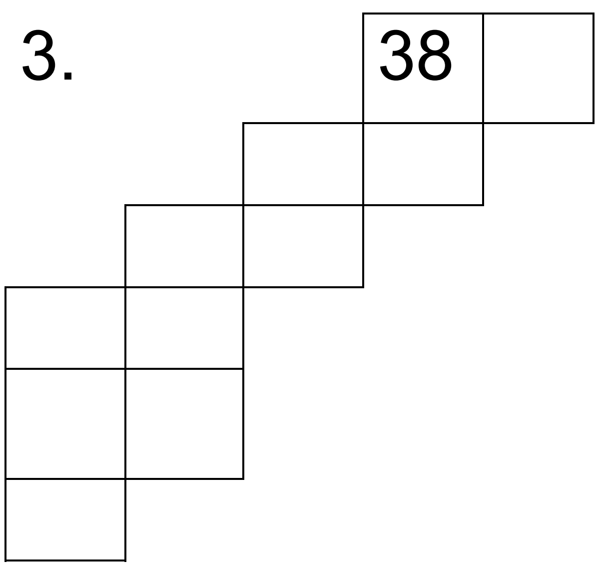
1.



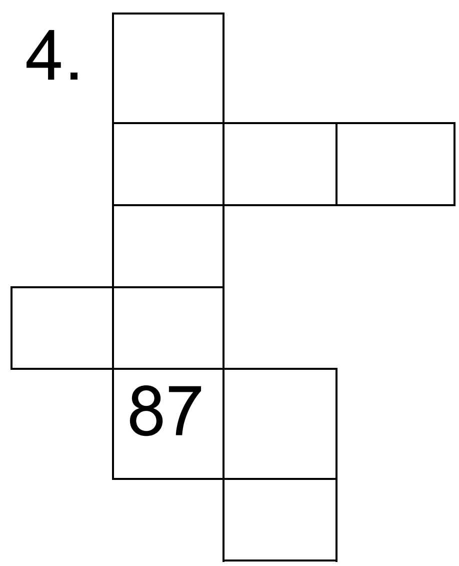
2.



3.



4.



Activity: Patterning on the 100 Chart

Format: Whole Group

Objective: Participants will be able to describe numerical and visual patterns made by skip counting on the 100 chart and connect these patterns to multiplication facts.

Related SOL: 1.21, 2.25, 3.24, 4.21, 5.20

Materials: 100 chart, linking cubes, recording sheets

Time Required: 15 minutes

Directions:

Skip counting can furnish practice with multiples while deepening students' understanding of multiplication facts. Participants are able to observe visual patterns resulting from identifying the multiples of a number.

1. Beginning with the number 2, participants will skip count by 2s marking the multiples of 2s with a linking cube. Participants will begin placing the linking cubes as they skip count, however, many participants will soon see the pattern and begin placing the cubes using the pattern rather than the skip counting. Have participants describe the pattern.
2. Participants will skip count by 3s, 4s, 5s, etc., up to 12s. It is important that participants be able to verbally describe the pattern.
3. At a later time, have participants repeat the previous activity. This time have participants record their findings on the Recording Sheet. After they have skip counted by 3, for example, ask the participants what number is under the 4th three. When they respond 12, reply that 4 threes is equal to 12. Continue this type of questioning so that participants understand the relationship between skip counting the multiples and multiplication.
4. Look at the Recording Sheet. What are the differences and similarities among the patterns? Is 239 a multiple of 6? How do you know? If a number is a multiple of 6, is it a multiple of 2? Of 4?

   Extension: Skip count by 2s and then by 3s, marking the multiples of 2 with one color and the multiples of 3 with a different color linking cube. Which numbers have two colors on them (6,12,18,...) Why?

Why is “Common Multiples” a good name for this set of numbers?

Looking at the numbers that are common multiples, which number is the Least Common Multiple?

Graphing 100 Chart Patterns

1. Look back at the recording sheets where you marked all of the multiples. Transfer these findings to the Multiplication Table worksheet.
2. We want to visually represent each group of multiples. Let's use the 3s as an example to model the process. Look back at the recording sheet where you marked the multiples of 3. The first multiple was 3, the second 6, the third 9 and so on. Record this information in a chart.

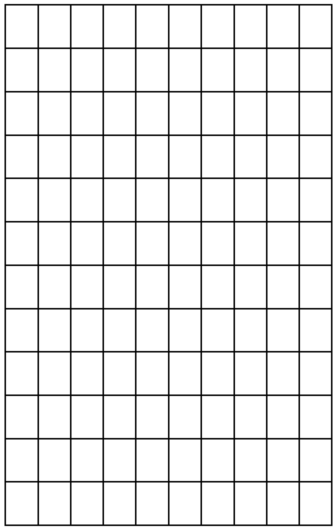
Multiples of 3	
Position of the Multiple	The Value of the Multiple
1	3
2	6
3	9
4	12
5	15

What is the relationship between the position of the multiple and the value of the multiple?

Value = 3 x position

What is the relationship between the chart and the 3 row or column on the multiplication table?

3. Graph the above information on a first quadrant graph where the position of the multiple is graphed on the x-axis and the value of the multiple is graphed on the y-axis. Lay a piece of spaghetti along the points to show that they are linear and can be connected with a straight line.



x-axis

5. Continue making charts and graphing this information for the other multiples on the same graph. Compare graphs with the multiplication table.

6. Participants should look for patterns and relationships on the graph. For example, to find common multiples on the graph, look for the times tables which have points located at 12 on the y-axis. Twelve is a common multiple for those numbers. Another way to state the relationship is that all numbers on the x-axis which have points located at 12 on the y-axis are factors of 12.

Closure

Journal Entry: Explain what is meant by the term "multiple”.

HUNDRED CHART

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

RECORDING SHEET

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Activity: Odd and Even

Format: Whole group

Objective: Participants will be able to represent even and odd numbers using concrete materials and make conjectures about adding and subtracting even and odd numbers.

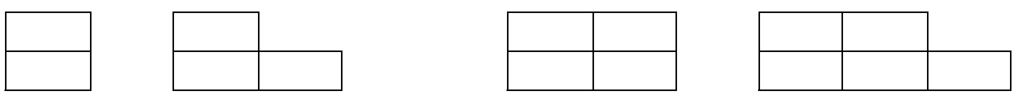
Related SOL: 2.5

Materials: color tiles

Time Required: 5 minutes

Directions:

1. Distribute color tiles to participants.
2. Participants will attempt to create rectangles where one side is two inches high using 2 tiles, then 3 tiles, 4 tiles etc. The pattern should look similar to the following:



2 **3** **4** **5**

3. What do you notice about the pattern shapes?

   Participants should notice that 2, 4, 6, 8, etc., are rectangles where all of the tiles have partners while 3, 5, 7, etc., there is a tile left over that does not have a partner. Use these models to explain why the numbers 2, 4, 6, 8, etc., are called even and 1, 3, 5, 7, etc., are called odd.

4. Model the following scenarios:

   What shape will result if you add two even numbers?

An even + an even will always equal an even number. Why?

What will result if you add two odd numbers?
What about an even and an odd number?

5. Have participants grab a handful of tiles. Count the tiles. Is this an even number or an odd number?

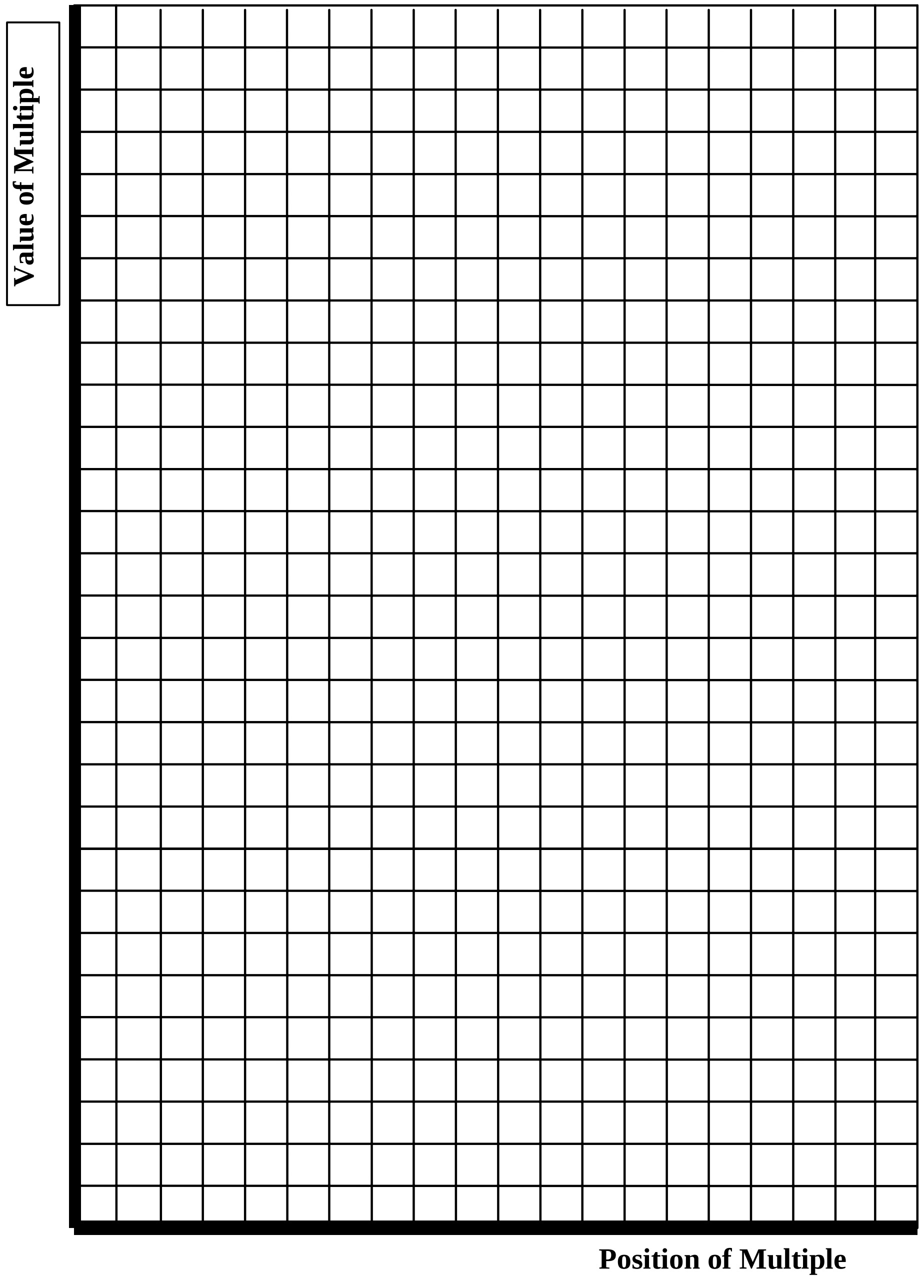
   Attempt to form a rectangle of height 2 to determine if the tiles will all have partners or if there will be an odd man out.

Multiplication Table Activity Sheet

X	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

GRAPHING HUNDRED CHART PATTERNS

Multiple of	
Position of multiple	The value of the multiple



<u>Activity</u>: Line Up!

<u>Format</u>: Whole group

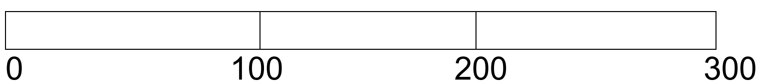
<u>Objective</u>: Participants will be able to identify and locate missing whole numbers on a given number line.

<u>Related SOL</u>: 4.21

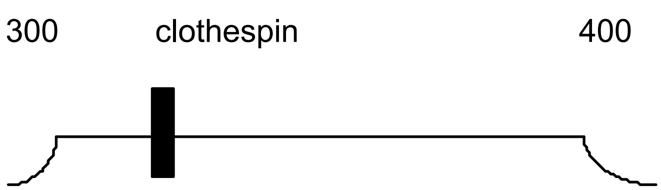
<u>Materials</u>: Line Up! Activity Sheet, meter sticks, yarn, and clothes pins

<u>Time Required:</u> 10 minutes

<u>Activity A:</u>
Tape three meter sticks to the blackboard. Participants identify points for numbers along the line and compare numbers. Play the game Guess The Number I Am Thinking About.



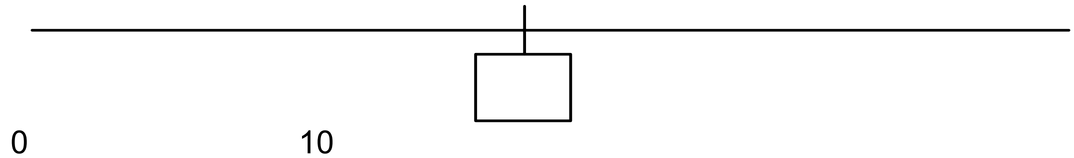
<u>Activity B</u>:
Attach yarn to the blackboard one meter apart and label the endpoints with two numbers that are multiples of 100. Place a clothespin on the yarn to represent the point in question. Participants estimate the number at the clothespin. Place a meter stick along the yarn to check the estimate.

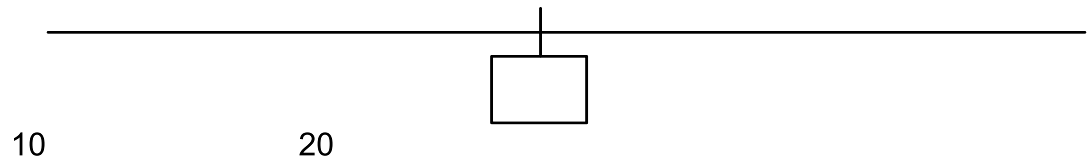


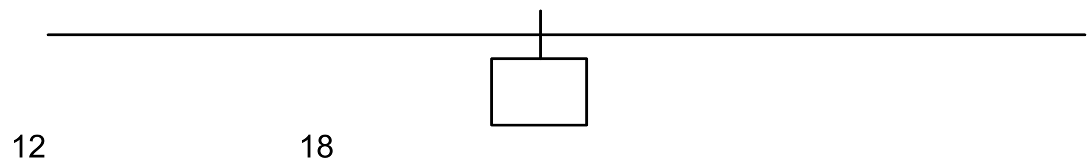
<u>Activity C:</u>
The Line Up Activity Sheet gives participants practice in identifying numbers that are half way between two other numbers. Have participants discuss the strategies they used to fill in the numbers. Additional problems should be used with the missing number in different locations.

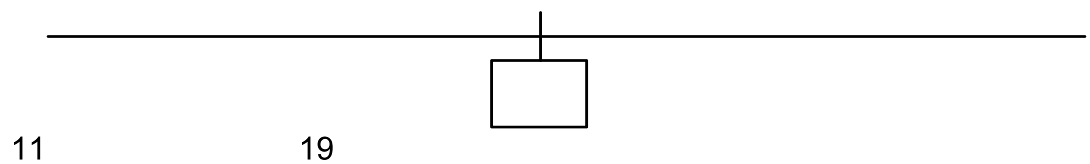
LINE UP!

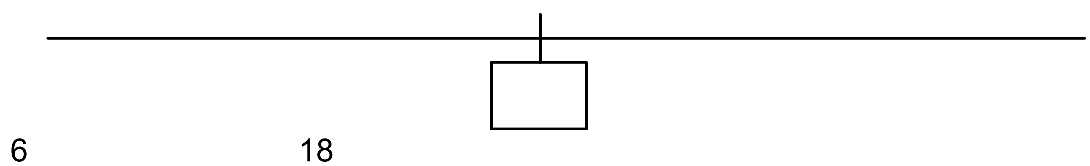
Each of the following segments has a missing number. Fill in the missing numbers. Be able to give a reason for your answer.

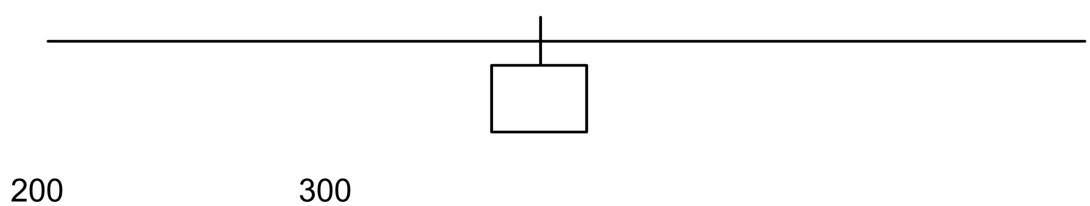












Activity: Arrow Math

Format: Whole Group

Objective: Participants will be able to describe numerical and visual patterns made by skip counting on the hundred chart and connect these patterns to multiplication facts.

Related SOL: 1.21, 2.25, 3.24, 4.21, 5.20

Materials: Arrow Math on the Hundred Chart Activity Sheet

Time Required: 5 minutes

Directions:

1. Have participants discuss this type of activity for developing mathematical reasoning with patterns. For example, if a student goes forward one and back one, what is the total effect; or up and down ten? Will students see the impact of arrow reversals? Will students begin to see how they can reduce the number of arrow steps by canceling out opposites? Further, ask participants how does arrow math reinforce place value concepts?

ARROW MATH ON THE HUNDRED CHART

34 ↓ → → ↓ → → **MYSTERY NUMBER:** ______

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

ARROW MATH ON THE HUNDRED CHART

28 ↓ ← ← ↓ ↓ → ↑ **MYSTERY NUMBER: _______**

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Activity: The King's Commissioners

Format: Whole Group

Objective: Participants will be able to recognize patterns that occur in the selection as they assist the Princess and the Royal Advisors by counting by 2s, 5s, and 10s. Participants will then be able to recognize, create, extend, and describe sequential numerical patterns on a hundreds chart.

Note: *The King's Commissioner* is a wonderful story that encourages students to examine not only place value but also the patterning that occurs in our number system. Participants are invited to count by 2s, 5s, and 10s as they attempt to solve the problems that besiege the royal kingdom. The story is mathematically sound because it reinforces the important mathematical concept that there are different ways to think about and solve the same problem. Perhaps the greatest value of the selection is in the very fact that all the thinking and pondering of the Royal Advisors and the Princess are correct just different.

Related SOL: 3.24, 4.21, 5.20

Materials: *The King's Commissioners*, hundreds boards (1 - 100), centimeter cubes, crayons or markers

Time Needed: 20 minutes

Directions:

1. Read *The King's Commissioners* by Aileen Friedman in its entirety to the class in order to describe to participants how the process of counting from 1 to 100 can be simplified with the recall of basic multiplication combinations.
2. Explain to the participants that they are going to find multiples of the numbers 2, 3, 4, 5, 6, 7, 8, 9, and 10 on a 100 chart. Remind participants that they should look for patterns on their charts and anticipate the patterns they will find.
3. Instruct participants to place a 100 chart in front of them as well as 15 centimeter cubes.
4. Explain to the participants that they are going to locate all the multiples of 2 and may begin by placing a blue cube on the number 2.

5. Ask the participants to place a blue cube on every other consecutive square on the 100 chart starting with the number 2. They may begin by orally reciting 1, 2, 1, 2. Whenever they recite 2, they should place a blue cube on the chart. They should place all 15 cubes on the numbers 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, and 30.
6. Explain to the participants that the numbers covered with blue cubes are multiples of 2. Ask participants if they notice a pattern and if they recognize other numbers on the chart that are multiples of two.
7. The participants should then color with a blue crayon or marker all the multiples of 2.
8. Participants should then be provided with another 100 chart and different color cubes.
9. Explain to the participants that they are now going to locate the multiples of 3.
10. Instruct participants that they are to use pink cubes for 3, green cubes for 4, purple cubes for 5, and so on.
11. Participants then arrange the cubes on the chart and then subsequently color the chart to show the multiples. Remind participants to always look for a pattern and attempt to extend the pattern to show all possible multiples of the number on the 100 chart.
12. Arrange all the completed charts of the multiples where the participants can make observations. Discuss the patterns that have become obvious.

Activity: *The King's Chessboard*

Format: Whole Group

Objective: Participants will be able to create a table that will allow them to see the results of exponential growth.

Note: *The King's Chessboard* by David Birch enables participants to learn the effects of doubling numbers and the pattern that is created by this geometric expansion.

Related SOL: 3.24, 4.21, 5.20

Materials: *The King's Chessboard*, 100 chart (1-100), grains of rice

Time Required: 20 minutes

Directions:

1. Begin this lesson by reading the selection *The King's Chessboard* to the class.
2. Some of the participants may already be familiar with the story of the pauper who asked the king to repay his kindness by placing a grain of rice on the king's chessboard and then doubling it for every one of the 64 squares on the board. Without thought into the matter, the king assumed that this was a small request but it ended up costing him his kingdom.
3. Explain to participants that they are going to begin the activity by putting one grain of rice on square one and then doubling this to two grains of rice, then doubling this to four grains on square three.
4. Ask participants to estimate and record how many squares they will be able to cover effectively before they run out of rice.
5. Most participants will be amazed at how quickly these numbers multiply and depending upon the grains of rice each student has, few participants will be able to continue far on the hundreds board chart.
6. From this information, have participants create a table that will show the resulting number on the 64 squares of the king's chessboard. Allow participants to use the calculator to assist in their efforts.
7. Encourage participants to attempt to write a rule and to create a formula that will allow them to deduce the last number.

Variables
Session 3

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Variables	Patio Tiles	104	3.25, 4.22, 5.20, 5.21, 5.22	Patio Tiles Participant Sheet, Patio Tiles Possibilities Overhead Sheet	Patio Tiles overhead
	The Varying Uses of Variables`	108	5.21, 5.22	Varying Uses of Variables Activity Sheet, Concept of a Variable reprint, Sources of Confusion in Algebra	Blank overhead transparencies, overhead markers
	Open Sentences	116	1.8, 2.10, 2.26, 3.4, 5.20, 5.21	Variables in Open Sentences Activity Sheet	Scissors
	Number Magic	120	3.8, 3.10, 4.7, 4.8, 4.9		Overhead transparency, with large number 5 written in lemon juice, a large ashtray, matches
	Number Magic Explanation - Manipulatives	121	5.20		Manipulatives (raisin boxes/ color tiles, or place value rods/unit pieces
	Number Magic Explanation - Visual	123	5.20	Number Tricks Activity Sheets, Number Tricks Answer Sheet; Number Magic Answer Key, with Algebraic Column, Answer Key; Number Magic Problem #2, with Algebraic Column; Number Magic Problem #3, with Algebraic Column	Manipulatives

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Sample Assessments	Patterning and Variables	136		Patterning and Variables Activity Sheets	Sample Assessments
	Popping for Pennies	144	4.5, 4.22	Popping for Pennies Activity Sheet	Sample assessments, scoring rubric

Activity: Patio Tiles

Format: Whole group

Objective: Participants, given the visual representation of the patio problem, will generate equivalent number sentences to describe the method they used to calculate the total number of tiles needed to complete the border of a patio. The patio problem provides a meaningful context in which to generate equivalent numerical expressions and to lay the foundation for the concept of variables.

Related SOL: 3.25, 4.22, 5.20, 5.21, 5.22

Materials: Patio Tiles, Patio Tiles Participant and Possibilities Activity Sheets

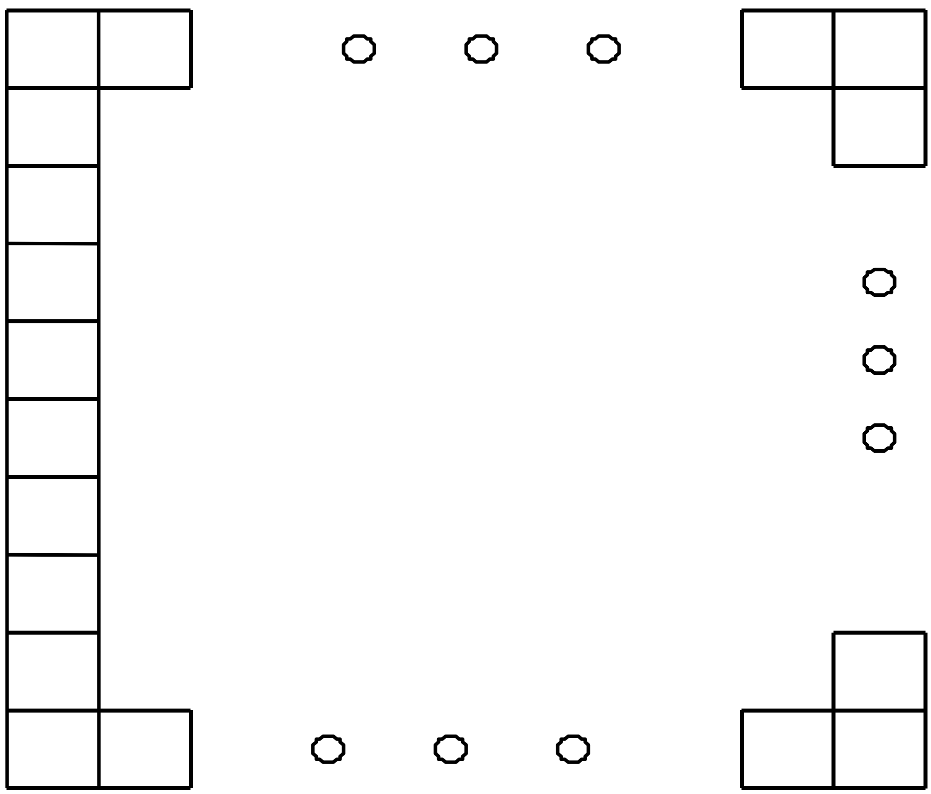
Time Required: 30 Minutes

Directions:

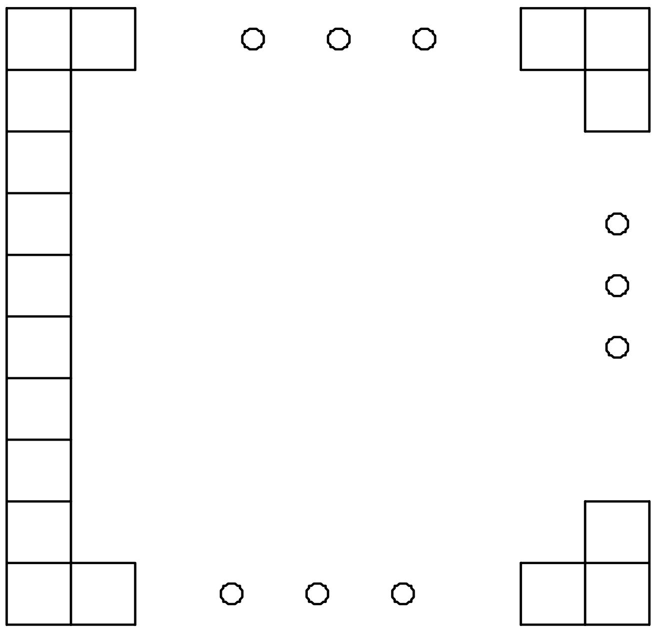
1. Distribute copies of the Patio Tiles Activity Sheet to the teachers.
2. Place the overhead of the Patio Tiles problem on the overhead and ask the teachers to write down the number sentence which represents the way in which they visualize the patio border as they work to determine the number of tiles needed to complete the border. Have teachers work individually on this problem, generating as many possible answers as possible.
3. Hand out transparencies with the printed image and select teachers to draw their picture solutions on the transparency and have them write the number sentence represented. Participants should present these to the other participants.
4. Display the overhead of the Possibilities Sharing equivalent expressions connecting the expressions of various participants' solutions. Translate some of the responses to algebraic expressions using variables.

PATIO TILES

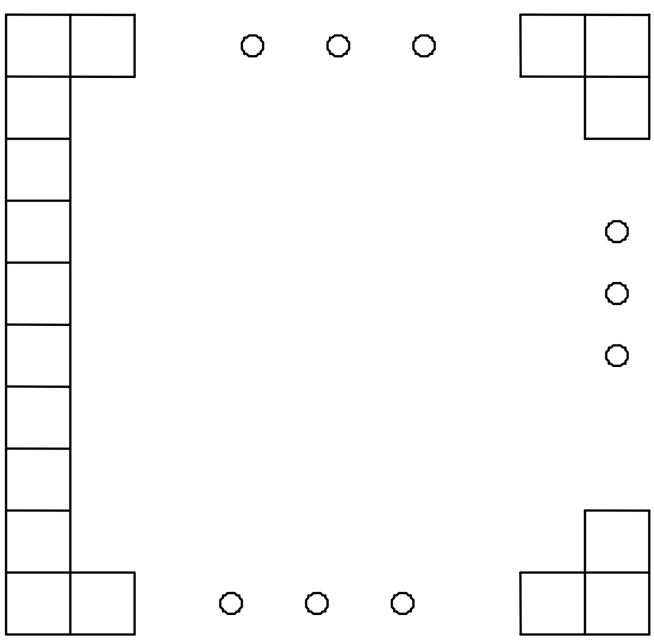
Tiles are placed to make the boundary of a square patio with 10 tiles on a side. How many tiles will be used to make the complete boundary of the square patio? Solve the problem in as many different ways as you can.



PATIO TILES



Numerical Expression: _________



Numerical Expression: _________

PATIO TILES

Answer: 36 tiles are needed to make the square

Number Sentences Representing the Approach Used to Calculate the Total Number of Tiles in the Square	Approaches to Calculating the Total Tiles in the Square
counting one-by-one to get 36	Counting
10 + 9 + 9 + 8 = 36	
(2 x 10) + (2 x 8) = 36	Grouping
(4 x 8) + 4 = 36	
(4 x 3) + (4 x 6) = 36	
2 x (10 + 8) = 36	Symmetry
(2 x 17) + 2 = 36 *...(17 = 9 + 8; plus two corners)*	
4 x { 8 + (2 x 1/2)} = 36 *...(half corners)*	
(4 x 10) - 4 = 36 *...(minus the corners)*	Generalization
4 x 9 = 36	

<u>Activity</u>: The Varying Uses of Variables

<u>Format</u>: Small groups

<u>Objective</u>: Participants will develop an understanding of the three ways in which a variable is used-- as specific unknowns, as generalizers, and as varying quantities. The first activity in this workshop provides an opportunity for participants to brainstorm about the various uses of variables and ways to address confusion that participants have about these uses.

<u>Related SOL</u>: 5.21, 5.22

<u>Materials</u>: Blank overhead transparencies, overhead markers, Varying Uses of Variables Activity Sheet, Concept of a Variable reprint, Sources of Confusion in Algebra

<u>Time Required</u>: 20 minutes

<u>Directions</u>:

1. Ask each group to generate examples of the different ways that a variable may be used. Have them write them on an overhead and a have group leader present them to the class.
2. Have participants read the reprint of Concept of the Variable (reprinted information from the <u>Partners in Change Project</u> at Boston University).
3. Use the ideas presented by the small groups to summarize the three types of varying uses of variables that are described in the article. Use the transparency of "The Varying Use of Variables" to guide the discussion.
4. Have participants discuss the points on a transparency of "Some Sources Of Confusion In Algebra".

Concept of the Variable

One of the most important ideas in algebra is the concept of variable. Historically, algebra has progressed through three major stages, each defined by the concept of variable prevalent during that period. Algebra in its earliest stage did not include symbols but rather used ordinary language to describe solution processes for certain types of problems. The second stage (ca. AD 250-1600) included the use of symbols to represent unknown specific quantities with the goal of solving problems for the unknowns. In the third stage (1600-present), symbols have been used to express general relationships, to prove rules, and to describe functions.

Francoise Viete (1540-1603) was the first to use letters in formal mathematics notation. Shortly thereafter, Rene Descartes used letters in a more systematic way: *a, b, c* for constants and *x, y, z* for the unknowns. When Descartes went to have his manuscript, *La Geometrie*, published in 1637, the printer found that Descartes was using too many of certain letters, so much so that the printer was running out of some of the type-set letters. He communicated with Descartes, asking if it mattered which letters represented the unknowns. Descartes replied that the specific letter was unimportant, as long as the unknown was represented by *x, y, z*. The printer, Jan Maire of Leyden, Holland, had plenty of the letter *x* on hand, so he used *x* to represent the unknown. Thus, Maire's decision contributed to the fact that we now "solve for *x*" in algebra!

As the concept of variable developed historically, uses of letters expanded. Experts categorize variables in different ways, but there is general agreement that any particular use of a variable is determined by the mathematical context. The most common uses of letters are as specific unknowns, as generalizers, and as varying quantities.

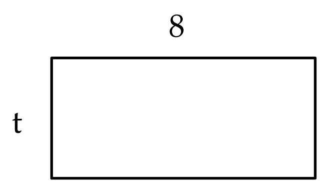
In recent decades, mathematics education researchers have begun investigating participants' usage of variables. Research by Dietmar Kuchemann (1978) on the use of letters in equations is presented for your information. Kuchemann reserved the term *variable* for letters that represent varying quantities.

- *Letter evaluated*. When a letter is evaluated, it is immediately (often mentally) replaced with a number. For example, in a + 5 = 8, participants would solve the equation by repeatedly substituting values for a (perhaps using trial and error) until a value of 8 was produced on the left side of the equation.

- *Letter ignored*. When a letter is ignored, computations are performed on the numbers in an equation as if avoiding the letter. For example, to solve 12a + 20 = 44, participants would use "undoing" or inverse operations (subtract 20 from 44 and divide the result by 12). Although the participants don't actually ignore the letter, they do "work around it." Participants who ignore the letter can solve equations such as ax + b = c but cannot solve equations such as ax + b = cx + d with letters on both sides.

- *Letter as object*. When a letter is used as an object, it is manipulated as a "thing". Participants often manipulate letters when solving equations by combining similar terms (e.g., 3x + 2x = 5x). When using algebra tiles to model polynomial operations or equations solutions, participants manipulate x tiles and x^2 tiles as objects. The fact that the letter x represents a specific unknown number does not have to be considered consciously in order to manipulate the symbols and correctly solve an equation.

- *Letter as specific unknown*. In this case, letter refers to a specific but unknown number. Participants are able to work with letter in an equation to represent the relationships between quantities without first replacing the letter with a particular numerical value. For example, participants could represent the area of the rectangle below by 8t, with the knowledge that t represents some specific amount.



  Familiar responses from participants such as, "I can't do it because I don't know both sides" and "There isn't an answer because you don't know what it is" reflects participants' struggles with the variable in this situation.

- *Letter as a generalizer*. When a letter is used as a generalized number, it represents the values of a set of unknown numbers rather than a single value. Letters in identities or properties such as 1 x a = a or 1 = n (1/n) convey relationships that are always true about multiplication of real numbers.

- *Letter as varying quantity*. In this case, the letter takes on a range of values and has a systematic relationship with another letter, as in a function of formula (e.g., y = 2x). Letters in a formula represent varying numbers that in the stated equation explain a real-world relationship. For example, the equation F = 32 + (9/5)C represents the relationship between Fahrenheit and Centigrade measures of temperature.

Kuchemann identified his first three categories of usage of variable (evaluated, ignored, as an object) as being the most elementary. At this level, participants may be able to get the correct answer to a symbolic equation without really thinking of the variable as representing a number. Such limited understanding of variables may be the cause of participants' inability to identify or interpret the variable in problem solutions, or to generate equations to represent relationships presented in prose.

When participants can use variables as specific unknowns, generalized numbers or varying quantities, they are exhibiting what Kuchemann considers a higher understanding of the concept of variable. They are able to accept the lack of closure of not knowing what number(s) are represented by a specific variable. Participants reach Kuchemann's highest level of use of variables-as varying quantities-when they are able to treat variables in many ways (object, specific unknown, generalizer, and varying quantity) based on the context and complexity of the situation.

In the process of learning algebra, participants often are unaware of or confused by the different meanings for letters in equations, inequalities, and functions. For example, if participants understand variables only as representations of specific unknowns, it not surprising that they have difficulty interpreting inequalities where variables represent sets of numbers.

Reprinted information from the Partners in Change Project at Boston University

The Varying Uses of Variables

Variable As Specific Unknowns:

- When Variables Are Immediately Evaluated

When a variable refers to a fixed but unknown quantity with only one possible value that is often immediately evaluated and replaced with a number.

Example: $t + 2 = 6$

- When Variables Are Ignored

When computations are preformed on the numbers in an equation as if avoiding the letter.

Example: $12a + 20 = 44$

$12a = 44 - 20$

$a = 24/12 = 2$

- When Variables Are Used As Objects

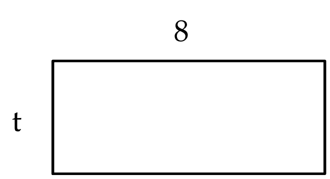
When a variable is used as an object, it is manipulated as a “thing”.

Example: $3x + 2x = 5x$

- When Variables Are A Specific Unknown

When a variable refers to a specific but unknown number.

Example: area of the rectangle = 8t



The Varying Uses of Variables

Variable As Generalizes

- When Variables Are Generalizes

When a variable represents the values of a set of unknown numbers rather than a single value.

Example: Letters in identities or properties

1 x a = a

1 = n (1/n)

Variable As Varying Quantities

- When Variables Are Varying Quantities

When a variable takes on a range of values and has a systematic relationship with another letter, as in a function of formula.

Example: y = 2x

F = 32 + (9/5)C

Some Sources Of Confusion In Algebra

- Variables Often Depend Upon Their Context

- The most familiar use of variables in algebra is as a specific unknown in an equation.
  Example: $20 + 2x = 15$

- A given variable represents the same number throughout an equation no matter how many times it appears.
  Example: $5x+ 2x = 14$

- Sometimes variables are used as a constant.
  Example: c = 186,000 miles per second
  where c represent the speed of light

- Different variables do not always equate to different solutions. In these equations the solutions are not different.
  Example: $3m + 4 = 19$
  $3x + 4 = 19$

Some Sources Of Confusion In Algebra

- Sometimes variables in a single equation can have different uses.

  Example: the standard form of the equation of a line in coordinate geometry, $y = mx + b$.

Here, b and m represent constants specific to the equation, and y and x are covarying quantities, with x being the independent and y being the dependent variable.

- Sometimes there are multiple values for x that will make the equation true.

  Example: $12 = Y * Y - Y$

  Solutions: $Y = 4$ and $Y = -3$ satisfy the equation

- Often in measurement, variables are used as labels.

  Example: 3F = 1Y

Here, the letters serve as abbreviations for the words they represent, three feet equal one yard. The variables DO NOT mean three times the number of feet equals one times the number of yards.

Activity: Open Sentences

Format: Partner Pairs

Objective: By identifying a number to replace the variable in an "open sentence", participants create what is often referred to as a "true sentence". Participants will review various types of open sentences and identify a sequence of steps for developing the concept of open sentences in grades K - 5.

Related SOL: 1.8, 2.10, 2.26, 3.4, 5.20, 5.21

Materials: Variables in Open Sentences Activity Sheet, scissors

Time Required: 30 minutes

Directions:
1. On the Variables in Open Sentences Activity Sheet, have the participants each individually solve the simple "open sentences" to make them "true sentences".
2. Then, have the participants work in partner pairs. Ask one of the participants in their partner pairs to cut-up their handout into sixteen cards.
3. Ask the participants to determine a sequence they would use when teaching and developing the concept of "open sentences" in grades K-5. Have them group the cards into a sequence for those they would use in Grades K and 1, in Grades 2 and 3, and in Grades 4 and 5. Have participants write the letters of the cards in the sequence they would use to develop the concept of variables in open sentences for each of the grade level groupings.
4. Although there is no specific order to the set of cards, the sequence of cards should be considered in groupings of cards, earlier cards being developmentally appropriate for Grades K and 1, middle being appropriate for Grades 2 and 3, and the more difficult and complex for those in Grades 4 and 5.

One Possible Grouping:

Grades K and 1 Cards:
F, J, I, B

Grades 2 and 3 Cards:
E, A, C, H

Grades 4 and 5 Cards:
D, L, G, K, M, N, O, P

Variables in Open Sentences

A.

$\square + \square + \square = 9$

$\square + \triangle = 8$

$\square =$ ____

$\triangle =$ ____

D.

$\square + \square = 12$

$\square + \triangle + \triangle = 10$

$\square =$ ____

$\triangle =$ ____

B.

$5 + \square = 8$

$\square + \triangle = 7$

$\square =$ ____

$\triangle =$ ____

E.

$\square + \square = 8$

$\square + \triangle = 9$

$\square =$ ____

$\triangle =$ ____

C.

$\square + \square = 8$

$\square + \triangle + \triangle = 10$

$\square =$ ____

$\triangle =$ ____

F.

$\square + 3 = 5$

$\triangle + 2 = 6$

$\square =$ ____

$\triangle =$ ____

G.

$\square + \square + \square = 12$

$\square - \square + \triangle = 3$

$\square$ = ____

$\triangle$ = ____

J.

$\square + 3 = 8$

$8 - \square = 3$

$\square$ = ____

$\triangle$ = ____

H.

$\square + \square + \square = 15$

$\square - \triangle = 1$

$\square$ = ____

$\triangle$ = ____

K.

$\square + \square + \square = 15$

$\square - \triangle - \triangle = 1$

$\square$ = ____

$\triangle$ = ____

I.

$\square + 4 = 5$

$\square + \triangle = 3$

$\square$ = ____

$\triangle$ = ____

L.

$\square + \square = 14$

$\square - \square + \triangle = 5$

$\square$ = ____

$\triangle$ = ____

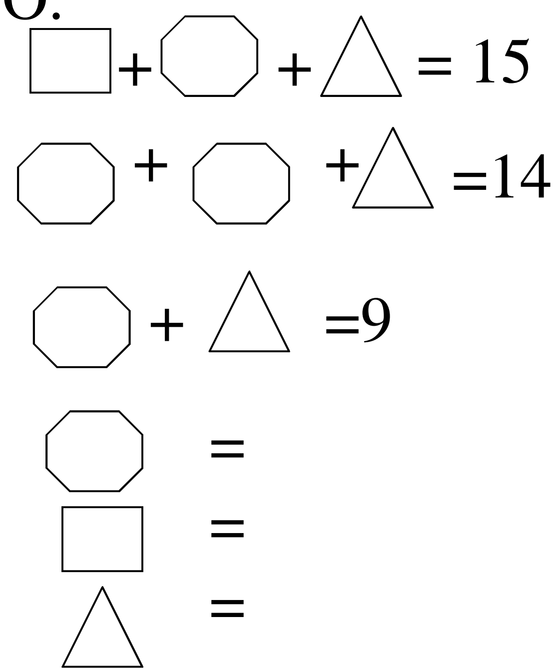
Variables in Open Sentences

M.

Which of these phrases describes the expression $3x - 12$?

A. twelve less than three times a number
B. twelve times three less than a number
C. three less than twelve times a number
D. twelve more than three times a number

O.



N.

Solve $12 + n = 21$

A. 33
B. 12
C. 9
D. 4

P.

Evaluate $3s - t$ for $t = 6$ and $s = 4$

A. 18
B. 6
C. 14
D. 9

Activity: Number Magic

Format: Whole group

Objective: Participants will use numbers to represent mathematical expressions in a number trick.

Related SOL: 3.8, 3.10, 4.7, 4.8, 4.9

Materials: Overhead transparency with a large number 5 written in lemon juice (the overhead must be prepared ahead so that the lemon juice can dry), a large ashtray, matches

Time Required: 10 minutes

Directions:

1. Announce to the participants that you are able to read their minds and that you are going to do an activity to prove it. Have a participant go to the board or overhead projector while you position yourself so that you cannot see the participant's work. As you call out instructions, the participant at the board will work the problem.

   Here are the instructions:

   Choose any whole number.
   Add 7.
   Multiply by 2. (Double it.)
   Subtract 4.
   Divide the number by 2.
   Subtract the original number.
   Write your answer on a small piece of paper.
   Fold the paper so that the answer is not visible.

2. Explain that, while you know what the hidden number is (you can read their minds), you don't want to make it too easy on yourself! Take the paper and carefully set fire to it while holding it over a large ashtray. Place the burning paper into the ashtray. After the flames are out, dump the ashes onto the prepared overhead transparency and turn the overhead on. As you rub the ashes across the overhead transparency, the heat from the ashes causes the number 5 to appear.

The obvious question is "How did you do that?" The following activities lead participants through a visual and pictorial explanation.

Activity: Number Magic Explanation - Manipulatives

Format: Small group

Objectives: Participants will be able to use manipulatives to model a series of mathematical statements in the form of a number trick.

Related SOL: 5.20

Materials: Manipulatives of choice such as raisin boxes and color tiles, or place value rods and unit pieces

Time Required: 10 minutes

Directions:

1. After seeing the number trick performed, participants are eager to know how the trick works. Have the class follow the instructions of the number trick with a person choosing a number of their choice so that they see that the answer is always 5. Have participants make observations and discuss their ideas about why everyone got the same number.
2. Provide manipulatives so that they may follow along as the instructor uses manipulatives to model the instructions. The following example will use boxes and color tiles. Participants must understand that the box holds a certain number of tiles and the number of tiles in the box can vary from problem to problem. However, in a given problem, once a number of tiles has been assigned to a box, that number cannot be changed, and every box in that problem must contain the same number of tiles.

   When participants hear the instruction “Choose any whole number” they will put a number of color tiles in the box. The number of color tiles that they use is their choice. Every box used in this problem however, must have the same number of tiles in it. Tiles cannot be added to or taken away from inside the box during this problem.

   After participants are comfortable with the concept of the box taking on different values for different problems they should be encouraged to remember the value of the box rather than always putting the tiles inside. Eventually, we want them to think of the box as “some number” rather than a specific number.

3. The following is a re-enactment of the magic trick problem with a picture of the manipulatives that the participants will be using.

Choose any whole number.

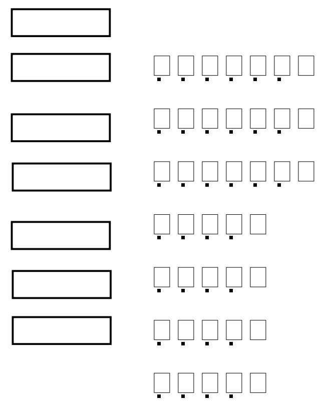
Add 7.

Multiply by 2. (Double it.)

Subtract 4.

Divide the number by 2.

Subtract the original number.



4. Participants are able to see that the box (the number they chose) is not the solution. They have a picture that shows that no matter what number you start with you will always end up with the number

Activity: Number Magic Explanation - Visual

Format: Individual

Objective: Participants will be able to draw a visual representation of a series of mathematical statements in the form of a number trick.

Related SOL: 5.20

Materials: Manipulatives; Number Tricks Activity Sheets, Number Tricks Answer Sheet; Number Magic Answer Key, Number Magic with Algebraic Column, Number Magic Answer Key; Number Magic Problem #2, Number Magic Problem #2 with Algebraic Column; Number Magic Problem #3, Number Magic Problem #3 with Algebraic Column

Time Required: 20 minutes

Directions:

1. Once participants are proficient with the modeling it is time to move to drawing visual representations of the modeling. This step is the link between the modeling with manipulatives that we did in the previous activity and the abstract algebraic method that we will use in the following activity. This is a very important step. Rectangles (▭) will be used to represent the boxes and squares (□) will be used to represent the color tiles.

2. Consider this number trick:
   Choose any whole number.
   Add the next consecutive number.
   Add 7.
   Divide the sum by 2.
   Subtract your first number.

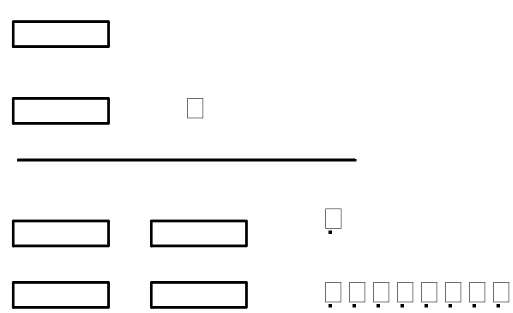
3. Have the participants model with their manipulative. Show how to draw a picture that will visually represent the actions that they have just completed. Here is the same example represented pictorially.

   Choose any whole number:

   Add the next consecutive number

   The result is:

   Add Seven



Divide by 2

Subtract the Original Number:

4. Overhead transparencies can be made from the Number Magic charts provided. These can be useful as you show participants how to move between the visual, verbal, and numerical models.
5. Some discussion should take place concerning vocabulary. For example, consider the different ways that instructions could be given:

   Double the number and take away two.
   Take twice the number, then minus two.
   Multiple by two, subtract two.

   These activities offer teachers an interesting way to teach vocabulary. Have participants state the instructions as many different ways as they can.

6. The Number Tricks Activity Sheet gives some additional tricks to try.

Closure

Write the clues to a number trick. Show how your trick works with numbers, and picture.

Number Tricks

1. Write down any number.

   Add 4.

   Multiply by 2.

   Subtract 4.

   Divide by 2.

   Subtract your original number.

   Write down your answer.

2. Write down any number.

   Multiply by 3.

   Add 6.

   Subtract the number you originally wrote down.

   Divide by 2.

   Subtract 3.

   Write your answer.

3. Write down any number.

Add 10.

Multiply by 2.

Subtract 8.

Add your original number.

Divide by 3.

Subtract 4.

Write down your answer.

NUMBER TRICKS

1.	Write down any number.	n
	Add 4.	n + 4
	Multiply by 2.	2(n + 4) = 2n + 8
	Subtract 4.	2n + 8 - 4 = 2n + 4
	Divide by 2.	(2n + 4)/2 = n + 2
	Subtract your original number.	n + 2 - n = 2
	Write down your number.	2
2.	Write down any number.	n
	Multiply by 3.	3n
	Add 6.	3n + 6
	Subtract the number you originally wrote down.	3n + 6 - n = 2n + 6
	Divide by 2.	(2n + 6)/ 2 = n + 5
	Subtract 3.	n + 3 - 3
	Write your answer.	n
3.	Choose any number between 1 and 9.	n
	Multiply by 5.	5n
	Add 3.	5n + 3
	Multiply by 2.	10n + 6
	Choose another number between 1 and 9 and add to the product.	10n + 6 + x
	Subtract 6.	10n + x
	What is your answer?	(2 digit number where the first number chosen is the tens digit and the second number chosen is the ones digit.)
4.	Choose and number.	n.
	Add the number of days in Nov.	n + 30
	Multiply by the number of days in a school week	5(n + 30) = 5n + 150
	Subtract the number of years in a century.	5n + 150 - 100 = 5n + 50
	Double the answer.	10n + 100
	Delete the “0” in the ones place.	n + tens
	Subtract the original number.	n + tens - n = tens
	What is the result?	tens

NUMBER MAGIC for Mind Reading Trick

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)
Think of a number from 1 to 10.		
Add 7.		
Multiply by 2 (double it).		
Subtract 4.		
Find ½ of the result.		
Subtract the original number.		
Write your answer.		

NUMBER MAGIC - Answer key for Mind Reading Trick

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)
Think of a number from 1 to10.	7	
Add 7.	14	
Multiply by 2 (double it).	28	
Subtract 4.	24	
Find 1/2 of the result.	12	
Subtract the original number.	5	
Write your answer.	5	

NUMBER MAGIC - Mind Reading Trick with Algebraic Column

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)	ALGEBRAIC (Abstract)
Think of a number from 1 to 10.			
Add 7.			
Multiply by 2 (double it).			
Subtract 4.			
Find ½ of the result.			
Subtract the original number.			
Write your answer.			

NUMBER MAGIC - Answer key for Mind Reading Trick with Algebraic

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)	ALGEBRAIC (Abstract)
Think of a number from 1 to 10.	7		n
Add 7.	14		n + 7
Multiply by 2 (double it).	28		2n + 14
Subtract 4.	24		2n + tens
Find 1/2 of the result.	12		n + 5
Subtract the original number.	5		n + 5 - n
Write your answer.	5		5

NUMBER MAGIC PROBLEM #2

Fill in the other columns

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)
Think of a number from 1 to 10.		
Multiply your number by 6.		
Add 12 to the result.		
Take half.		
Subtract 6.		
Divide by 3.		
Write your answer.		

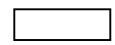
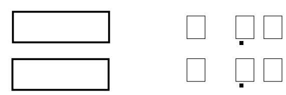
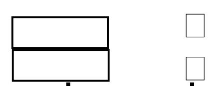
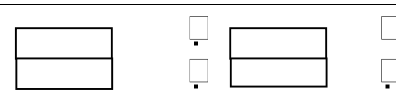
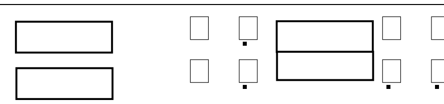
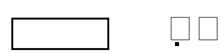
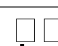
NUMBER MAGIC PROBLEM #2 WITH ALGEBRAIC COLUMN

Fill in the other columns

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)
Think of a number from 1 to 10.		
Multiply your number by 6.		
Add 12 to the result.		
Take half.		
Subtract 6.		
Divide by 3.		
Write your answer.		

NUMBER MAGIC PROBLEM #3

Fill in the other columns

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)
		
		
		
		
		
		
		
		

NUMBER MAGIC PROBLEM #3 WITH ALGEBRAIC COLUMN

Fill in the other columns

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)	ALGEBRAIC (Abstract)

Activity: Patterning and Variables

Format: Partner Pairs

Objective: Participants will match assessments with SOL and describe the action verbs that the assessment is measuring.

Related SOL: All Standards of Learning in the Patterns, Functions and Algebra strand.

Materials: The sample assessments, Patterning and Variables Activity Sheets

Time Required: 20 minutes

Directions:

1. Distribute the handouts and have participants work in partner pairs to review the assessments.
2. Have participants determine what action is being required by the assessment - to extend a pattern, to create a pattern, etc.
3. Have participants identify the SOL that could be assessed with the assessment.
4. The following assessments have been adapted from TIMSS (Third International Mathematics and Science Study) Population 1 Item Pool.

Here is part of a wall chart that lists numbers from 1 to 100.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25					

Below is part of the same wall chart. What number should be in the box with the question mark inside?

43	
53	
	?

A. 34
B. 44
C. 54
D. 64

Which pair of numbers follows the rule, "Multiply the first number by 5 to get the second number?"

A.	**15**	**3**
B.	**6**	**11**
C.	**11**	**6**
D.	**3**	**15**

Here is the beginning of the pattern of tiles.

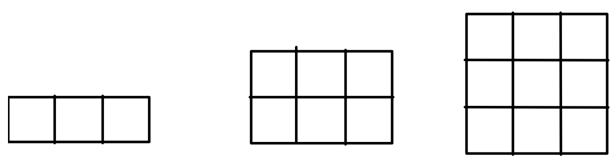


Figure 1 Figure 2 Figure 3

If the pattern continues, how many tiles will be in Figure 6?

A. 12
B. 15
C. 18
D. 21

These numbers are part of a pattern.

50, 46, 42, 38, 34, ...

What do you have to do to get the next number?

Answer: ______________________________

A. Add 4

B. Subtract 2

C. Add 2

D. Subtract 4

These shapes are arranged in a pattern.

O ∆ O O ∆ ∆ O O O ∆ ∆ ∆

Which set of shapes is arranged in the same pattern?

A. ★□★□★★□□★★□□

B. □★□□★□□□★□□□

C. ★□★★□□★★★□□□

D. □□★★□★□□★★□★

If c - 4 = 7, where c = the number of cars in the parking lot this morning, which of the following is true?

A. There were three cars in the parking lot this morning.

B. After 4 cars arrived, there were 7 cars left.

C. After 4 cars drove away, there were 7 cars left.

D. After 7 cars arrived, there were 4 cars left.

Tanya has read the first 78 pages in a book that is 130 pages long. Which number sentence could Tanya use to find the number of pages she must read to finish the book?

A. $130 + 78 = \square$

B. $\square - 78 = 130$

C. $130 \div 78 = \square$

E. $130 - 78 = \square$

What do you have to do to each number in Column A to get the number next to it in Column B?

Column A	Column B
10	2
15	3
25	5
50	10
10	2

A. Add 8 to the number in Column A.

B. Subtract 8 from the number in Column A.

C. Multiply the number in Column A by 5.

E. Divide the number in Column A by 5.

Activity: Popping For Pennies

Format: Small Group

Objective: The participant will discuss performance based assessments and their merits as a component of classroom assessments.

Related SOL: 4.5, 4.22

Materials: Sample assessments, scoring rubric, Popping for Pennies Activity Sheet

Time Required: 10 minutes

Directions:

1. Instruct the participants to read and complete the assessment.
2. Have the participants compare their results and discuss the scoring rubric.
3. Discuss the merits of using performance-based assessments as part of classroom assessment. How do these type of assessments support the mathematics goals of the Virginia Standards of Learning that include Mathematical Problem Solving, Mathematical Reasoning, Mathematical Communication, Mathematical Connections, and Mathematical Representation.

♦ Assessment Description:
The participant will construct a chart to compare two methods of computing salary: straight per hour rate versus a doubling method.

♦ Assessment Purpose:
To assess the participant's ability to:
- identify, describe, and continue numerical patterns;
- find decimal sums and products;
- create charts to represent the pattern; and
- write a letter of resignation in business letter format.

♦ Background Knowledge or Skills:
- recognizing and describing a pattern of adding and multiplying with money;
- creating charts;
- writing a business letter.

♦ Equipment or Materials:
- copy of task, 2 extra sheets of paper (one for charts and one for letter) and pencil
- calculator (optional)

♦ Directions for Administering Task:
- Distribute task copies and review task requirements with participants.

- Extension
  - Study the mathematical relationship between the day number and the salary as a power of 2 (use of calculator or spreadsheet optional) (e.g., Day 4= \$.08 = $2^{(4-1)}$ = 2^3 =8; Day 50 = $2^{(50-1)}$ = 2^{49}).
    Participants can develop this on a spreadsheet.

- ## Scoring Rubric:

3+ Exceeds Expectations

- Salary chart organized and correctly computed for second, third, and fourth week.
- Pattern described correctly with elaboration.
- Total earnings correctly computed for both situations - doubling and minimum wage.
- Letter clearly explains resignation and follows business letter format.

3 Meets Expectations

- Salary chart organized but may contain minor errors in calculation.
- Pattern described incorrectly.
- Total earnings may contain minor errors.
- Letter follows business letter format and explains resignation.

2 Partially Meets Expectations

- Salary chart is poorly organized with frequent errors in calculations.
- Pattern is not described accurately.
- Total earnings may be inaccurate.
- Letter not in business letter format and does not explain resignation.

1 Inadequate Response

- Salary chart incomplete.
- Total earnings missing.
- Letter is only a short statement with no explanation.

0 No Response

Name:______________ Date:_______________

POPPING FOR PENNIES

You have applied for a job at a movie theater as the popcorn popper, but the owner does not want to hire you because you have never had a job. In desperation, because you need money, you offer to sell popcorn for just pennies a day. Instead of paying you the minimum wage of $4.35 per hour for the three hours you will work each day, the owner eagerly accepts your proposal and starts you at one cent the first day and agrees to pay you in the pattern established by the chart below. Here is a chart showing you how he calculated your paycheck of $1.27 for the first week.

Day	Salary
1	1¢
2	2¢
3	4¢
4	8¢
5	16¢
6	32¢
7	64¢
Week One	$1.27

A. The owner is very pleased with your work and believes he is getting a lot of popcorn popped for very little money. He quickly agrees to continue the payment pattern for another three weeks (28 days total). On a separate piece of paper, make a chart like the one above showing your wage for each day (days 8 - 28), and the total of your paycheck for the second, third, and fourth weeks. Label your work and describe the pattern.

B. What was the total of your earnings for four weeks using the pattern method of payment? ____________

C. What would the movie theater owner have paid you for four weeks of work if you had earned the minimum wage for three hours a day? __________

D. Which method of payment would have been better for the movie theater owner and why?

E. Over the past four weeks, you have made so much money you decide to resign from your job. On a separate piece of paper, write a letter to the theater owner telling him you no longer need your job and thanking him for hiring you.

FUNCTIONS
Session 4

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Function Overview		149			
Function Machines	I am a Wizard!	150	K.1, K.6, 1.8		Large cardboard box to create a magic box or screen, a variety of concrete materials (i.e., buttons, chalk, popsicle sticks)
	Fun with Function Machines	151	K.1, K.2, K.6,1.8	Fun with Function Machines Activity Sheet	Input/output cards, recording chart for the instructor, input/output mats and manipulatives for each participant
Functional Relationships	Build the Rule	155	1.4, 1.8, 2.26, 3.4, 3.25	Function Machine Activity Sheet, 10-Strip Activity Sheet	Sets of cards for the instructor, color tiles in two colors,
	Guess My Rule	157	2.26, 3.4,, 3.25 4.22	Function Machine Activity Sheet	Sets of cards for the instructor, index cards (1 per participant)
	Other Function Machines	161	2.25, 2.26, 3.24, 4.22, 5.19	Two-function Machines Activity Sheet, Fish Function Machines Activity Sheet	Calculator, overhead calculator when possible
	Toothpick Patterns	167	3.24, 3.25		Toothpicks
Graphing Functions	Graphing Patterns	169	4.18		Tennis shoes, graph paper
	Black plus White Equals Five	171	4.18		Two different colored counters, graph paper transparency
	Graph the Rule	172	4.18, 5.20	What's My Rule? Activity Sheets 1-6	Function machines, graph paper
	Figurate Numbers	180	4.21, 5.20	Square Numbers Activity Sheet, Triangular Numbers Activity Sheet, Tower of Cubes Problem Activity Sheet	Colored tiles and cubes,

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Reflections	Reflections on Functions	187	All of the Above		Blank sheet of paper

Key Idea: Function Overview

Description:

One of the most important concepts in mathematics is that of a function. A function is a relationship that pairs each number in a given set of numbers with exactly one number in a second set of numbers. While participants in the elementary grades are not exposed to the formal definition of function, participants begin to develop this pairing concept as they work with function machine models. In fact, participants become familiar with models of function machines in the real-world; for instance, when they insert a quarter in a gum machine which gives them two pieces of gum.

In the following activities, participants will explore instructional approaches to developing the concept of a function. They will start with a concrete representation of a function rule and gradually make the representations more abstract by going from physical objects to pictures to the standard representation followed by connections to their graphical representations. By initially exposing participants to concrete representations of function models, the use of the standard representation is usually an easy transition for participants.

Activity: I am a Wizard!

Format: Whole group

Objective: Participants will be able to understand the concept of an input and an output related by a rule.

Related SOL: K.1, K.6, 1.8

Materials: A magic box or a screen made out of a large cardboard box and a variety of concrete materials (i.e., buttons, chalk, popsicle sticks)

Time Required: 15 minutes

Directions:

1. Decorate a large backless cardboard box or a three-fold project screen so that it looks like a weird machine. There should be an IN slot and an OUT slot. The slots should be big enough for participants to put their hand filled with objects into the box.
2. The instructor whispers instructions to the participant such as "Take the objects that are given to you, add 1 more to them and then return the objects through the OUT slot." The participant then goes into the box to play the wizard in the Magic Box. They will need buttons, chalk, popsicle sticks, etc.
3. Another participant places some objects such as popsicle sticks into the IN slot announcing how many objects they are putting into the machine. If they put in 4 sticks, the wizard will return 5 sticks when the rule is add 1. Another classmate might put in 2 cubes and the wizard will return 3 cubes or input 3 pieces of chalk to get back 4 pieces of chalk. Continue letting participants take turns putting in various objects. When appropriate, have another participant record the inputs and outputs.

   When participants think that they know the rule, then they may not tell anyone in the class but must prove their knowledge by telling what the output will be when a certain number is input. The other participants continue taking turns until most of the participants have discovered the rule.

4. Let participants take turns being the wizard in the box. Each turn will be a different rule. The rule should be limited to adding or subtracting a small number such as 0, 1, 2, or 3. Participants, however, may wish to stump the others when used in the classroom by using more difficult rules.

Activity: Fun With Function Machines

Format: Whole group

Objective: Participants will use concrete objects to model the concept of input and output.

Related SOL: K.1, K.2, K.6,1.8

Materials: Fun with Function Machines Activity Sheet, input/output cards, recording chart for the instructor, input/output mats and manipulatives for each participant

Time Required: 20 minutes

Directions:

A function machine is a concrete approach to demonstrating relationships between numbers and serves as an initial exploration of the concept of a function. In this activity participants model the input and output numbers using manipulatives while trying to observe the rule that defines the relationship. Function machines always follow a rule.

Note: In these beginning explorations of function in the classroom, it is important to give the input numbers in sequential order. (i.e., 1, 2, 3…).

IN	OUT
1	2
2	3
3	4

1. Find the card with the number 1 on it. Show the card to the participants and ask what they see on the card. Record the number on a chart under "IN". Tell them to watch carefully as you put the number into the box (the 1 side up) and observe what number comes out.
2. Record that number on the chart under "OUT". Continue with the next card, showing the participants the card and recording the number on the chart under "IN.
3. Continue with the cards in sequence, recording the "IN" number. Ask the participants to predict what number they think will come out each time.
4. While the instructor is recording the inputs and outputs, the participants should be modeling the process with manipulatives, such as beans, junk, cubes, etc. Each participant works on a 2-column mat with labels IN and OUT. As the instructor shows the card with the 1, the participants put 1 object in the first box on their paper. When they see 2 come out, the participants put 2 objects in the square next to the first one on their mat and continue as each new card goes in and comes out.

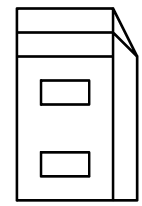
In the classroom, using this "hands on" experience will give the participants a better understanding of the concepts of more, less, before, and after as well as seeing a pattern develop. It gives meaning to the abstract numbers the instructor is recording.

5. Extension - Ask participants ahead of time to bring a half-gallon cardboard milk carton. Have participants create milk carton function machines during the staff development program.

FUN WITH FUNCTION MACHINES

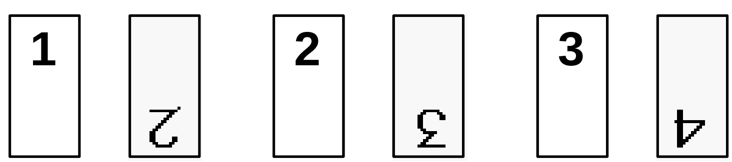
Instructions for Creating a Function Machine

Cut a slit across the top of a half-gallon cardboard milk carton about half an inch high and three inches wide. Cut another slit across the bottom. Cut a piece of tagboard 3.5 inches wide and eight inches long. Fold under about .5 inches at the top and bottom of the tagboard. This is the slide the card will travel along. The tag board should go inside the box with the folded flange taped to the outside of the top slit. The slide should curve from the top slit, touch the back of the carton and come out the bottom slit. The cards will be placed into the machine with the input number facing up. It is important that the card "flip" on its way down the slide so that the output number is facing up when the cards slides out of the box. Adjust the length of the slide if necessary before taping the bottom folded flange to the bottom slit. Decorate your function machine.



Construction of Cards

Prepare cards as shown. You will want to create about 10 cards per rule. The cards should have a notation about which is the input side. The size of the cards depends upon the size of the opening of slits of your function machine. The cards below would be appropriate for the classroom; however, cards representing more challenging functions should be made for the participants.



INPUT	OUTPUT

Activity: Build the Rule

Format: Whole group

Objective: Participants will use concrete objects to model the concept of input and output and will identify the function rule.

Related SOL: 1.4, 1.8, 2.26, 3.4, 3.25

Materials: Function Machine Activity Sheet, sets of cards for the instructor, color tiles in two colors, 10-Strip Activity Sheet

Time Required: 10 minutes

Directions:
1. The instructor shows the participants the first card. The participants build that number on their 10-strip using one color of tiles. For example, the input card is 3.
2. The instructor inserts the number into the Function Machine. When the card comes out, the instructor shows the participants the number and they build UP TO that number using the other color of tiles.

3	□	□	□							

To continue the above example: The Output card says 7. So the participants add 4 color tiles in a color different to the original 3.

7	□	□	□	■	■	■	■			

3. Discuss what you had to do to get the new number. This is the rule (i.e., input plus four equals output).
4. Record the input and output numbers on the chart.
5. Use more interesting rules to challenge the participants (i.e., □ X 2 + 1).
6. Discuss the value of using manipulatives to discover the rule.

IN	OUT

10-Strip to accompany Build the Rule activity

To make linear 10-strips, copy this page, and tape two strips together to create a strip of 10 squares.

Activity: Guess My Rule

Format: Whole group

Objective: Participants will solve problems involving functional relationships.

Related SOL: 2.26, 3.4, 3.25, 4.22

Materials: Function Machine Activity Sheet, sets of cards for the instructor, index cards (1 per participant)

Time Required: 15 minutes

Directions:

1. Show participants the function machine and cards.

   Point out the input slot and the output slot. Discuss the terms input and output. Explain to participants that this machine takes numbers in, performs operations on the number and then spits them out. For example, if the machine's rule is to add 3 then when the number 4 is input a 7 is output; if the number 6 is input a 9 is output; if the number 12 is input a 15 is output, etc.

2. Begin feeding cards into the machine. Record the input and output numbers.
3. Continue with additional cards until the rule is guessed.
4. When making cards you should have about 10 cards per rule. Extra cards can be put through the Function Machine to verify the rule. Make sets of cards for +2, +3, +4, +5, -2, -3, -4, -5, etc. As participants become comfortable with recording data in numerical order, shuffle the cards and use the cards out of order .
5. The Function Machines handout may be used for additional questions
6. Pass out one index card to each participant. Have participants make up a rule and write the Inputs, respective Outputs and the rule on their card. Break into partner pairs. Have partner pairs challenge another set of partner pairs to guess their rule.

Background Information:

How Partners Can Discover Rules

There are a variety of methods for participants to discover the rule in functional relationships. A few suggestions follow as to some of the ways in which participants can discover rules. Successful guessing of rules depends on experience and practice.

Looking at Differences:

If the participants cannot immediately identify a number pattern, have the participants order the numbers consecutively from least to greatest in the input box, placing the respective numbers in the output box. Have the participants then look at the difference between the output numbers. Have the participants compare the difference between the input number and the output number. Have them do some detective work to determine what happens to a number after it goes into the function. If the number increases slightly, perhaps the participants are dealing with a "The number plus something? " situation. If the number differences are large, consider multiplication and division.

Recognizing Famous Numbers:

Participants should be familiar with various types of number such as odd number (2n-1) and the even numbers (2n). They should learn the rules for detecting whether a given number is a multiple of each of the whole numbers 3, 4, 5, 6, 8, 9, 11 (i.e., multiples of 9: the sum of the digits is always a multiple of 9, etc.).

Participants should be able to recognize the numbers related to the powers of numbers and their differences (i.e., powers of 2: 1, 2, 4, 8, 16, 32, 64, 128, 256, 512), and that the differences between the powers of 2 are the same as the powers of 2.

Recognizing Figurate Numbers:

By fifth grade participants should be familiar with figurate numbers including triangular numbers, square numbers, etc.

Input - Output Charts

1.

Rule	
INPUT	OUTPUT

2.

Rule	
INPUT	OUTPUT

3.

Rule	
INPUT	OUTPUT

4.

Rule	
INPUT	OUTPUT

5.

Rule	
INPUT	OUTPUT

6.

Rule	
INPUT	OUTPUT

Function Machines

This is a function machine. It takes the number that you feed in, processes it and then creates the output. The processing takes place according to some rule.

Input

Rule

Output

---

1. 4 — Subtract 3 — a.? =____

   3 — Subtract 3 — b. ? = ____

   ?= ______ — Subtract 3 — c. 9

2. 2 — Multiply by 3 — a.? =____

   0 — Multiply by 3 — b. ? = ____

   ? — Multiply by 3 — c. 9

3. Find a rule that connects the inputs and outputs in problem 3. One rule fits all.

   12 — *Rule* — 4

   3 — *Rule* — 1

   24 — *Rule* — 8

   Rule: __________

Activity: Other Function Machines

Format: Small group

Objective: Participants will explore function machines using calculators and other function machine formats

Related SOL: 2.25, 2.26, 3.24, 4.22, 5.19

Materials: Calculator, overhead calculator when possible, Two-Function Machines Activity Sheet, Fishy Function Machines Activity Sheet

Time Required: 30 minutes

Directions:

PART I: Calculator activities (calculators with a built-in constant function can be used as a function machine).

1. For example, with the TI-108 clear the calculator, then press "+", "6", "=". Then, without touching any other keys, press "4", "=" and the display will show 10. The calculator is now a "+6" machine.
2. Hand the calculator to a participant with instructions not to touch any key until told to do so. Invite the participants to select a number. If the number is 7, instruct the participants to enter "7" and then press "=". The display will show 13.
3. The participant can continue to choose numbers to enter followed by "=" until the class predicts the rule that the calculator is using. Usually participants get the rule after only one or two examples as long as only addition and subtraction are allowed.
4. Have the participants explore functions in their groups using the calculator. Call upon participants to challenge the class to guess their function by placing the function in the calculator and then calling upon the participants for their inputs.

PART II: Two-Function Machines

1. Have the participants complete the Two-Function Machine Activity Sheet. They will be required to apply function rules to determine the output when given the input and to apply function rules to determine the input when given the output. Discuss.
2. Discuss what strategies to use to “undo” the operations from right to left to get the answers. (Guess and check, inverse operations, etc.)
3. Have participants develop a set of two-function machines appropriate for their students on the blank Two-Function Machine Activity Sheet. Include problems where students must “undo” the operations.

PART III: Fishy Functions

1. Have the participants deduce the function rule from the examples at the top of Fishy Function Machines Activity Sheet. Ask participants how they determine the rule and what strategies they would expect students to use to determine the function.
2. Ask participants to apply the function rule to determine the output when given the input and vice versa (determine the input when given the output).
3. Ask the participants to design a few problems on their own that follow the rule and then finally to write the rule, e.g., the fish multiplies the first number by 2 then adds the second number to that product.
4. Finally, have participants design their own "Fishy Functions" on the blank Fishy Function Machines Activity Sheet and challenge others to their "Fishy Function".

Two-Function Machines

IN → → → OUT

3 → + 4 → x 2 → 14

Write what comes out.

5 → + 2 → x 2 →

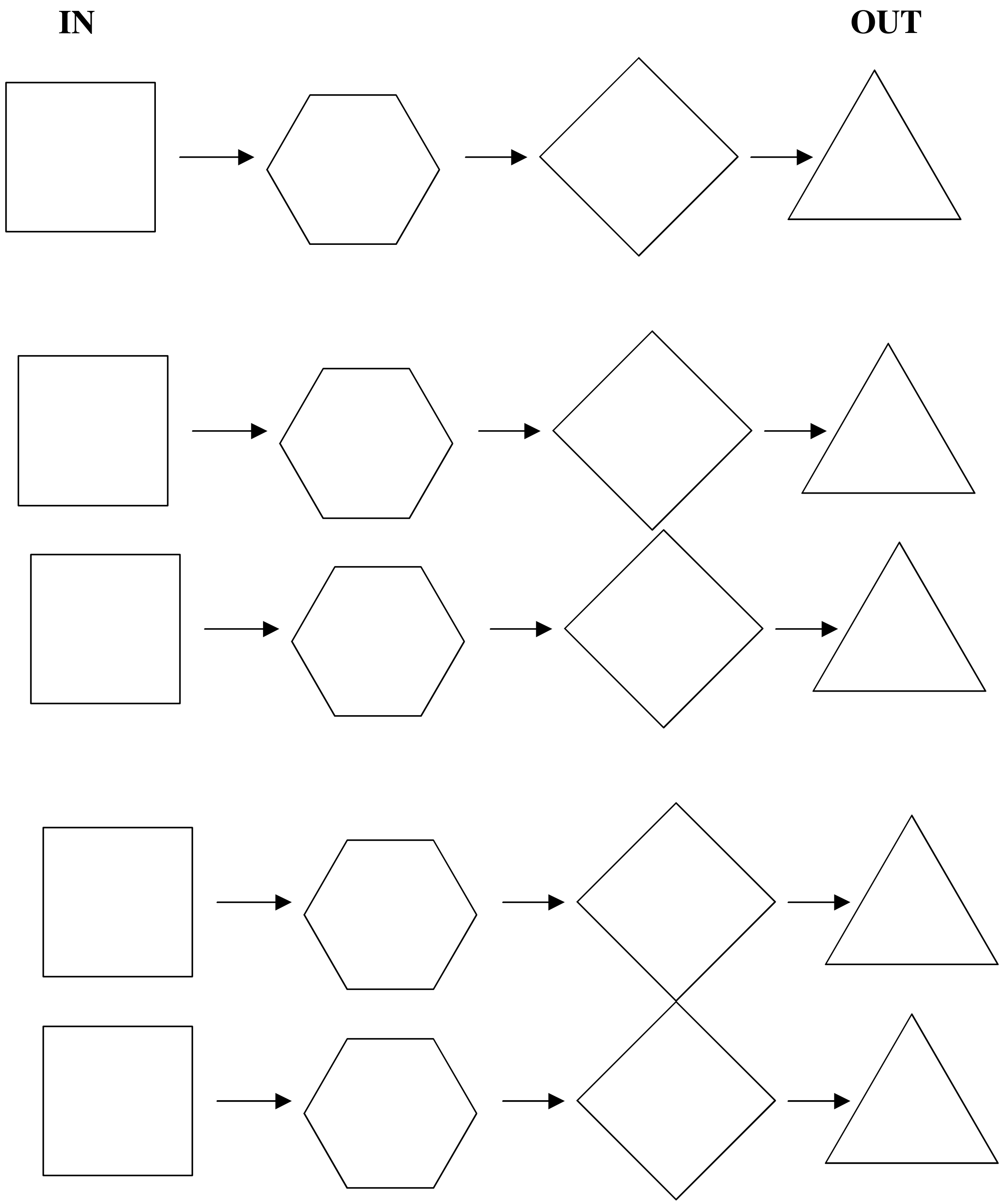
4 → + 3 → x 3 →

Write what goes in.

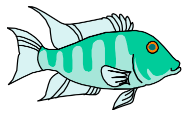
→ + 4 → x 2 → 18

→ + 5 → x 3 → 18

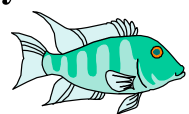
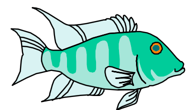
Two-Function Machines

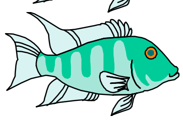
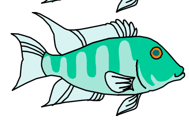


Fishy Function

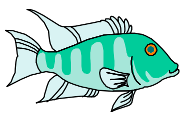
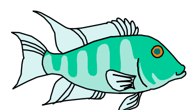
What is  doing?

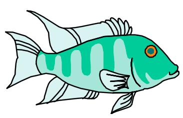
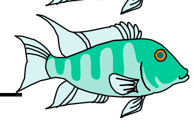
Study theses examples.

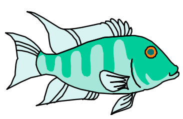
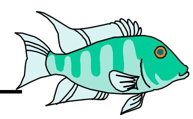
2  3 = 7   4  0 = 8

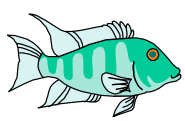
1  1 = 3   0  3 = 3

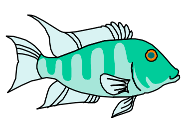
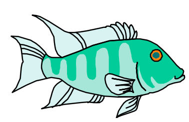
Fill in the numbers.

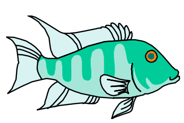
3  3 = ___   5  1 = ___

4  3 = ___   ___  1 = 7

1  5 = ___   ___  5 = 13

Complete these  problems using different numbers.

_  _ = 8   _  _ = 8

Tell what  is doing. ____________________

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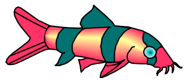
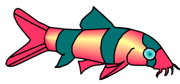
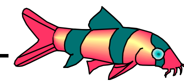
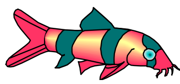
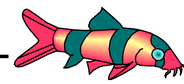
Fishy Function

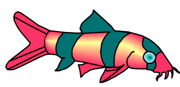
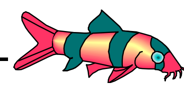
What is  doing?

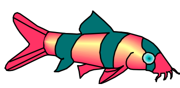
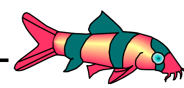
Study theses examples.

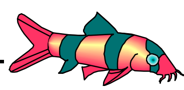
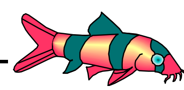
___  ___ = ___ ___  ___ = ___

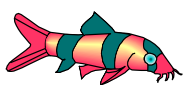
___  ___ = ___ ___  ___ = ___

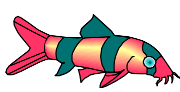
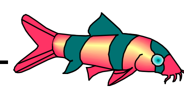
Fill in the numbers.

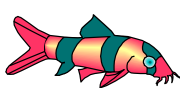
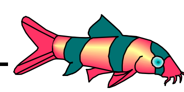
___  ___ = ___ ___  ___ = ___

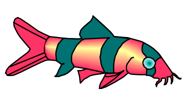
___  ___ = ___ ___  ___ = ___

___  ___ = ___ ___  ___ = ___

Complete these  problems using different numbers.

___  ___ = ___ ___  ___ = ___

___  ___ = ___ ___  ___ = ___

Tell what  is doing. ______________________

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Activity: Toothpick Patterns

Format: Whole group

Objective: Participants will be able to analyze and make predictions about patterns.

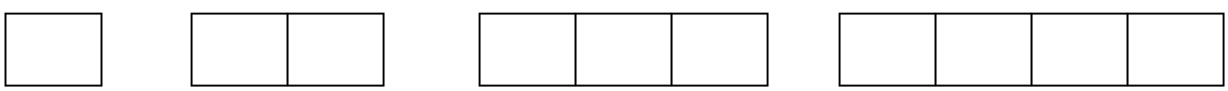
Related SOL: 3.24, 3.25

Materials: Toothpicks

Time Required: 10 minutes

Directions:

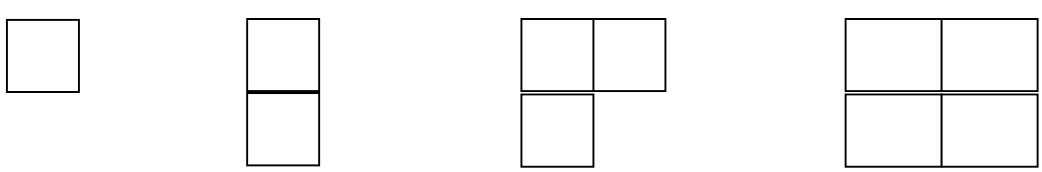
1. Distribute toothpicks so that participants may create the strip pattern.
2. The first term in the sequence is a square made with the toothpicks. The second term in the sequence is 2 connected squares. The third term is 3 connected squares and so on.



3. How many toothpicks does it take to make 5 connected squares? First organize the data in table form.

Number of squares	Number of toothpicks
1	4
2	7
3	10
4	13

4. How can you determine the number of toothpicks required to make 5 connected squares? What is the rule (e.g., (n x 3) + 1)?
5. Would the answer be the same if the squares were attached differently? What would the next term look like?



6. What would the answers be if the squares were attached to make a staircase (e.g., $n^2 + 3n$) ?
7. What about triangles? Hexagons?

Activity: Graphing Patterns

Format: Whole group

Objective: Participants will make patterns using concrete objects, represent the objects visually, organize data in a chart and then represent the pattern on a graph.

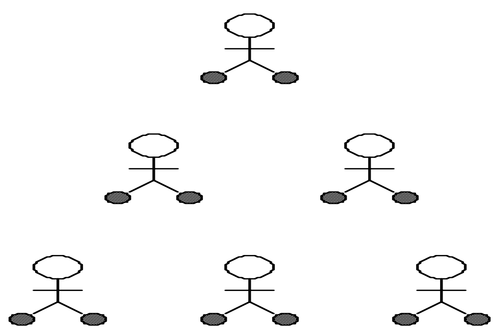
Related SOL: 4.18

Materials: Tennis shoes, graph paper

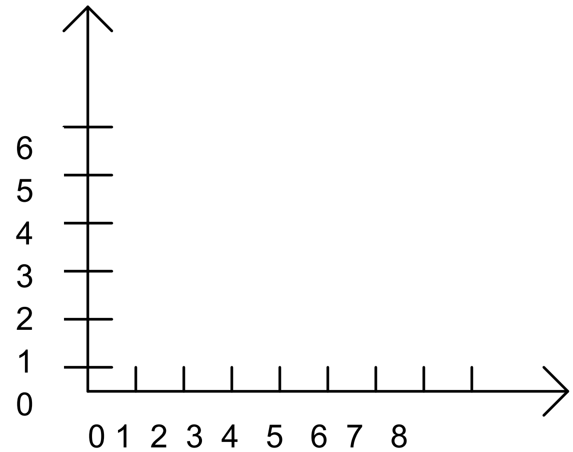
Time Required: 10 minutes

Directions:
Tennis Shoes Pattern

1. Have the participants that are wearing tennis shoes model this pattern while the instructor records the data



Participants	Tennis Shoes
1	2
2	4
3	6



2. Represent this data using ordered pair notation. (1,2) means 1 student and 2 shoes, (2,4) means 2 students and 4 shoes, etc.
3. Tell participants that in the classroom instructors should explain how to record both pieces of data by using one number line to record the number of participants and another number line to record the number of shoes. Instructors should show the participants how these number lines are connected together at right angles to give us the x-axis and the y-axis.
4. Have the participants plot the point on a coordinate graph. Label the horizontal line (x-axis) “Number of Participants” and the vertical line (y-axis) “Number of Tennis Shoes.” Ask participants if they notice a pattern on the graph. Can they predict from the graph how many shoes there will be for 5 students? Is (6,16) in this pattern? Why or why not? (The second coordinate number is twice the first.) Is (5,11)? Why or why not? (All second coordinate numbers are even since they are multiples of two.)
5. If you have 20 tennis shoes, how many participants will you have? (10)
   If you have 32 tennis shoes, how many participants will you have? (16)
   If you have 8 participants, how many tennis shoes would there be? (16)
   Will the number of participants and tennis shoes ever be the same? (No, except for 0 participants, 0 tennis shoes)
   Will the number of tennis shoes ever be three times the number of participants? (No)
6. Have the participants identify relationships that will create linear functions such as participants, fingers; packs of gum, sticks of gum; buttons, number of holes in buttons. Create a list for participants to copy.

Activity: Black Plus White Equals Five

Format: Whole group

Objective: Participants will be able to graph ordered pairs.

Related SOL: 4.18

Materials: Two different colored counters, graph paper transparency

Time Required: 5 minutes

Directions:

1. Model all the possible combinations of five, using beans or cubes.

5 black 0 white	4 black 1 white	3 black 2 white	2 black 3 white	1 black 4 white	0 black 5 white

2. When this concrete information is put into a graph, the horizontal axis will represent white beans and the vertical axis will represent black beans.
3. The first situation, 0 white beans and 5 black beans, is represented by the point (0,5).

   The second situation, 1 white bean and 4 black beans, is represented by the point (1,4). The last point on the coordinate axis represents 5 white beans and 0 black beans (5,0).

4. A table could be made of this data.

number of white beans	number of black beans
0	5
1	4
2	3
3	2
4	1
5	0

5. Have participants model combinations for 6, 4, 8, 10, 9. Have them review each others work. Ask them to describe what they have learned.

Activity: Graph the Rule!

Format: Whole group

Objective: Participants will be able to represent functional relationships both graphically and in tables using ordered pairs in the first quadrant of the coordinate plane.

Related SOL: 4.18, 5.20

Materials: What's My Rule? Activity Sheets 1-6, function machines, graph paper

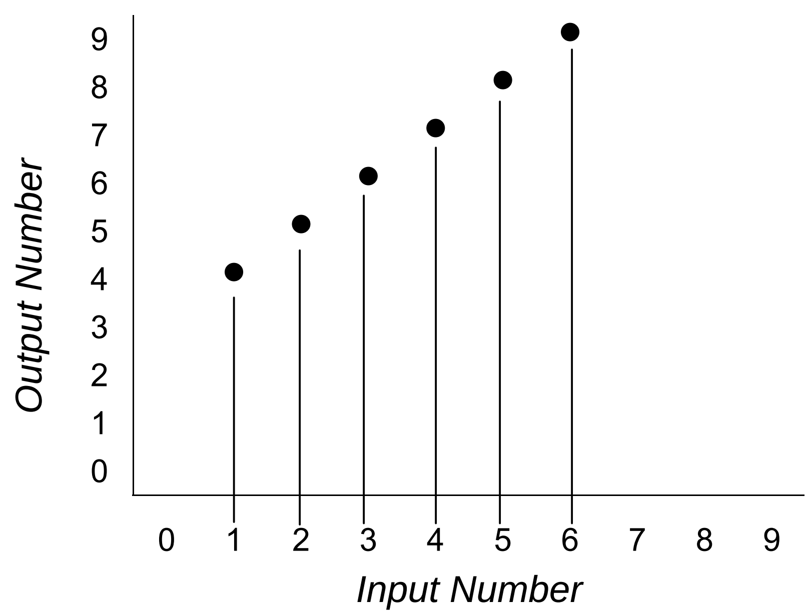
Time Required: 15 minutes

Directions:

1. Review activities with function machines, emphasizing the relationship between the input/output numbers and the rule.
2. Lead participants in an activity that graphs the rule.

   For example, if the rule is "Add 3", participants will locate the input number on the horizontal axis and then apply the rule to locate the output number vertically.

Graph of the Rule "Add Three"



3. Can you predict what the output number will be when 5 is the input number just by looking at the graph?

4. Inform participants that the Input and the Output numbers can be written as an ordered pair where the Input number is the first number and the Output number is the second number. These numbers are separated by a comma and enclosed in parentheses.

Record your findings in a table:

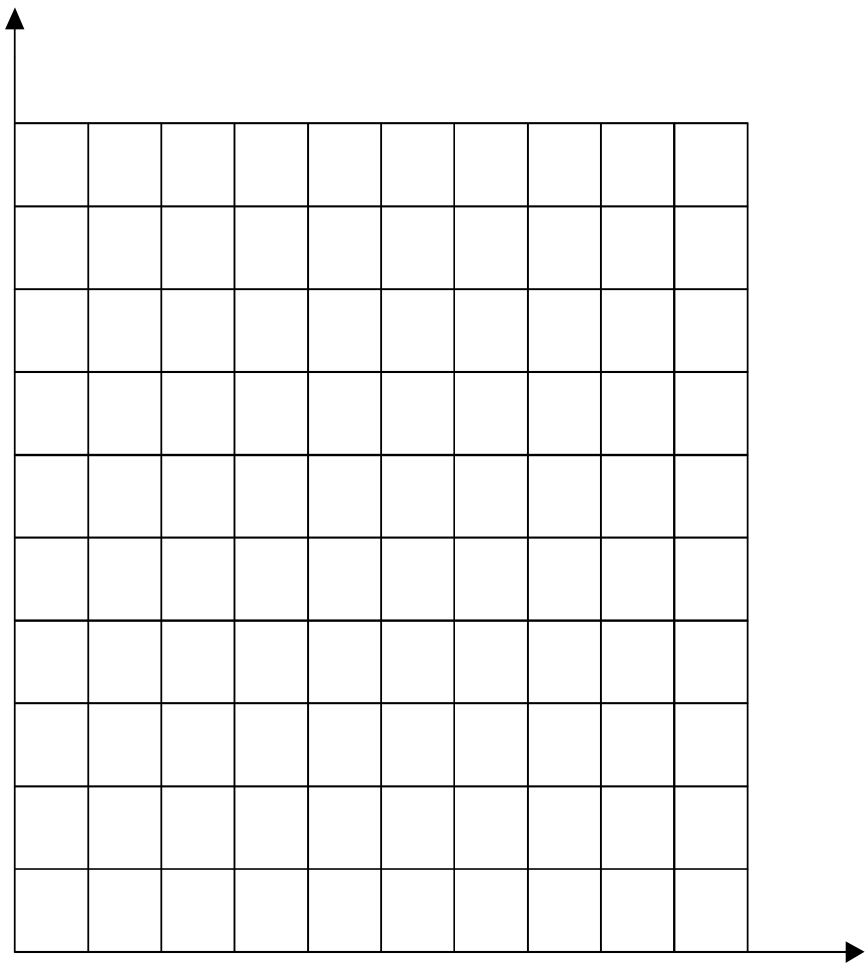
Rule: Add three to the input number to get the output number		
Input Number	Output Number	Ordered Pair (Input Number, Output Number)
1	4	(1,4)
2	5	(2,5)
3	6	(3,6)
4	7	(4,7)
5	8	(5,8)

5. Do the ordered pairs (3,5) and (5,3) follow the same rule? Are they located at the same point on a graph? Why or why not?
6. Provide several examples to participants where the instructor uses the Function Machine while the participants graph the corresponding ordered pairs. The instructor should use consecutive input numbers beginning with "1" .
7. Have participants record the input/output numbers in a table.
8. Have participants complete the tables in What's My Rule? problems #1 - #6 that follow and construct the graph for each rule. Additional problems similar to those presented can be made by participants.

WHAT'S MY RULE? #1

1. The table shows some inputs and some outputs.
2. Complete the table.
3. Use words to write a rule for finding the output when you know the input.
4. Optional: Look at the graph of the rule. Use graph paper to graph the first few input/output numbers in the table.

Input	Output
0	0
1	3
2	6
3	9
4	12
	21
9	
10	
12	
	42
	48
	51

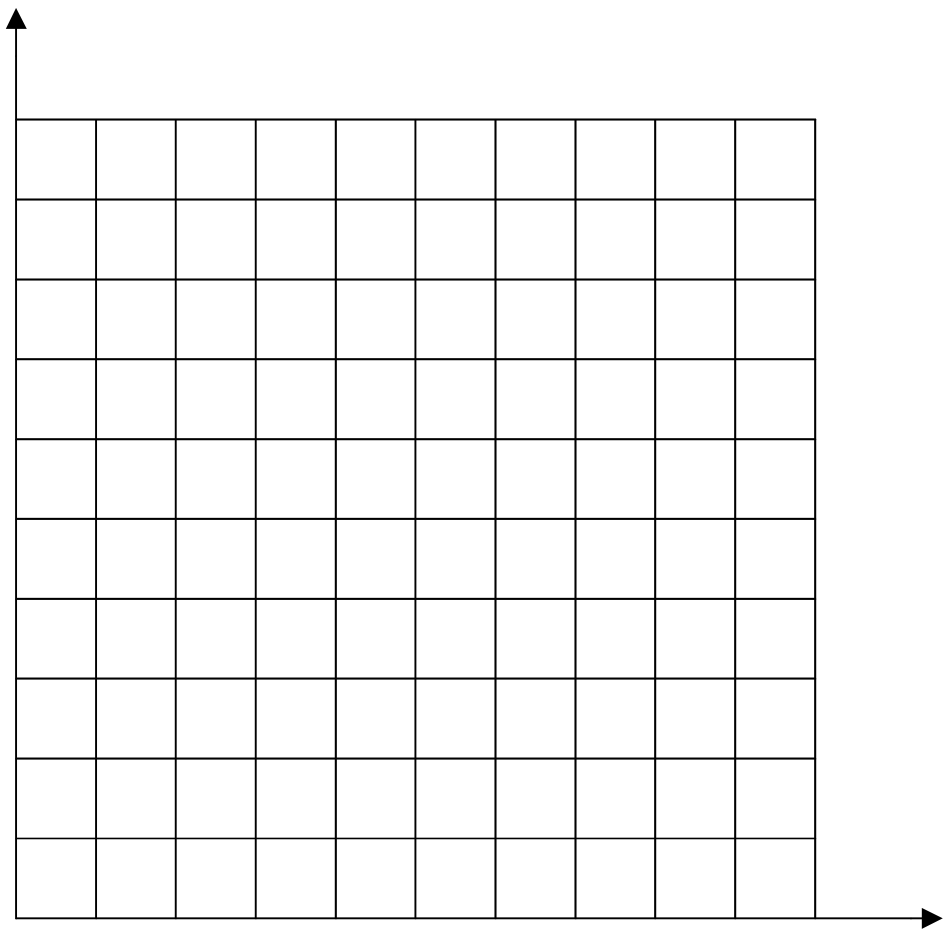


WRITE THE RULE:

WHAT'S MY RULE #2

1. The table shows some inputs and some outputs.
2. Complete the table.
3. Use words to write a rule for finding the output when you know the input.
4. Optional: Look at the graph of the rule. Use graph paper to graph the first few input/output numbers in the table.

Input	Output
0	1
1	4
2	7
3	10
4	13
	19
9	
10	
15	
	52
	61
	151

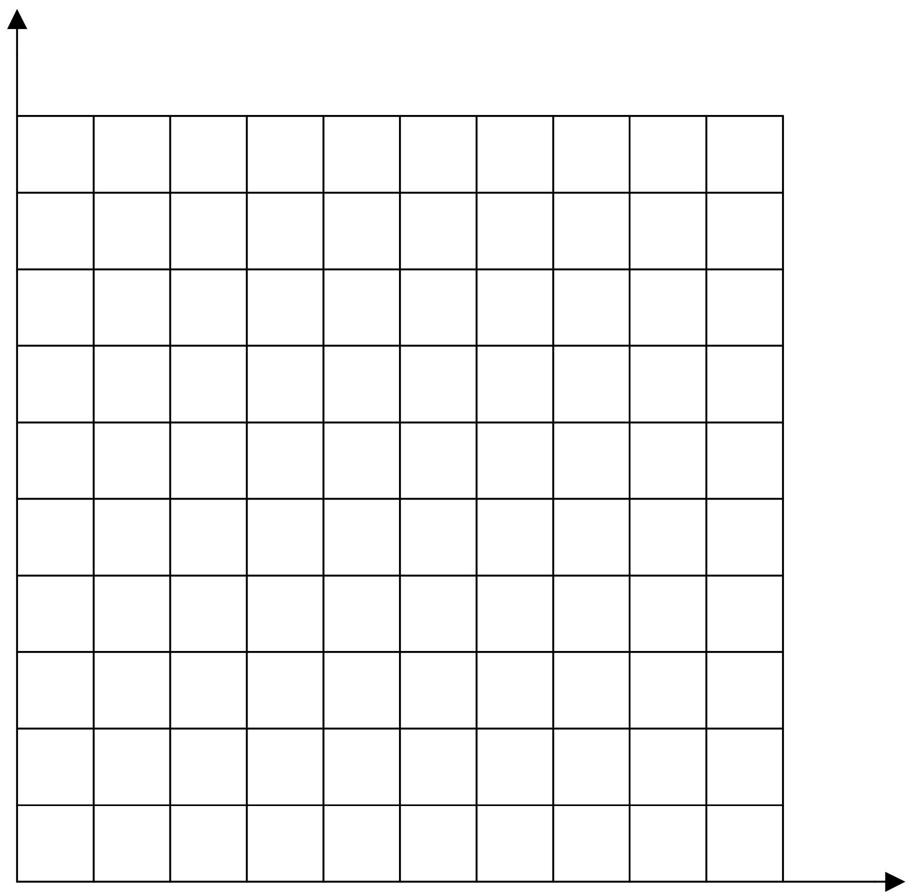


WRITE THE RULE:

WHAT'S MY RULE? #3

1. The table shows some inputs and some outputs.
2. Complete the table.
3. Use words to write a rule for finding the output when you know the input.
4. Optional: Look at the graph of the rule. Use graph paper to graph the first few input/output numbers in the table.

Input	Output
0	2
1	6
2	10
3	14
4	18
	30
9	
10	
15	
0	94
1	126
2	162

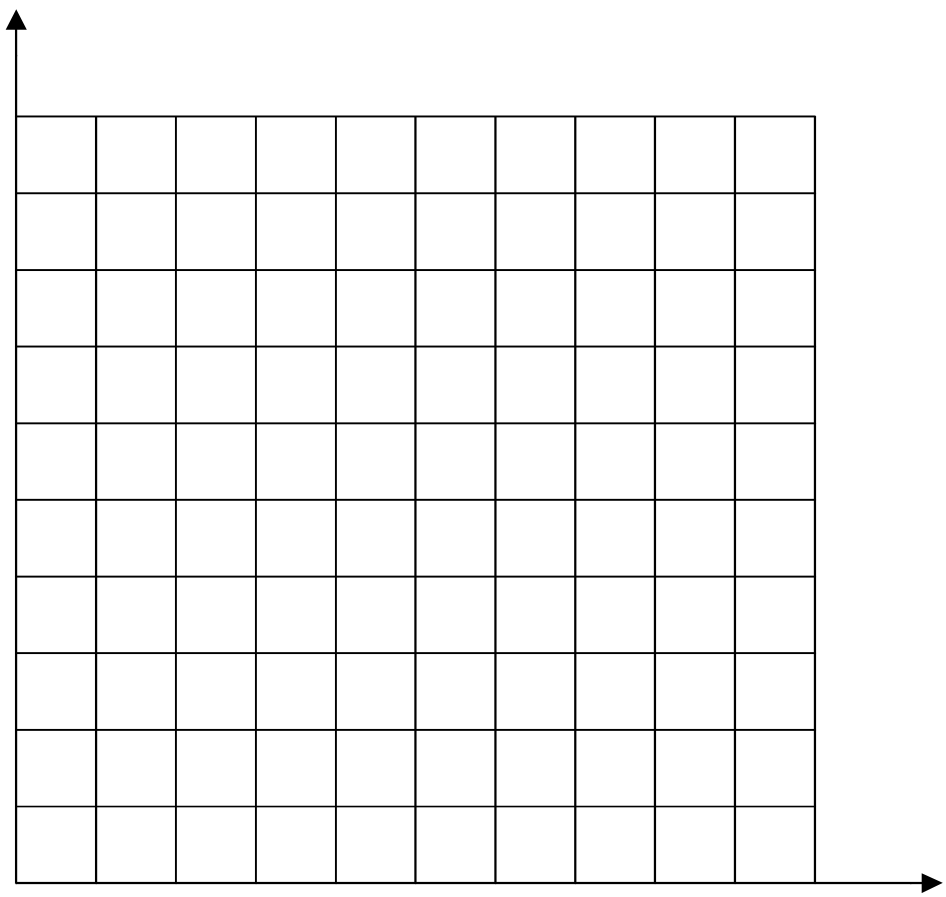


WRITE THE RULE:

WHAT'S MY RULE? #4

1. The table shows some inputs and some outputs.
2. Complete the table.
3. Use words to write a rule for finding the output when you know the input.
4. Optional: Look at the graph of the rule. Use graph paper to graph the first few input/output numbers in the table.

Input	Output
0	0
1	4
2	8
3	12
4	18
	20
8	
9	
10	
0	80
	100

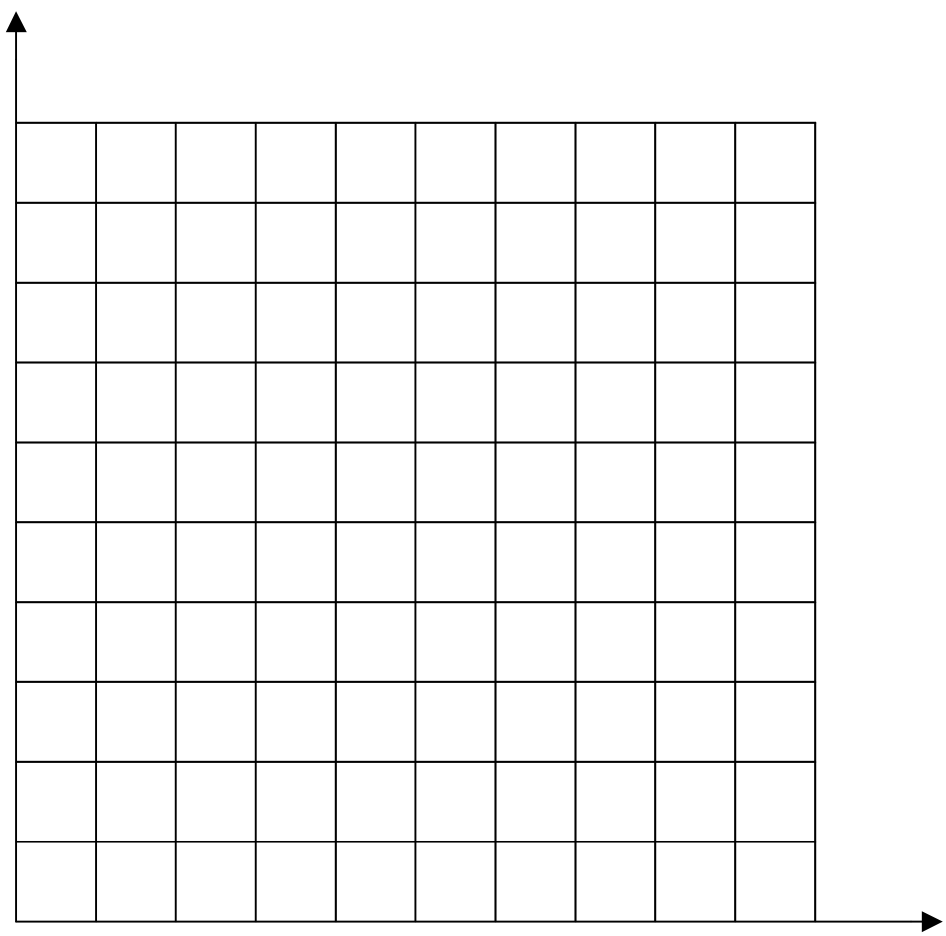


WRITE THE RULE:

WHAT'S MY RULE? #5

1. The table shows some inputs and some outputs.
2. Complete the table.
3. Use words to write a rule for finding the output when you know the input.
4. Optional: Look at the graph of the rule.
   Use graph paper to graph the first few input/output numbers in the table.

Input	Output
0	0
1	4
2	8
3	12
4	18
	20
8	
9	
10	
0	80
	100

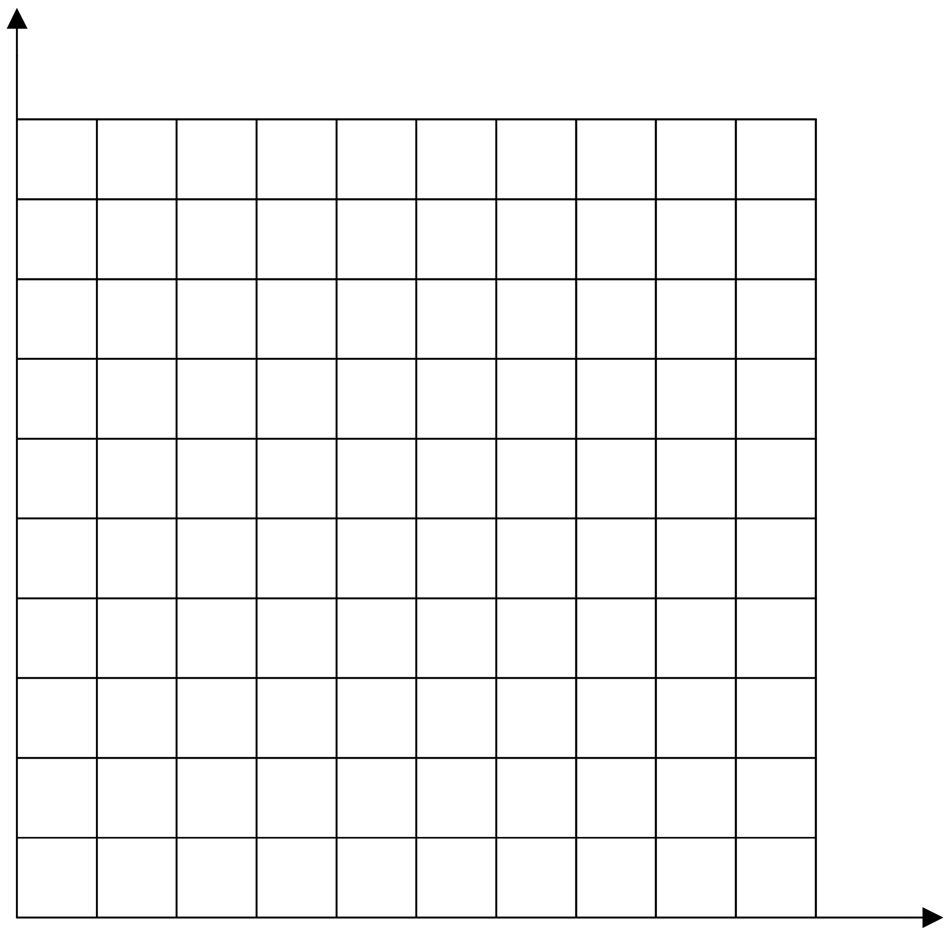


WRITE THE RULE:

WHAT'S MY RULE? #6

1. The table shows some inputs and some outputs.
2. Complete the table.
3. Use words to write a rule for finding the output when you know the input.
4. Optional: Look at the graph of the rule.
   Use graph paper to graph the first few input/output numbers in the table.

Input	Output
0	
1	3
2	5
3	7
4	
	13
7	
10	
10	
	25
	81



WRITE THE RULE:

Activity: Figurate Numbers

Format: Partner Pair

Objectives: Participants will investigate figurate numbers, including square and triangular numbers.

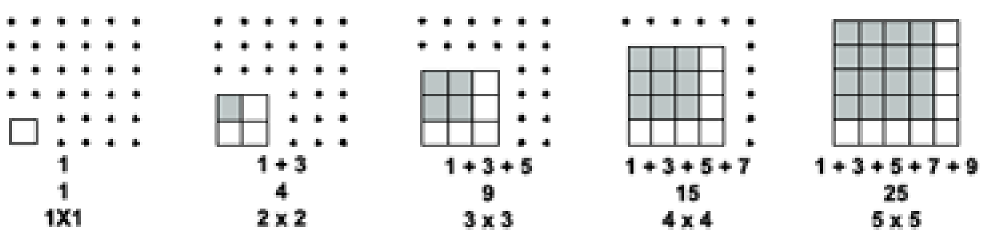
Related SOL: 4.21, 5.20

Materials: Colored tiles and cubes, Square Numbers Activity Sheet, Triangular Numbers Activity Sheet, Tower of Cubes Problem Activity Sheet

Time Required: 30 minutes

Directions:

1. For each problem, have participants work through the process of:
   a. Building the model of the pattern using manipulatives.
   b. Completing the function table.
   c. Explaining the pattern verbally and make predictions about what will happen later in the sequence.
   d. Justifying their explanations through reasoning about the structure of the relationship.
2. For example, with "Square Numbers," participants might use color tiles or inch graph paper to show squares, beginning with a 1-by-1 square, then a 2-by-2 square, and so forth. With a set of five or six squares they can experiment with their relative size and with ways of representing the patterns they notice in the models.



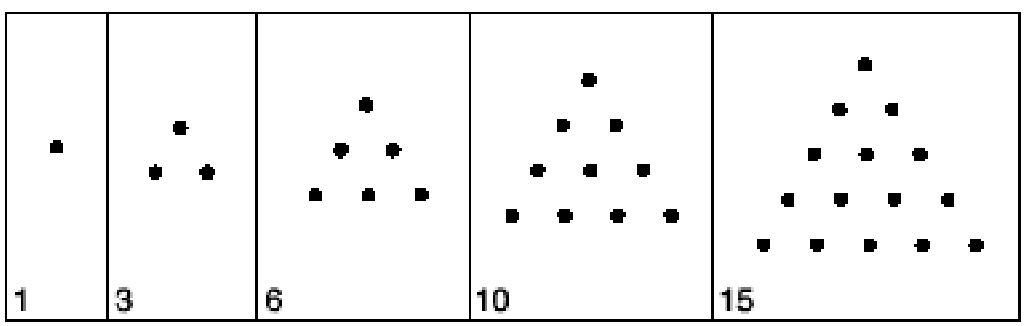
One participant might notice that a square always fits into the "corner" of the next larger square leaving an L-shaped empty space. They my verbalize this pattern and described the area of the larger square as equal to the area of the smaller plus "two sides" and one more. They may have represented their solution by writing "Area of a 5 x 5 square = area of a 4 x 4 square +4 +4 +1".

Another participant may note that the area of the shaded square was increasing by an odd number with each new square. They may have generated the numbers for each square number by writing “Area of a 3 square = 1 + 3 + 5”

3. Discuss what participants may do for the “Triangular Numbers” problem. (Discussion from the NCTM Standards 2000)

   When students are asked to explain the relationships involved in the triangular numbers, they often pursue the development of mathematical concepts through the study of patterns. When searching for a generalization, students can consider or generate a set of specific instances, organize them, and look for a pattern. The identification of regularities underlying patterns or sequences is typically based on inductive reasoning. On the basis of observations made about portions of a pattern, conjectures can be made and then tested using inductive or deductive inference.

   Have students investigate patterns in figurate numbers such as the triangular numbers. First ask students to generate the first five numbers in the set of triangular numbers by using the visual structure of the numbers. Knowing that triangular numbers could be represented by rows of dots in the form of equilateral triangles, students progressing from the first to the fifth noticed that they added a row at the bottom of the triangle with one more dot than the bottom row in the previous triangle.



First five triangular numbers

Secondly, students should be asked by their teacher to predict (without drawing) how many dots would be needed for the next triangular number. Reflecting on what they did to generate the sequence thus far, students will quickly concluded that the sixth triangular number would have six more dots than the fifth triangular number. Further, they see that this would seem to go on forever in the sequence of numbers. In fact, this led to a generalization about triangular numbers: Each triangular number is the sum of consecutive counting numbers. Thus, students can engage in recursive reasoning about the structure of this sequence of numbers, using the previous numbers to generate the next number. When the teacher asks the students to find the 100th term in the sequence, they knew that it would be 100 more than the 99th term. However, students will not usually know what that would be. If students are not using a computer, it is not expedient to list the first 99 terms and we would expect that students would need another way to think about the pattern.

To facilitate their work, the teacher might suggest that they make a chart to record their observations about the triangular numbers and the differences between numbers that they had detected thus far. After studying the table for a few minutes, students may see a relationship between the differences and the number. Perhaps they will note that the 2nd term is half of the product of 2 and 3, the 3rd is half of the product of 3 and 4, etc. For instance, the third term in the sequence is 6, the differences associated with 6 are 3 and 4, and 1/2 of the product (12) is 6.

Term	1st		2nd		3rd		4th		5th		6th
Number	1		3		6		10		15		21
Difference		2		3		4		5		6	

The teacher may then ask the group to check to see if this method would work to find the next number. Using their original sequence and the recursive pattern noted, the students could test their conjecture for the 7th term as see if it is correct. Then the teacher may ask for a conjecture regarding the 100th triangular number. Using the conjecture, the 100th number should be (100)(101)/2. Finally the teacher may decide to press the students to think about how they might verify the generalized rule for the sequence-half the product of the term number and the next term number, or n(n + 1)/2.

To help the students understand the validity of their generalization, the teacher may have students look at a simpler, related problem. The teacher may ask students to use "little Gauss's method" to find the sum of the first ten counting numbers. That is, ask students to add matched pairs of numbers: 1 + 10, 2 + 9, 3 + 8, 4 + 7, 5 + 6, from which student can quickly see that the sum of each pair equals 11. Students would then conclude that the sum of the five pairs is (10)(11)/2 or 110/2 or 55. Next, the teacher can challenge the students to connect this problem to their work with the triangular numbers. After a few minutes, students should observe that the pattern they had first seen with the triangular numbers meant that each of these numbers was the sum of consecutive (counting) numbers. Therefore, they can see that 55 must be the 10th triangular number, since it is the sum of the first ten (counting) numbers. After some discussion, be sure that everyone sees the connections among the original recursive pattern and the triangular numbers.

4. Discuss what participants may do for the “Tower of Cubes” problem. (Discussion from the NCTM Standards 2000)

For example: “A fourth grader might make a table and decide from examining the table that multiplying the number of cubes by 4 and adding 2 yields the number of unit squares "Because it works every time."

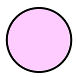
Number of cubes **n**	Surface area (square units) **f(n)**
1	6
2	10
3	14
4	18

Solution to the tower of cubes problem

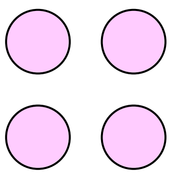
A fifth grader should be challenged to justify the rule with reference to the geometric model. For example, "It's 4 times the number of cubes plus 2 more because there are always 4 square units around each cube and one extra on each end of the tower," or "It's 5 for the end cubes, then 4 more for all the cubes in between, because the end cubes have 6 but one is covered up, and the cubes in between have 6 but two are covered up." Participants might also note the iterative nature of the pattern: "You add four to the previous number." Once a pattern is established, participants can use it to answer questions like, "What is the surface area of a tower with 50 cubes?" or "How many cubes would there be in a tower with a surface area of 242 square units?"

In this example, some participants may use a table to organize and order their data, whereas others use concrete objects (connecting cubes) to model the growth of an arithmetic sequence. Some participants may use words, whereas others use numbers and symbols to express the generalization they find. All these are important and appropriate ways of organizing and expressing ideas about function, and participants in grades 3-5 should feel comfortable moving among them.

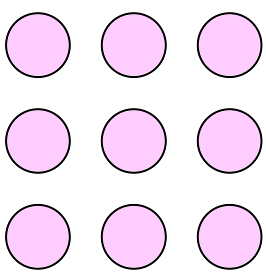
Square Numbers



n=1



n=2

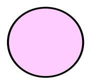


n=3

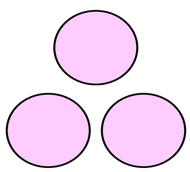
n	f(n)

RULE:______________________

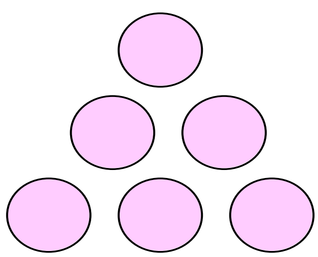
Triangular Numbers



n=1



n=2



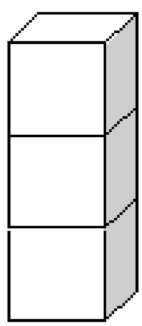
n=3

n	f(n)

RULE:____________________________________

Tower of Cubes Problem

What is the surface area of the tower of cubes? As the line of cubes is extended, how does the surface area change?



N	f(n)

RULE:____________________________________

Activity: Reflections on Functions

Format: Partner Pair

Objectives: Participants will leave class with an action plan for developing the concept of function with their students.

Related SOL: All Patterns, Functions, and Algebra Standards of Learning.

Materials: Blank sheet of paper

Time Required: 10 minutes

Directions:

1. Have teachers choose a partner who works at the same grade level as they work.
2. Have the teachers reflect on the activities in the module and develop a plan for teaching the concept of function appropriate to their grade level. The plan should include conceptual development using manipulative and activities. Ask them to try some of the activities, if appropriate, and report back to the class on the results at the next session.

EQUALITY/BALANCE
Session 5

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Equivalence	How Many Baby Bears	189	K.17, 1.20		Balance scale, Teddy Bear counters, or other manipulatives
	Missing Addends	190	2.26, 3.4		Counters, linking cubes, two-pan balance scales
	Seesaw Balance	192	2.26,3.4, 5.21	Seesaw Balance Activity Sheets	Overhead projector
Solving Equations	Can You Make The Scale Balance	197	3.25, 4.22, 5.21		Cubes or Counters, balance scale worksheet, overhead projector, transparency of Balance Scale
	Weighty Problems	199	2.26, 4.22, 5.21	Weighty Problems Activity Sheets	Stationary physical scale, geometric solids
	Balance Those Blocks	226	2.26, 4.22, 5.21		Laminated balance scale, stationary physical scale, unit blocks, rectangular solids
	Coat Hanger Balances	228	2.9, 5.21	Shape Pattern Activity Sheets	One coat hanger per participant, yarn or string, tape, scissors

Activity: How Many Baby Bears?

Format: Whole Group

Objectives: Kindergarten and first graders will begin to formalize emerging algebraic thinking as they study the relationships between and among quantities. A balance scale will reinforce two-for-one equivalence as students weigh Mama Bear and Baby Bear counters.

Related SOL: K.17, 1.20

Materials: Balance scale for each pair of participants, Teddy Bear counters (the three size set), or other manipulatives where the weight relationship is 1-2-4 (1 papa = 2 mamas = 4 babies)

Time Required: 10 minutes

Directions:

4. Ask participants to select a set of Mama Bear and Baby Bear counters and place them on their table.
5. Instruct participants to place the Mama Bear on the balance scale and discover that the scale does not balance but, instead, tips.
6. Ask participants to place a Baby Bear on the other pan of the balance to discover that the scale still does not balance.
7. Next, ask participants to place another Baby Bear counter in the same pan as the original Baby Bear and discover what happens.
8. Ask participants to decide upon a rule that they discover about the balance, such as, "The number of Baby Bears is twice the number of Mama Bears."
9. Question the students as to this occurrence:
    - What do you think would happen if you put another Mama Bear on the pan?
    - What would you need to do to balance two Mama Bears?
    - What would you need to balance three Mama Bears?
    - What would you need to balance four Baby Bears?
10. Ask participants to think of a way they can record what they discovered to share with the class.
11. Be prepared to guide participants to write: One to two, two to four, three to six; or m = 2b (one Mama Bear is equivalent to two Baby Bears).

Activity: Missing Addends

Format: Small Group

Objective: Participants will be able to move from a concrete understanding of missing addends to conceptual understandings of the simplest of equations through investigations with two-pan balances.

Related SOL: 2.26, 3.4

Materials: Teddy Bear Counters or other appropriate counters, linking cubes, two-pan balance scales

Time Required: 10 Minutes

Directions:

1. Explain to the children that two blue bears and three red bears have decided to go on a camping trip.

2. Direct the children to use their counters to model exactly what this looks like.

3. Invite the participants to tell you the addition equation you would write to explain exactly what is happening with the Teddy Bears and the camping trip.

4. Repeat this same procedure with several other equations.

5. Explain to the participants that some of the bears wondered off the path on the camping trip and erase one addend in each equation

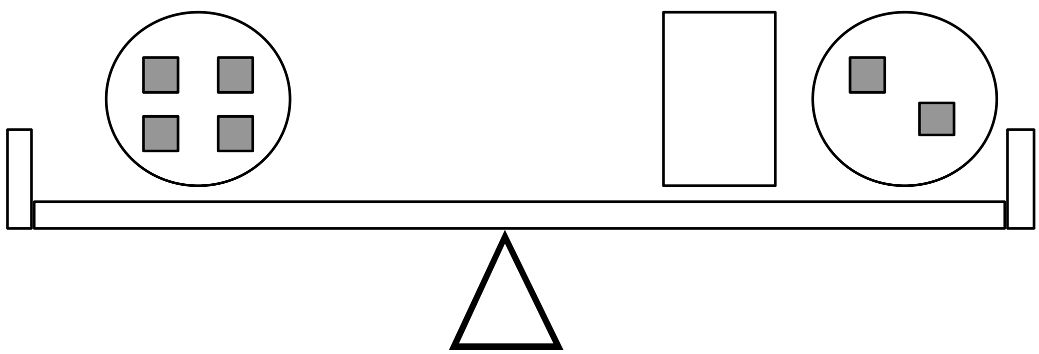
6. Ask the participants to assist you in discovering exactly how many bears might be lost on the camping trip.

7. Once you are sure that the participants are comfortable with the concrete manipulation of the counters, assist them in transferring that understanding to the two-pan balance scale.

8. Say:
   Now boys and girls, we are going to work with the two-pan balance scales to discover more about missing addends. This time instead of using Teddy Bears to discover about missing addends, we are going to use linking cubes.

   When you put one linking cube in one pan, what happens to the balance? If you put one cube in the other pan, what will happen? If each pan on the balance has five cubes, what will happen if two cubes are added to each pan? What will happen if five cubes are added to the left pan and only one is added to the right pan? What will happen if three cubes are removed from each pan? What will happen if four cubes are removed from the right pan?

9. Assist participants in transferring the learning by directing them to work with a partner to complete numerical representations of missing addends on balance scales.



$$4 = \_\_ + 2$$

$$4 = \underline{\ ?\ } + 2$$

$$4 = \underline{\ x\ } + 2$$

Presenter's Notes:

Activity: Seesaw Balances

Format: Small Group

Objective: Participants will be able to balance a scale by assigning numerical values to characters on a balance scale and by adhering to prescribed rules.

Related SOL: 2.26, 3.4, 5.21

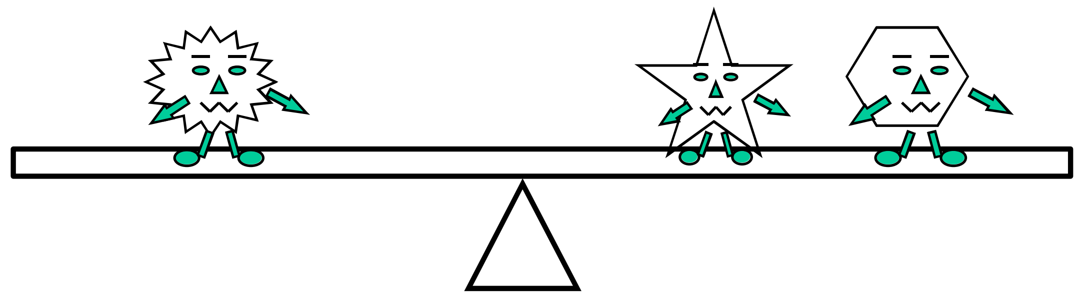
Materials: Overhead projector, Seesaw Balance Activity Sheets

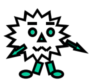
Time Required: 15 Minutes

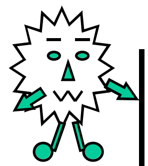
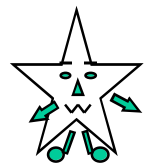
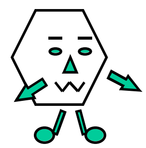
Directions:

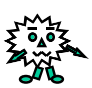
1. Display the overhead transparency of a seesaw balance on the overhead projector.
2. Explain to participants that they will be assigned weights for some of the characters represented on the seesaw.
3. Instruct the participants that they are to calculate a numeric value for each of the characters.
4. Remind participants to follow all the rules:
   - *Balance the seesaw.
   - *Gobots that are the same have the same weight.
   - *Gobots that are different have different weights.
   - *All gobots weigh more than zero pounds.
5. Encourage participants to work with a partner to discover as many solutions as possible for each problem.

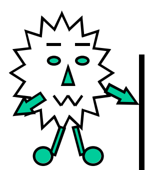
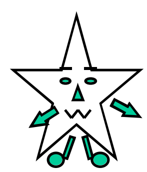
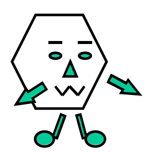
SEESAW BALANCES



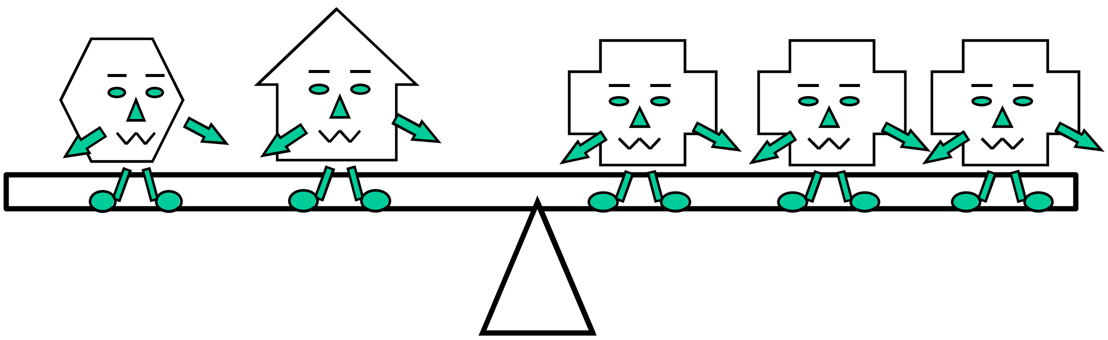
If  on the seesaw weighs four pounds, what could the other gobots weigh?

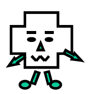
		
4	1	3
4	3	

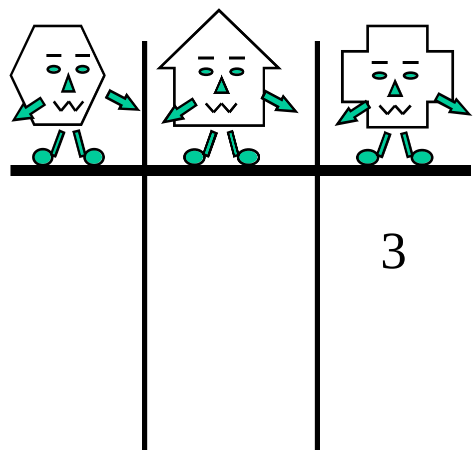
If  on the seesaw weighs seven pounds, what could the other gobots weigh?

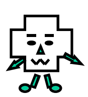
		

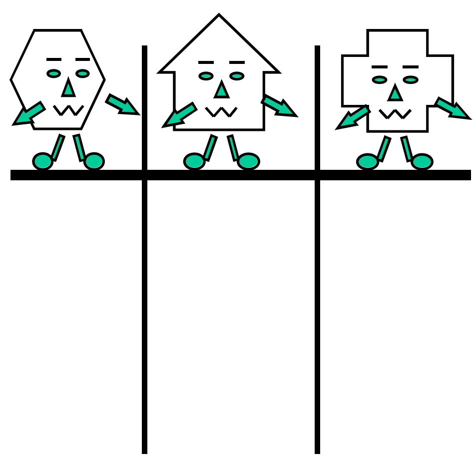
SEESAW BALANCES



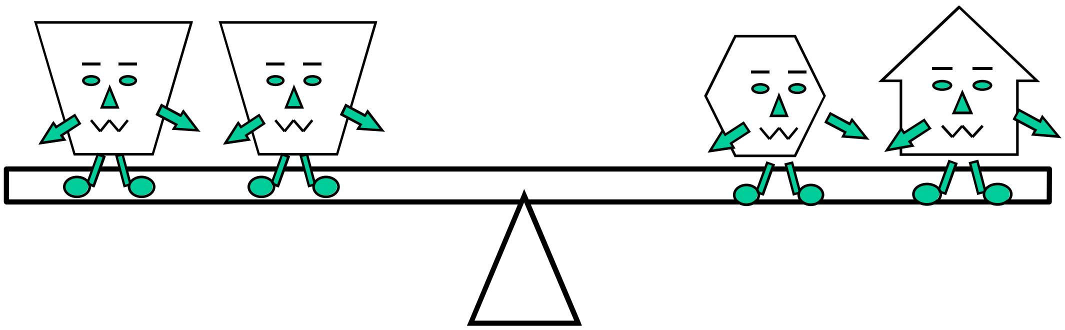
If  on the seesaw weighs three pounds, what could the other gobots weigh?

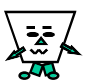


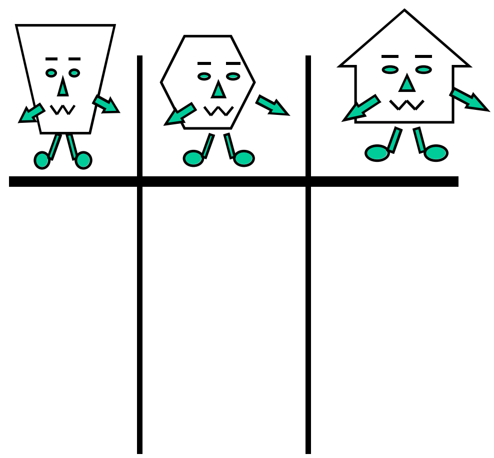
If  on the seesaw weighs four pounds, what could the other gobots weigh?

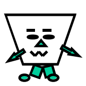


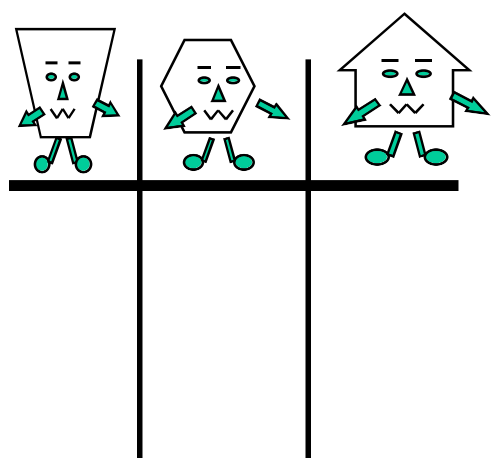
SEESAW BALANCES



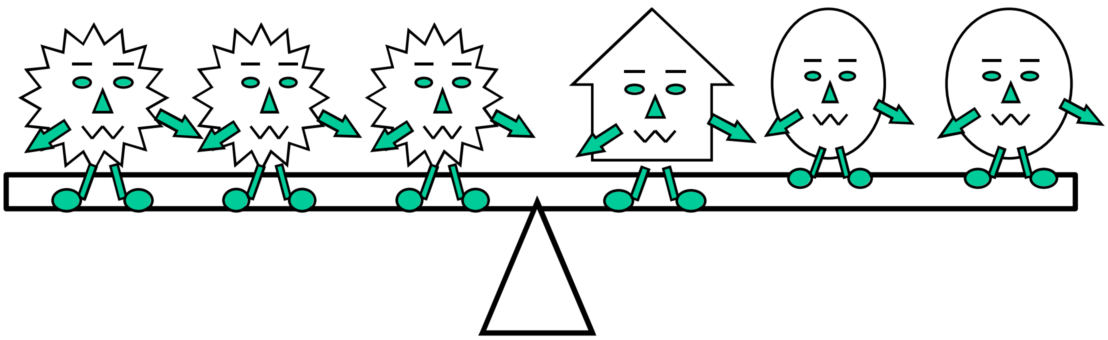
If  on the seesaw weighs three pounds, what could the other gobots weigh?

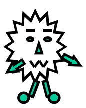


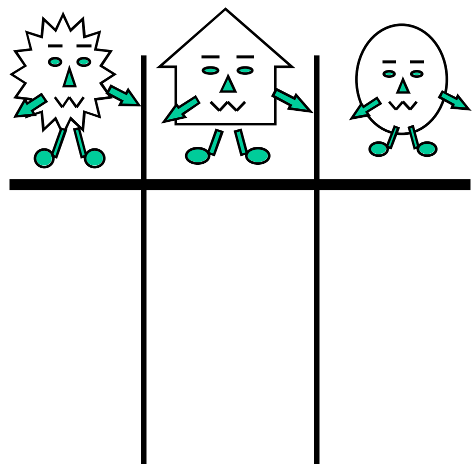
If  on the seesaw weighs five pounds, what could the other gobots weigh?

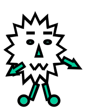


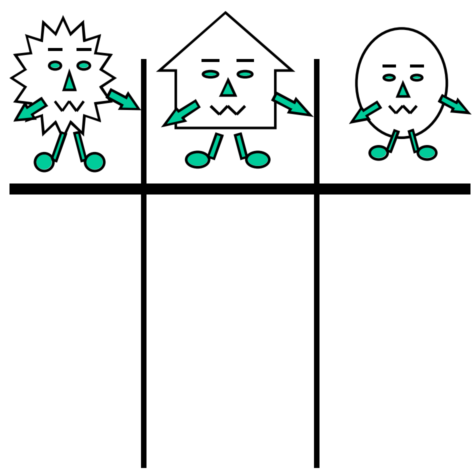
SEESAW BALANCES



If  on the seesaw weighs five pounds, what could the other gobots weigh?



If  on the seesaw weighs six pounds, what could the other gobots weigh?



Activity: Can You Make This Scale Balance?

Format: Small Group

Objective: Participants will be able to balance the scale with a variety of different combinations of cubes by placing the colored cubes on provided shapes.

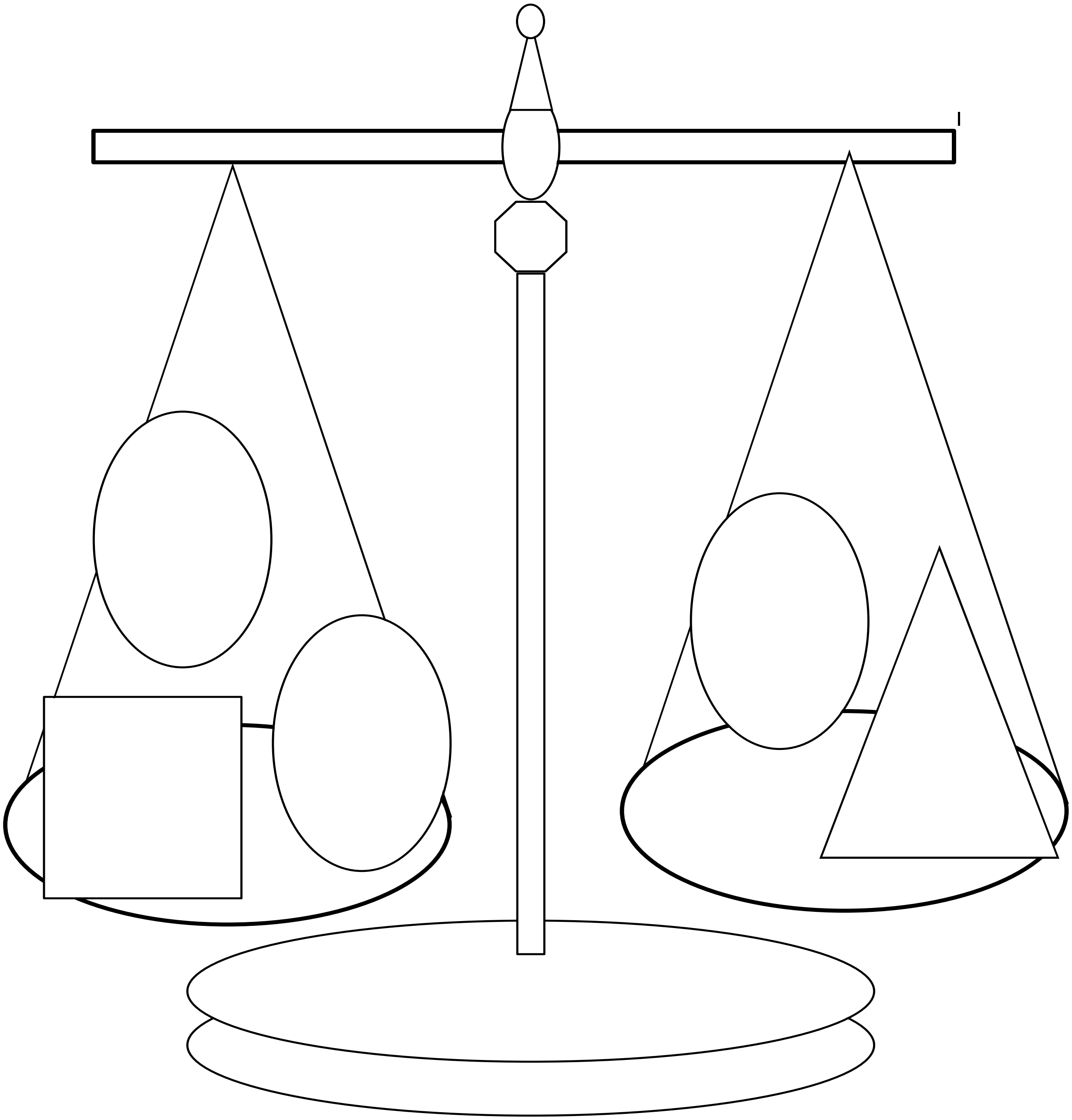
Related SOL: 3.25, 4.22, 5.21

Materials: Colored cubes or counters, balance scale worksheet for each student, overhead projector, transparency of the Balance Scale

Time Required: 20 minutes

Directions:

1. Display the Balance Scale transparency on the overhead projector.
2. Explain to participants that they can balance the scale by placing colored cubes in the square, circle, and triangle shapes on the scale
3. Instruct participants that they must follow the rules that you will list on the board or on chart paper.
   - Shapes that are the same must hold the same numbers of cubes.
   - Shapes that are different must hold different numbers of cubes.
   - All shapes must hold some cubes.
   - The two sides must balance by holding equal numbers of cubes all together.
   - You may use a total of 10, 15, or 20 cubes.
4. Assign each participant a partner and ask them to balance their Balance Scale using eighteen rainbow cubes.
5. Be sure to remind the participants to follow all the rules.
6. Work on the chalkboard or overhead to create a table that will track participants' differing solutions.
7. Instruct participants to again attempt to balance the scale this time using ten, fifteen, and finally twenty colored cubes.
8. Encourage participants to record their findings so that they will be able to explain how they balanced the scale each time.



Same Shapes Must Hold Same Numbers

Activity: Weighty Problems

Format: Small Group

Objective: Participants will explore the concepts of equality among variable expressions to develop an understanding of equality or balance. This is a necessary foundation for solving equations in algebra.

Related SOL: 2.26, 4.22, 5.21

Materials: Weighty Problems Activity Sheets: Primary and Upper Elementary , stationary physical scale, geometric solids

Time Required: 50 Minutes

Directions:

1. Present the weighty problems to the participants as transparencies on an overhead projector. (Teachers can use these problems with participants and may wish to reproduce them as handouts or as laminated problem cards.) Recognize that concrete experiences with a balance scale can provide the foundation for understanding the concepts of equality and inequality.

2. Discuss the concepts that participants would be required to develop, including weight relationships, equality, and equivalent expressions.

Weighty Problem Cards
Primary Set #1- #11:

Objective: Deduce weight relationships
Recognize that a balanced scale represents equality

Teacher Questions Appropriate to the Different Situation Might Include :
What does the scale show? or What does the first (or second) scale show?
How are the scales alike? How are they different?
Which scale would you use first to solve the problem? Why?
How can you find the weight of the geometric solid (sphere, cylinder, cone)?

3. Have the participants develop a few cards of their own. Suggest that they use common objects such as pictures of toys like dolls, trucks, shoes, etc.

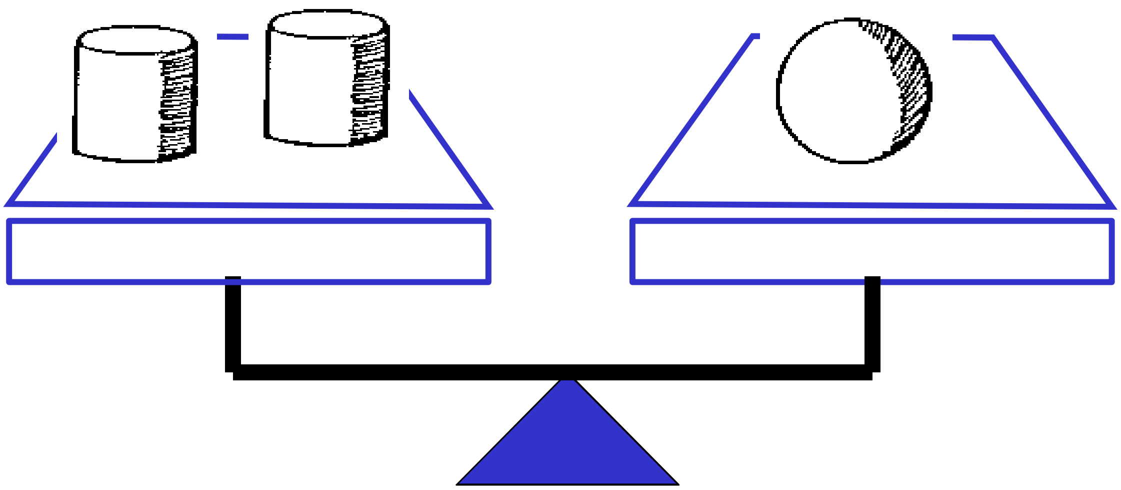
Weighty Problem Cards
Upper Elementary Set #1- #10:

Objective: Deduce relationships among the mass of objects from visual cues.
Recognize that a balanced scale represents equality
Identify collections of objects with equal mass.
Recognize that there are a variety of ways to solve each problem.

Teacher Questions Appropriate to the Different Situation Might Include :
What do the scales show? or What does scale A (or B) show?
How are the scales alike? How are they different?
Do any scales have only one type of object on them? What can you deduce about the weight of each object on a scale like this?
Do any scales differ by only one object? What does this tell you about the scale?
What object weighs more? What object weighs the least?
Which scale would you use first to solve the problem? Why?
How can you find the weight of the geometric solid (sphere, cylinder, cone)?

4. Have the participants develop a few cards of their own using the blank. Suggest that they use common objects such as action figures, cars, beads, etc.

Weighty Problems
Primary #1



Circle the block that weighs more.

Explain how you know.____________________

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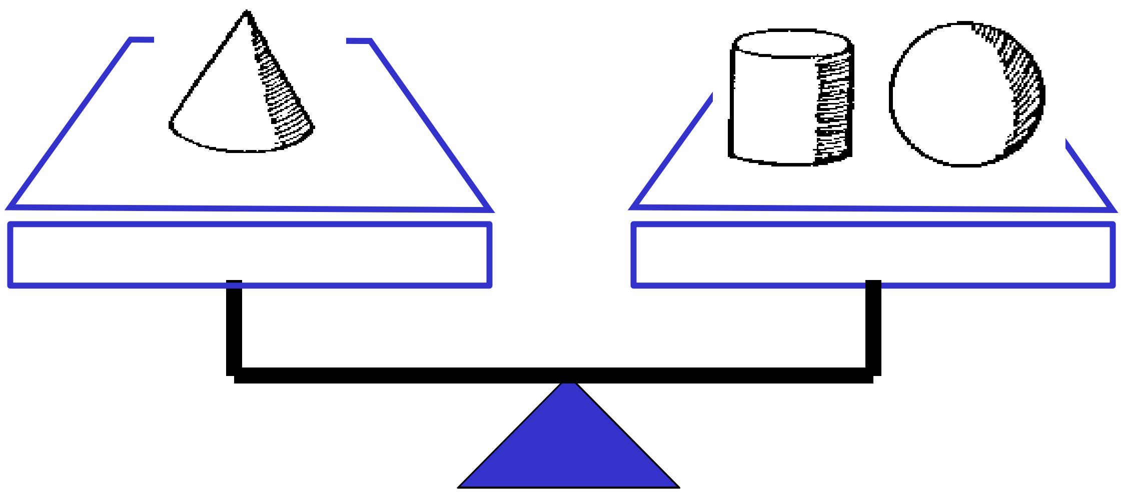
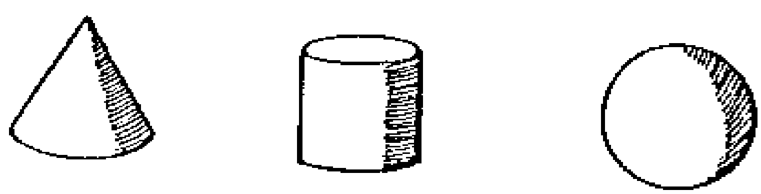
Weighty Problems
Primary #2



Circle the block that weighs more.



Explain how you know.___________________

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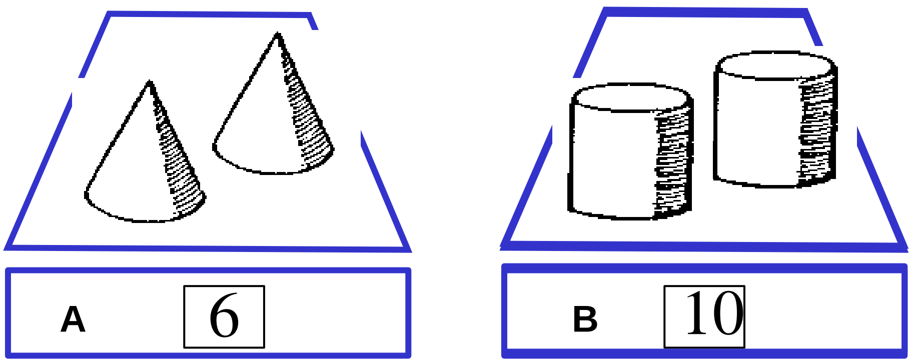
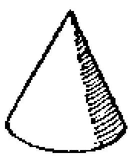
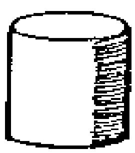
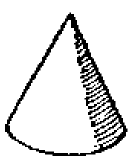
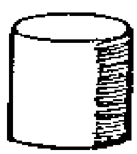
Weighty Problems
Primary #3

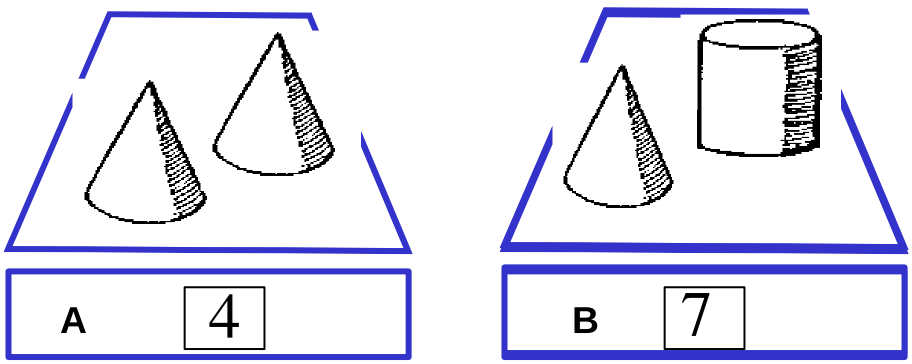


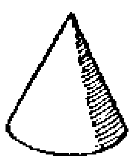
1.  weighs ____________ pounds.

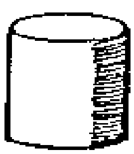
2.  weighs ____________ pounds.

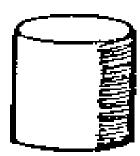
3.  and  weigh _________pounds.

Weighty Problems
Primary #4



1.  weighs ___________ pounds.

2.  weighs ___________ pounds.

3. How did you find the weight of  ? ______

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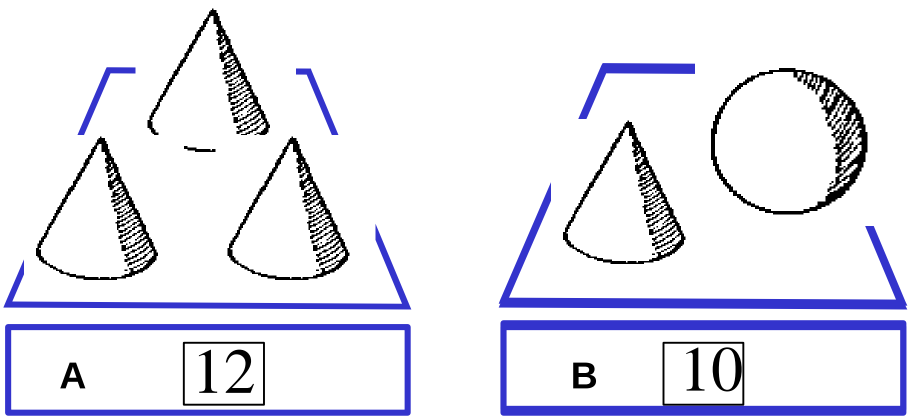
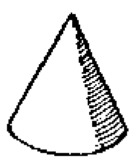
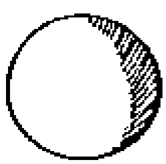
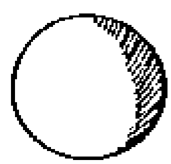
Weighty Problems
Primary #5



1.  weighs __________ pounds.

2.  weighs __________ pounds.

3. How did you find the weight of  ? _____

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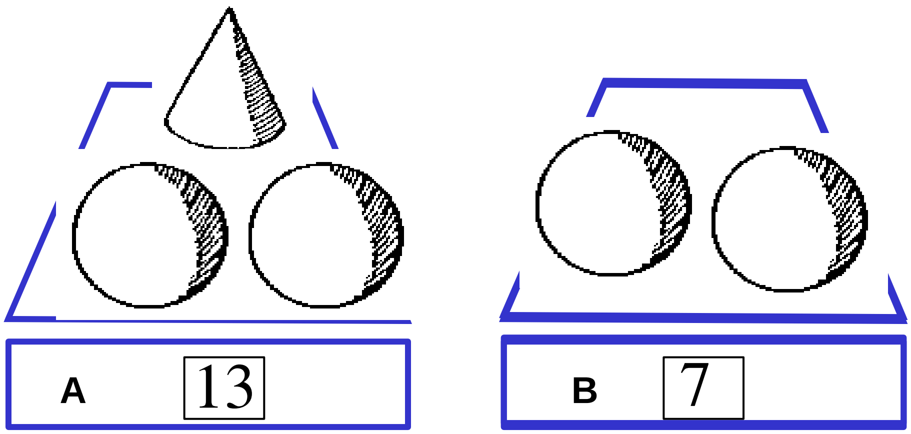
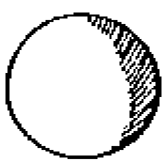
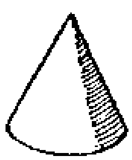
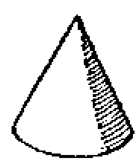
Weighty Problems
Primary #6



1.  weighs ____________ pounds.

2.  weighs ____________ pounds.

3. How did you find the weight of  ? ______

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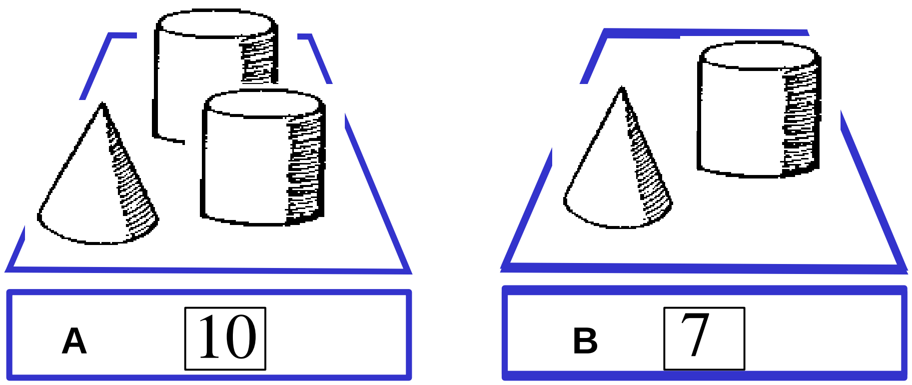
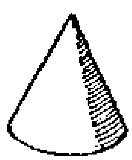
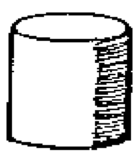
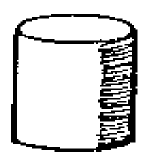
Weighty Problems
Primary #7



1.  weighs ____________ pounds.

2.  weighs ____________ pounds.

3. How did you find the weight of  ? _______

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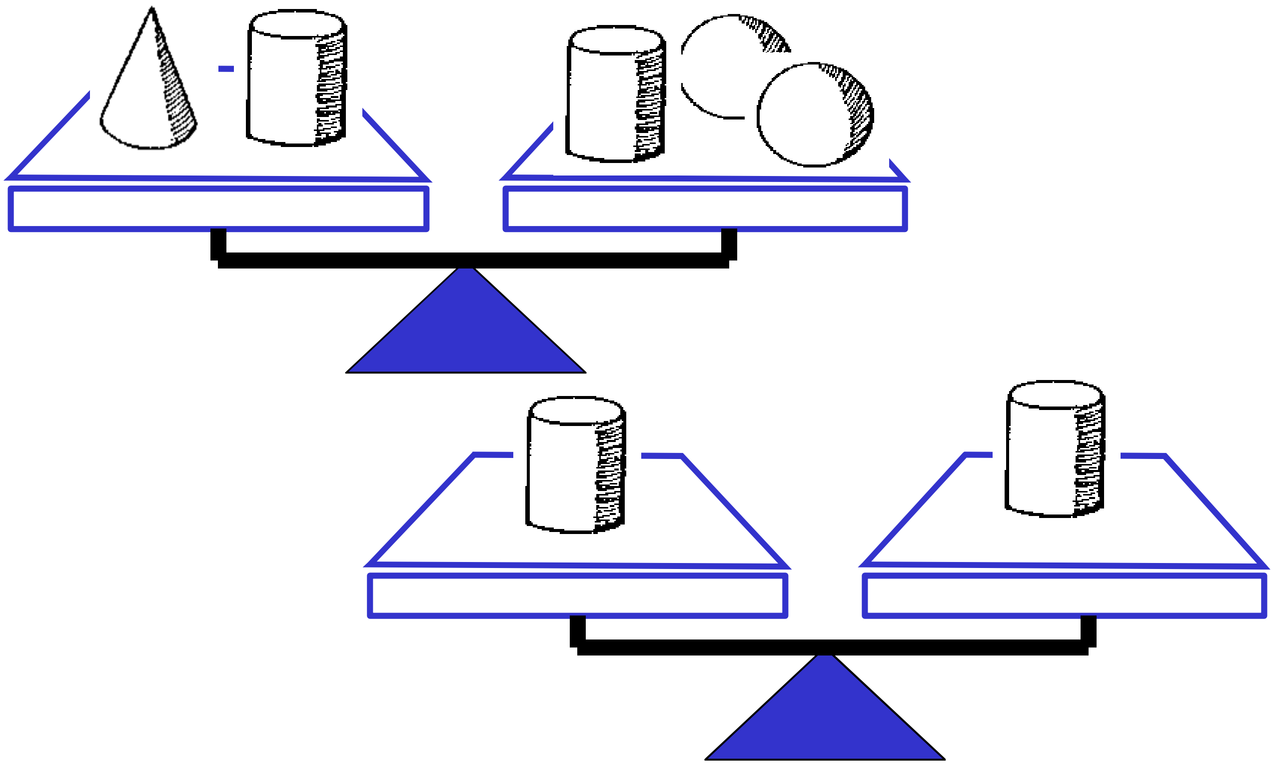
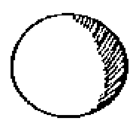
Weighty Problems
Primary #8



Circle the block that weighs more.

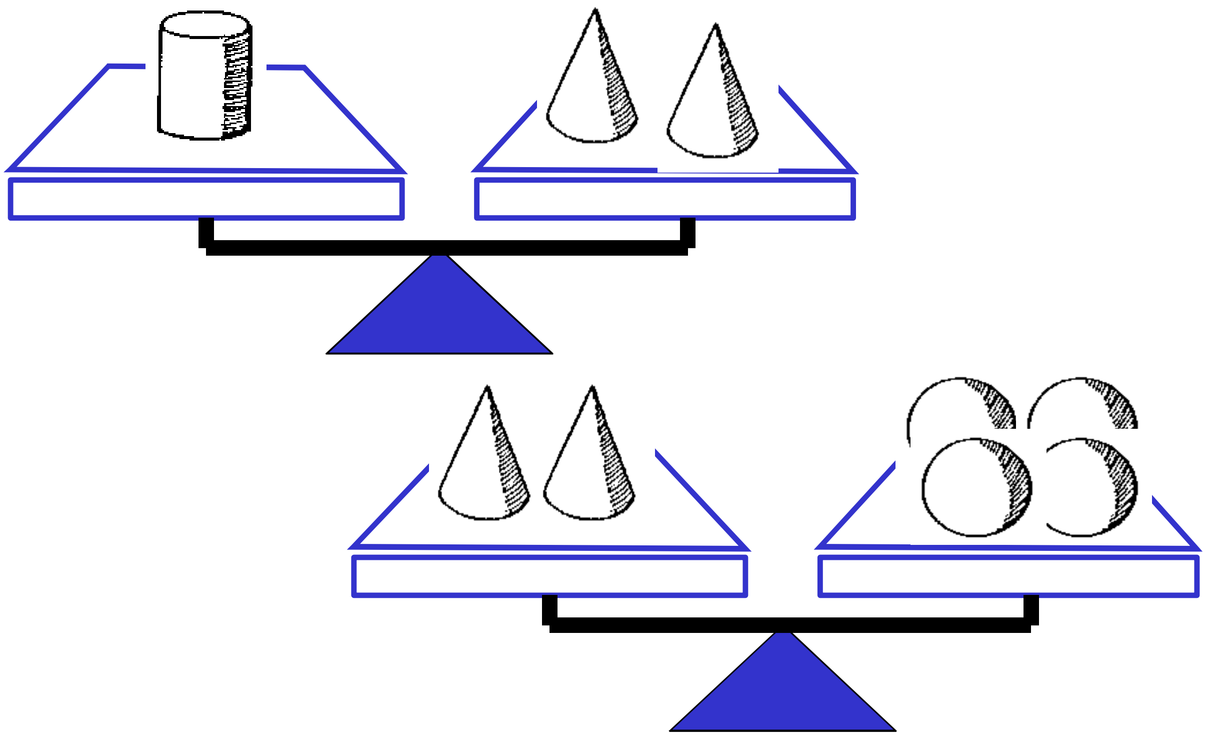


Explain how you know.___________________

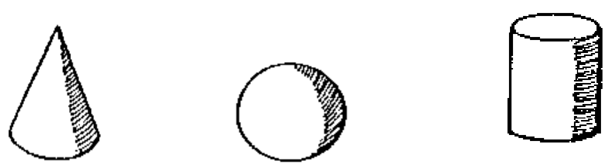
____

____

Weighty Problems
Primary #9



Circle the block that weighs the most.
Put a check mark on the one that weighs the least.



Explain how you know.________________________

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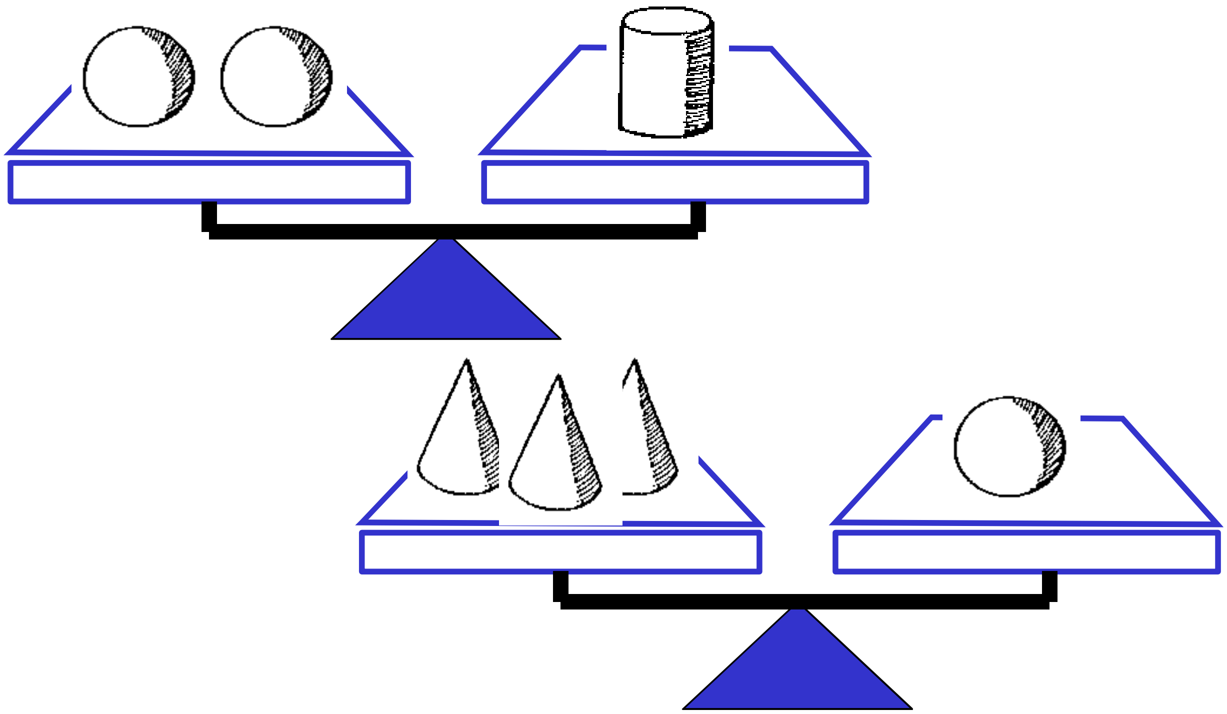
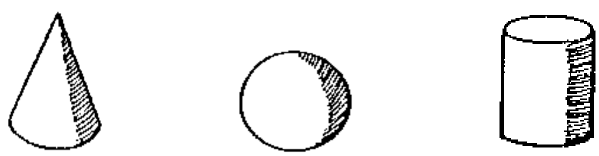
Weighty Problems
Primary #10



Circle the block that weighs the most.
Put a check mark on the one that weighs the least.



Explain how you know.________________________

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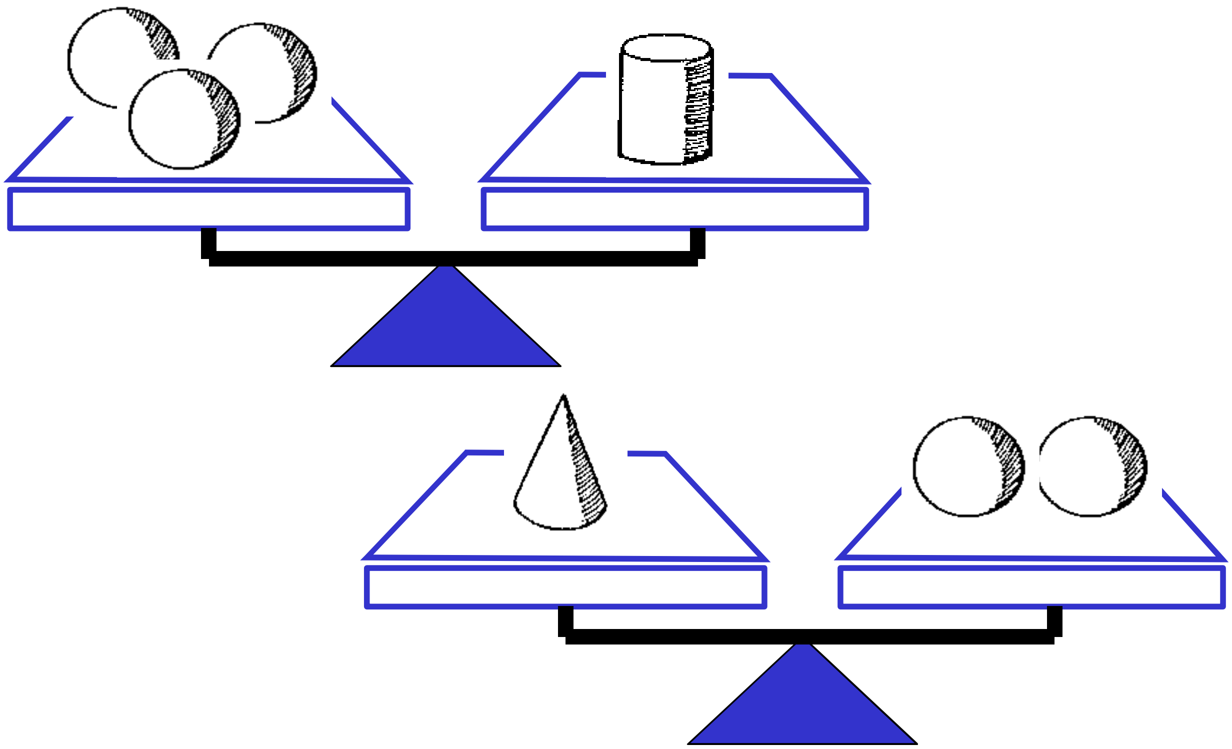
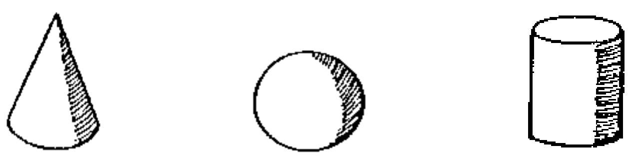
Weighty Problems
Primary #11



Circle the block that weighs the most.

Put a check mark on the one that weighs the least.



Explain how you know.____________________

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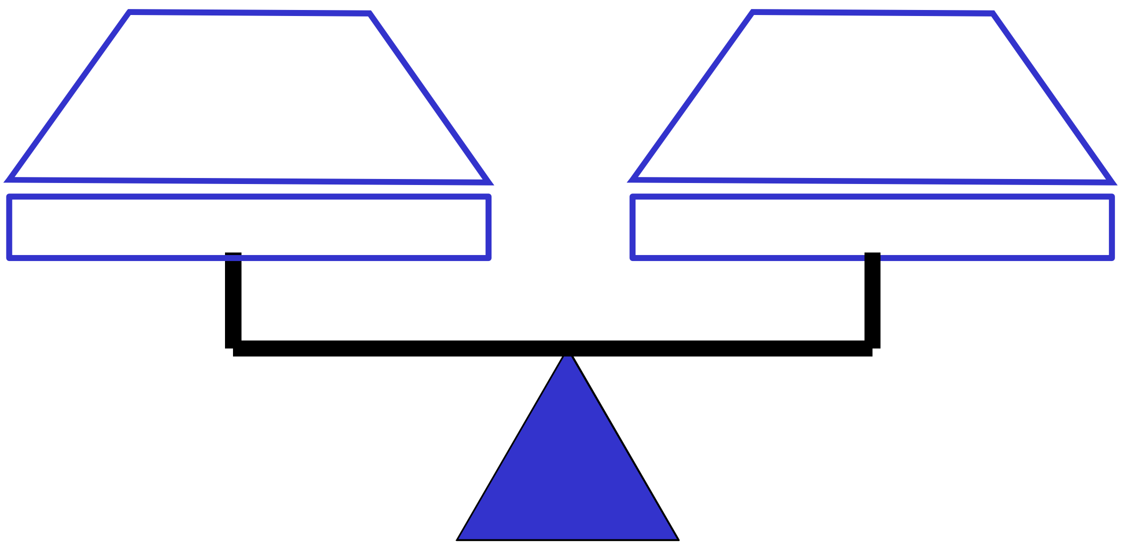
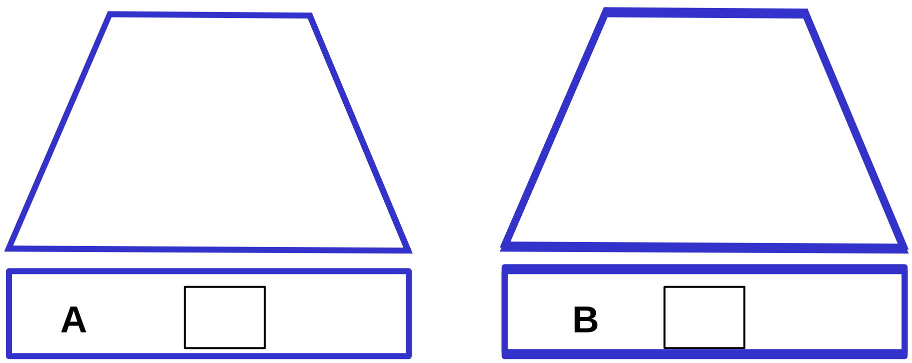
Weighty Problems
Primary



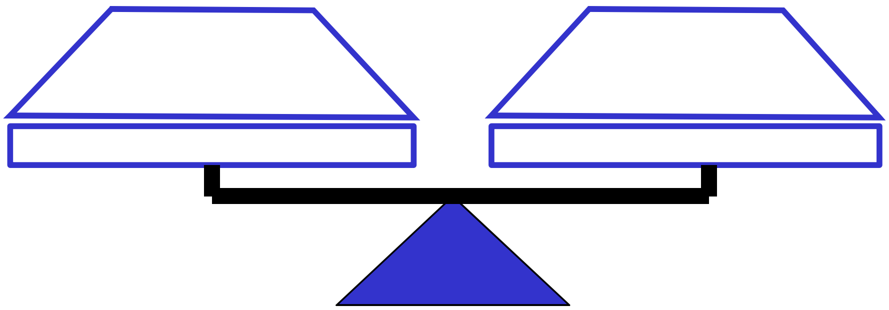
Weighty Problems: Primary

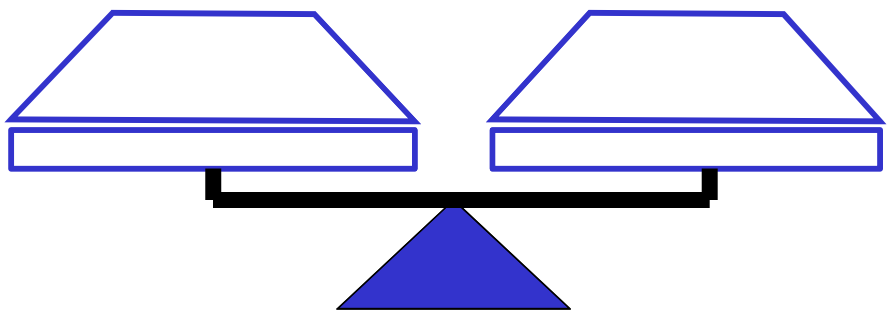


1. weighs ____________ pounds.

2. weighs ____________ pounds.

Weighty Problems: Primary





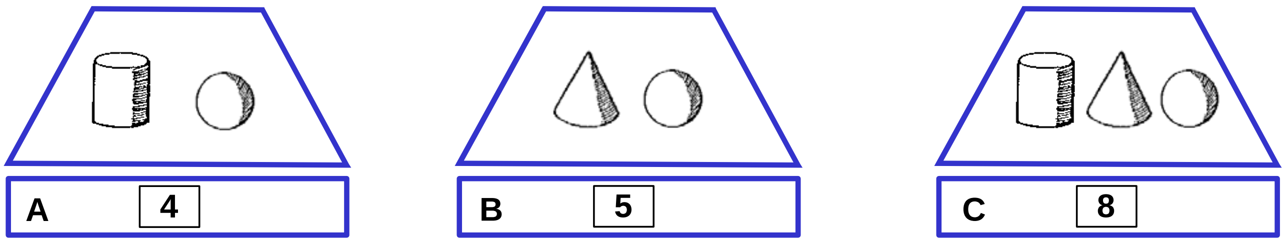
Circle the one that weighs the most.
Put a check mark on the one that weighs the least.

Explain how you know.____________________

____

____

Weighty Problems #1



1. **Find the weight of each block.**

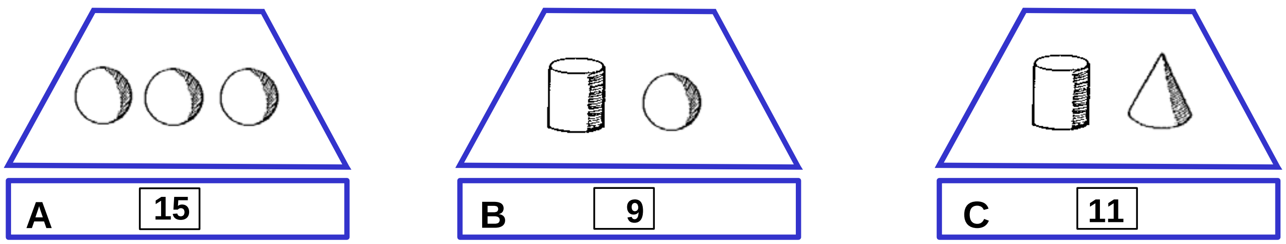
   cylinder = ________ pounds

   sphere = _________ pounds

   cone = ________ pounds

2. **List or draw the steps you followed to solve the problem.**

Weighty Problems #2



1. Find the weight of each block.

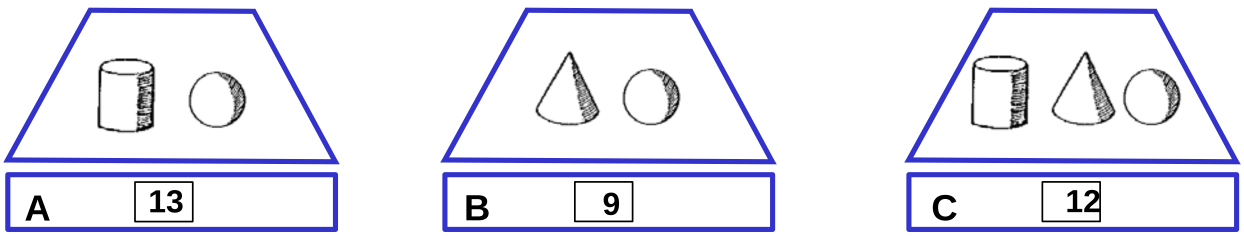
   cylinder = _________ pounds

   sphere = __________ pounds

   cone = _________ pounds

2. List or draw the steps you followed to solve the problem.

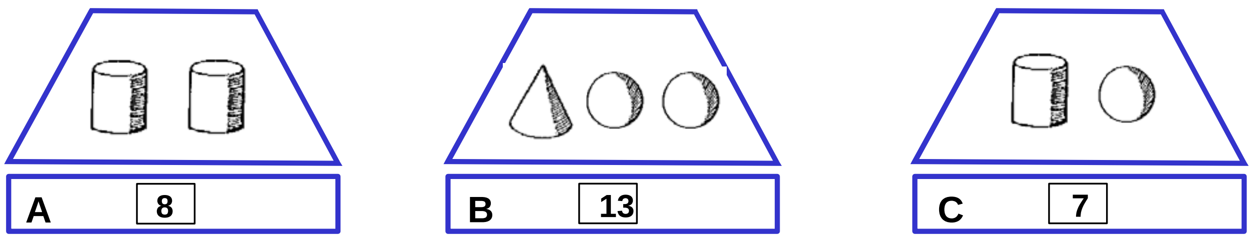
Weighty Problems #3



1. Find the weight of each block.
   cylinder = _________ pounds
   sphere = __________ pounds
   cone = _________ pounds

2. List or draw the steps you followed to solve the problem.

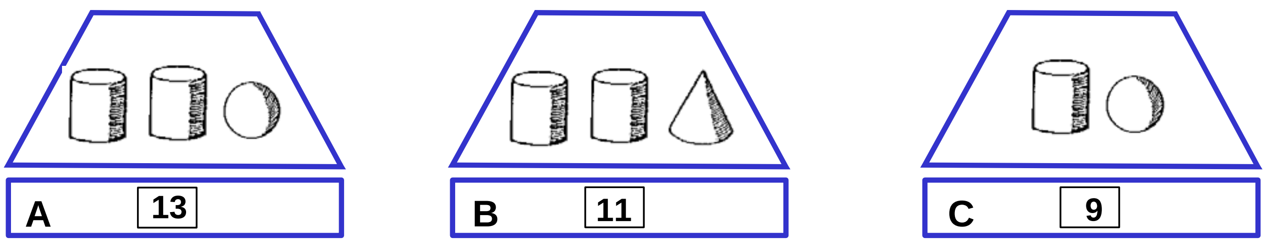
Weighty Problems #4



1. **Find the weight of each block.**
   cylinder = _________ pounds
   sphere = __________ pounds
   cone = _________ pounds

2. **List or draw the steps you followed to solve the problem.**

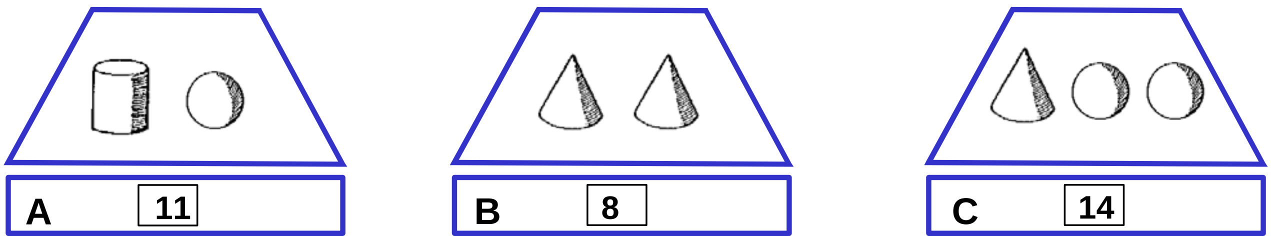
Weighty Problems #5



1. Find the weight of each block.
   cylinder = ________ pounds
   sphere = _________ pounds
   cone = ________ pounds

2. List or draw the steps you followed to solve the problem.

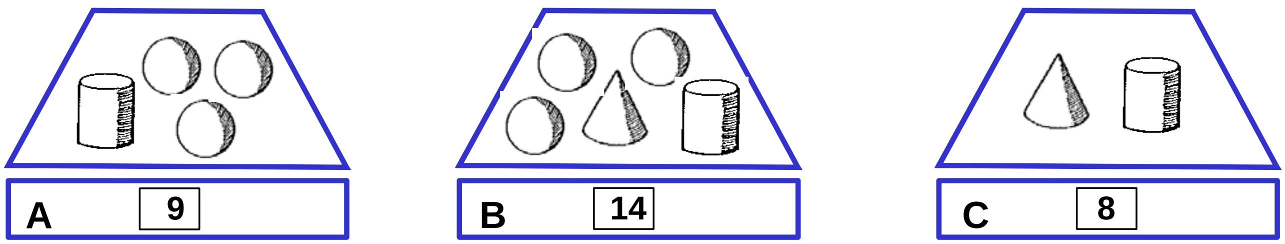
Weighty Problems #6



1. **Find the weight of each block.**
   cylinder = _________ pounds
   sphere = __________ pounds
   cone = _________ pounds

2. **List or draw the steps you followed to solve the problem.**

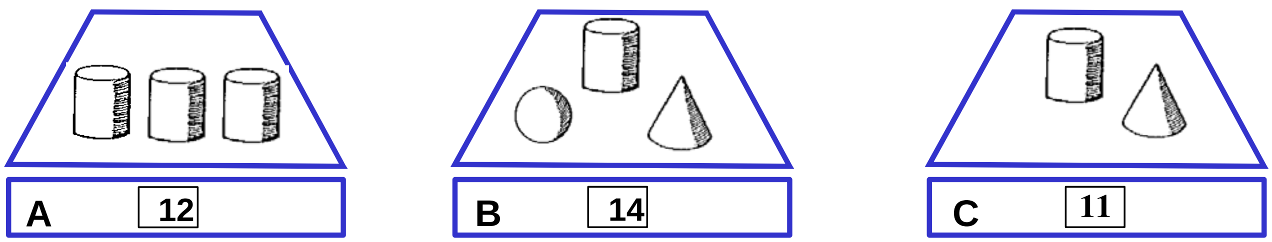
Weighty Problems #7



1. **Find the weight of each block.**
   cylinder = _________ pounds
   sphere = __________ pounds
   cone = _________ pounds

2. **List or draw the steps you followed to solve the problem.**

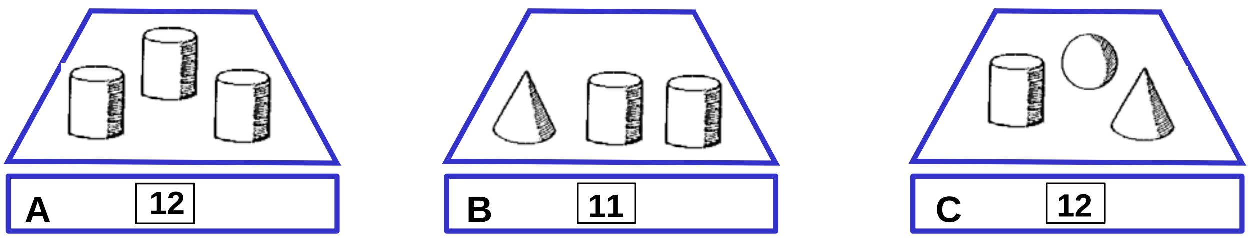
Weighty Problems #8



1. **Find the weight of each block.**
   cylinder = _________ pounds
   sphere = __________ pounds
   cone = _________ pounds

2. **List or draw the steps you followed to solve the problem.**

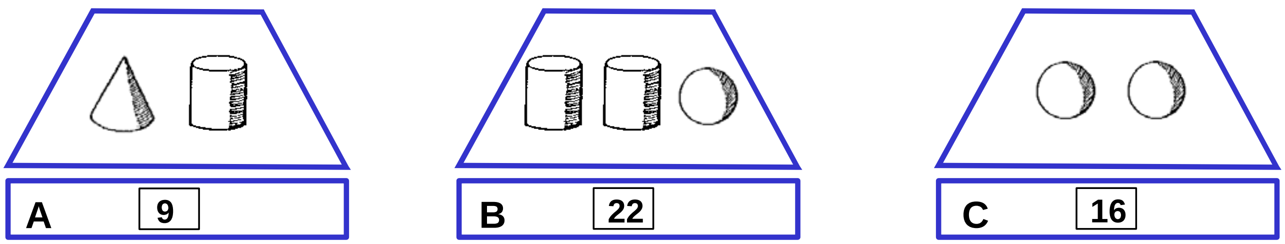
Weighty Problems #9



1. **Find the weight of each block.**
   cylinder = _________ pounds
   sphere = __________ pounds
   cone = _________ pounds

2. **List or draw the steps you followed to solve the problem.**

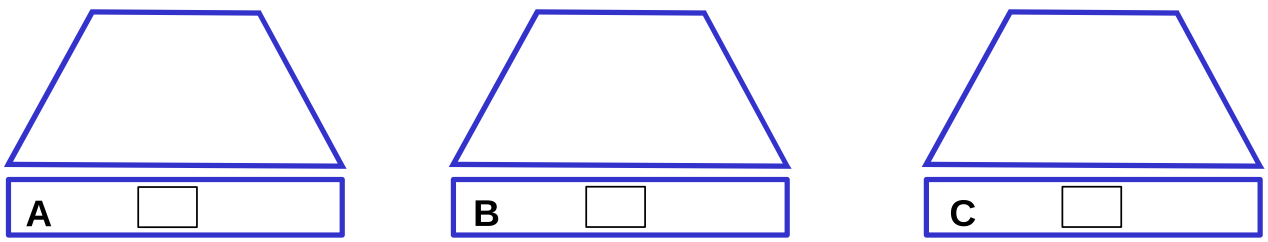
Weighty Problems #10



1. Find the weight of each block.
   cylinder = _________ pounds
   sphere = __________ pounds
   cone = _________ pounds

2. List or draw the steps you followed to solve the problem.

Weighty Problem



1. Find the weight of each block.
   cylinder = _________ pounds
   sphere = __________ pounds
   cone = _________ pounds

2. List or draw the steps you followed to solve the problem.

Activity: Balance Those Blocks!

Format: Small Group

Objective: Participants will be able to use blocks to balance pictorial equations.

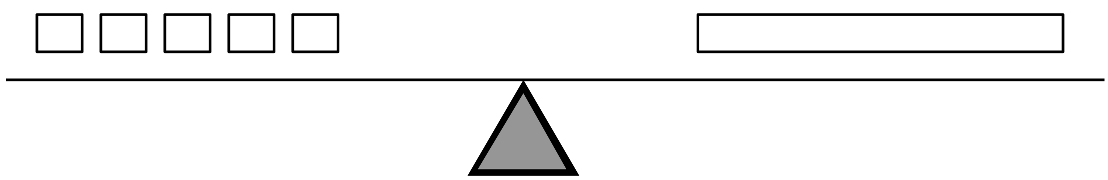
Related SOL: 2.26, 4.22, 5.21

Materials: Laminated balance scale, stationary physical scale, unit blocks, rectangular solids

Time Required: 15 minutes

Directions:

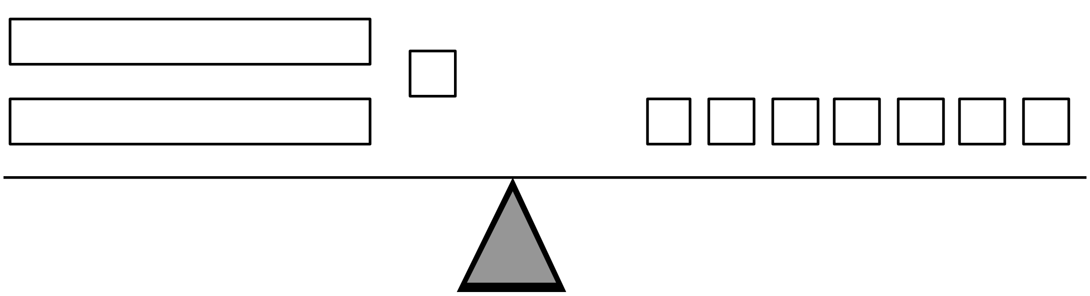
1. The teacher displays the equation on the physical scale in front on the class.



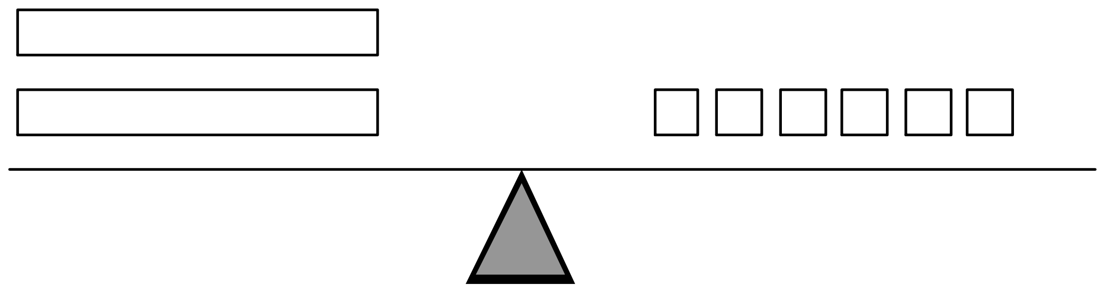
The teacher assists the participants in grasping the concept that both sides of the scale must have the same value for the scale to balance.

Participants begin to understand that the rectangular solid in the equation is valued at five in order to balance the five unit blocks .

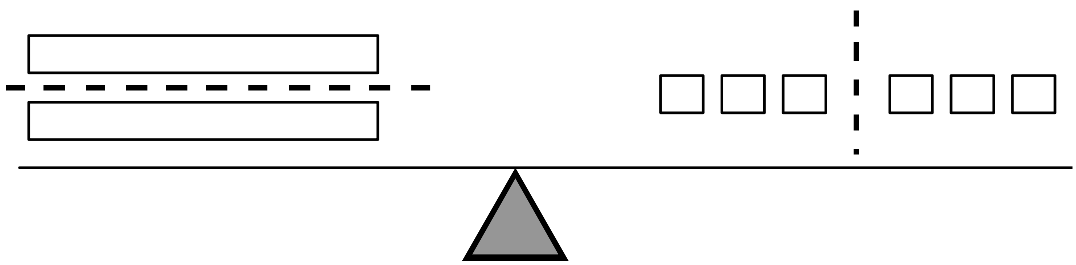
2. The teacher then displays the equation 2x + 1 = 7 on the physical scale.



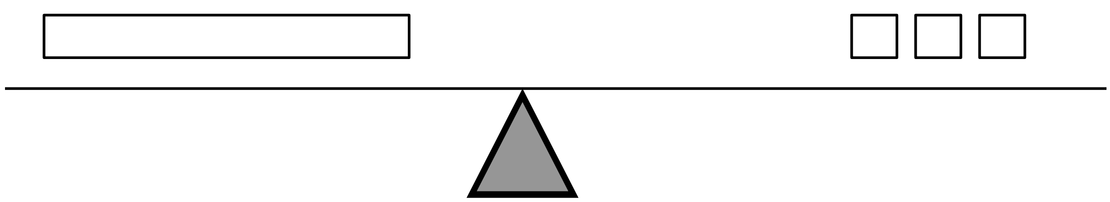
The teacher points out to the class that, in keeping the scale balanced, one unit block can be removed from each side of the balance scale.



The teacher assists the class in determining that the balance scale has two groups of blocks on each side. In order to successfully solve the equation, however, only one variable is needed. Thus, the teacher models for the class as she divides each side by two.



Participants again recognize that in order for the scale to balance the rectangular solid must be valued at three.



3. Assign student partners to work together on their laminated balance scale mats to solve the equation: $3x + 1 = 13$ and $x + 4 = 2x + 3$.

Assess student understanding.

Activity: Coat Hanger Balances

Format: Individuals

Objective: Participants will use their knowledge and understanding of the concepts of balance and equality to create their own coat hanger balance. These balances are foundation knowledge to the concept of equality found in algebraic thinking.

Related SOL: 2.9, 5.21

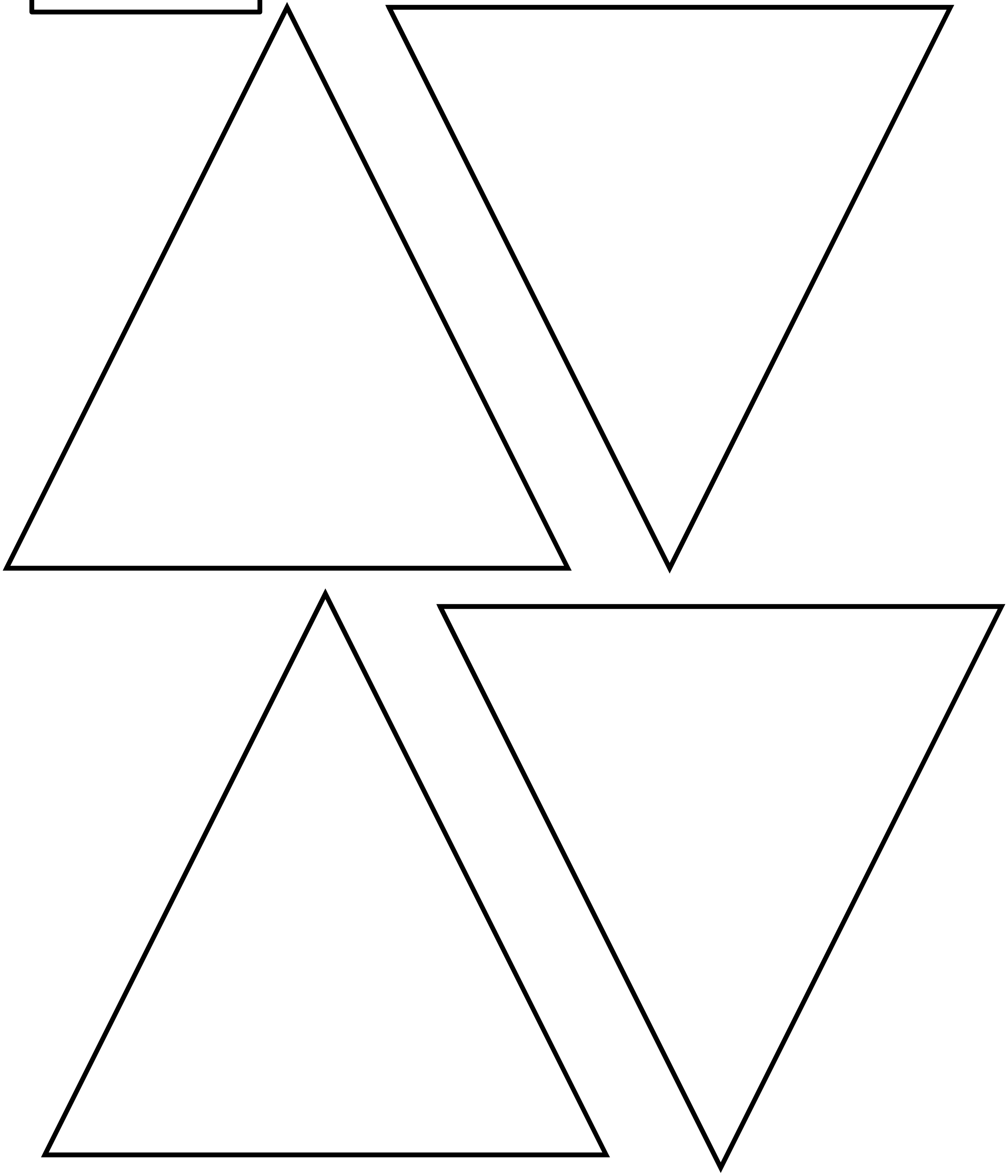
Materials: One coat hanger per participant (ask participants to supply their own), Shape pattern Activity Sheets printed on various colored cover stock (e.g., triangles on yellow, squares on blue, rectangles on green, etc.), yarn or string, tape, scissors

Time Required: 25 Minutes

Directions:

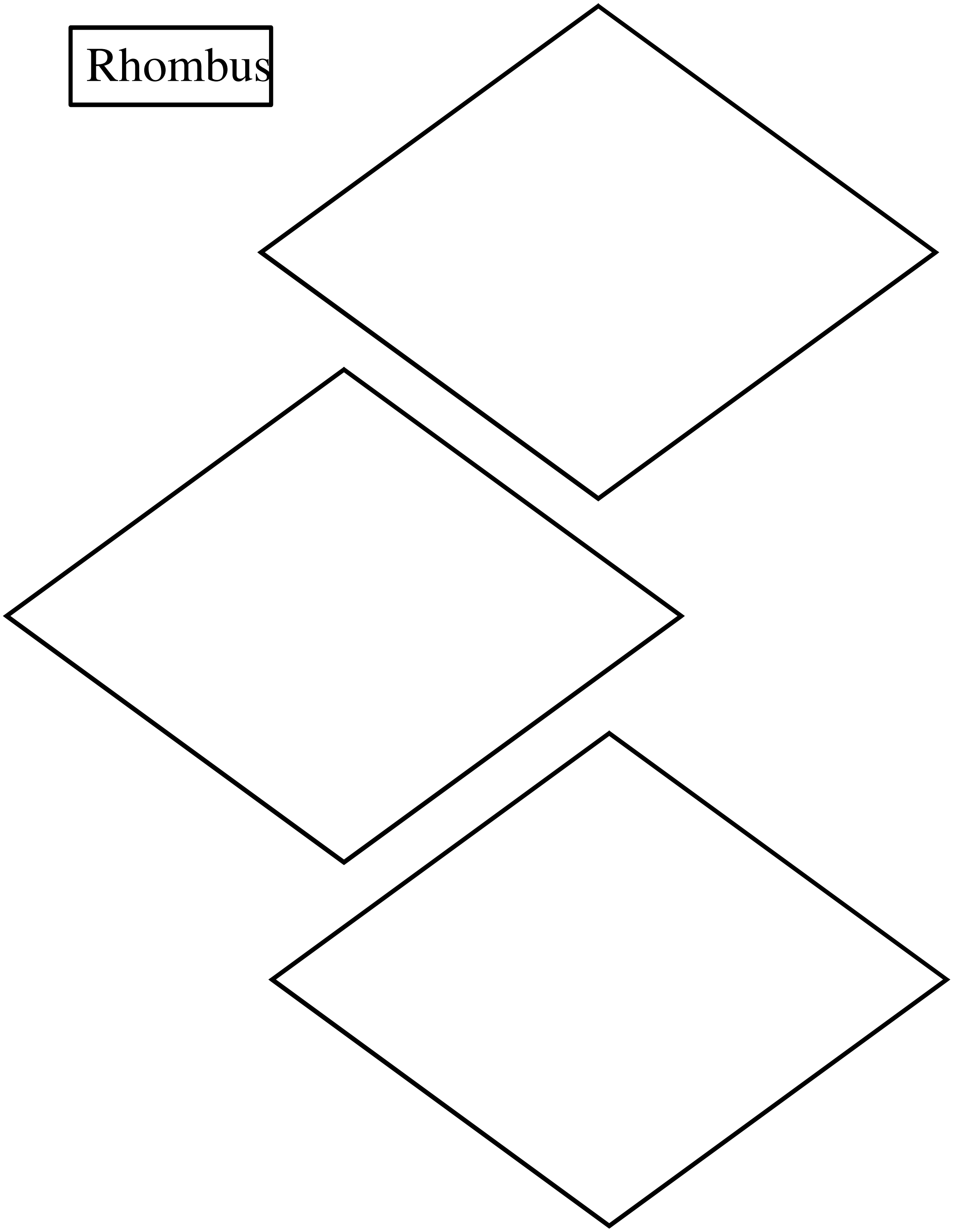
1. Participants are asked to create their own coat hanger balance using a variety of shapes where each shape stands for a number value or "weight." Each shape should have a different value, just as each variable in an equation has a different value. The total value can be taped to the neck of the hanger.
2. Participants should exchange their balances with others and solve their balance problem, checking to see whether their solutions match. In cases where they do not match, participants should redo their problems.
3. Participants should pay attention to the following rules:
   - The right and left sides of the coat hanger must balance;
   - Each shape has a unique and consistent weight within the balance, and no shape weighs zero;
   - Clues may be attached at the neck of the hanger and should be checked to ensure that the weights are properly distributed;
   - Weights can be positive whole numbers, fractions or decimals; depending on the grade level taught by the participant;
   - A shape hanging directly below the fulcrum (center) does not affect the balance between the left or right arms of the balance;
   - Size of the shapes has no relation to the weight;
   - These are problems in balancing numbers and do not take into consideration distances from the fulcrum or any other principles of physical science.

Triangles



Squares

Rhombus



Hexagon