

The Redistribution Impossibility Theorem:

An open economy exposition

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Chamley and Judd found that in standard economic environments it's literally impossible to tax capitalists, give the proceeds to workers, and leave workers better off as a result. This is the RIT, the Redistribution Impossibility Theorem. Using standard tools of macroeconomics, they found that any tax on capital shrank the capital stock so much that it left a smaller tax base and fewer machines for workers to use. In their entirely conventional framework, fewer machines to tax + fewer machines to use = less disposable income for the proletariat.

But the finding still isn't that intuitive: Am I really supposed to believe that even a low tax rate on capital shrinks the economy so much that any attempt to help workers actually hurts workers? Using a constant returns Cobb-Douglas production function for simplicity, and holding labor supply and technology fixed at unity, we have a simple illustrative production function:

$$GDP = Y = K^\alpha, 0 < \alpha < 1.$$

Each period in this competitive economy, capitalists will earn the the marginal product of capital MP_K on each unit of capital K , but before receiving that income, fraction t of it will be handed over to the workers. Let's consider the crucial Chamley-Judd equation, the fact that the after-tax return on capital—the net marginal product of capital returned to the capitalist—has to equal the capitalist's opportunity cost rate of return, $r > 0$:

$$r = MP_K(1 - t) = \alpha K^{\alpha-1}(1 - t),$$

The literature typically treats r as the steady-state rate of return, determined by the capitalist's time preference and perhaps other parameters; to make it more realistic, let's think of r as the global after-tax rate of return, since capital today can move quickly across borders in search of the highest reward. Any one country is too small to noticeably change r , so any one country takes the

after tax rate of return as given just as any one farmer takes the price of corn as given. That means that the nation's supply of capital is a horizontal line, perfectly elastic at the price $r/(1-t)$, while the national demand for capital is a typical downward-sloping demand curve. The above equation can readily be solved for the equilibrium capital stock,

$$K^* = \left(\frac{\alpha(1-t)}{r} \right)^{\frac{1}{1-\alpha}}$$

which obviously shrinks as the tax rate rises: Indeed, capital shrinks by more than 1% for every one percentage point increase in the tax rate. The Chamley-Judd finding is that in the steady-state, or the long run, the disposable income for workers (i.e., wages (w) plus transfers (τ)) always shrinks whenever the tax on capital increases. And in this open economy model—as so often in the real world—the long run arrives quickly. Let's define disposable income in this setting, a competitive economy where α is the capital share and $(1-\alpha)$ is the worker share of GDP:

$$\text{wages} + \text{transfers} = w + \tau = (1-\alpha)K^\alpha + t\alpha K^\alpha$$

At this point, it just seems obvious that there's got to be some level of capital taxation that could help workers out. Chamley and Judd must have made a math mistake: a rising tax rate might shrink the capital stock, sure, but surely there's some level of taxation where we won't shrink the capital stock all that much, workers can get a check from the government, and the proletariat will be better off. Let's try to prove Chamley and Judd wrong. Substituting in the equilibrium capital stock K^* and simplifying, we find that:

$$\text{wages} + \text{transfers} = w + \tau = \left(\frac{(1-t)\alpha}{r} \right)^{\frac{\alpha}{1-\alpha}} (1 - (1-t)\alpha)$$

The second set of parentheses says that workers get the whole pie except for the part that the capitalists are allowed to keep. If it's possible for $\partial(w + \tau)/\partial t$ to be positive, ever, then the Redistribution Impossibility Theorem is false. To

investigate the possibility, it's easiest to take logs first, then the derivative with respect to the tax rate, then simplify a bit:

$$\partial \ln(w + \tau) / \partial t = \frac{\alpha}{(1-\alpha+t\alpha)} - \frac{\alpha}{(1-\alpha+t\alpha)-t}$$

This derivative is negative whenever $t > 0$. That means that cutting taxes on capitalists—or should we say globalists?—is always good for workers. For a Cobb-Douglas production function in an open economy, Chamley and Judd are right and the Redistribution Impossibility Theorem holds.

A square root production function: A simpler example

If output is the square root of capital, it's even easier to see that the optimal tax rate on capital is zero. In this case, worker income including transfers becomes:

$$w + \tau = 0.5K^{0.5} + 0.5tK^{0.5}$$

And the equilibrium national capital stock now becomes:

$$K^* = \left(\frac{0.5(1-t)}{r} \right)^2$$

Combine these two expressions. The square root of a squared expression is the expression itself, while a touch of factoring yields a formula for the disposable income of the proletariat:

$$w + \tau = 0.5(1+t) \left(\frac{0.5(1-t)}{r} \right) = \frac{(1-t^2)}{4r}$$

So the best way to maximize the disposable income of the proletariat in a small open economy is to set the tax rate on capital exactly equal to zero. That means, incidentally, that making workers subsidize capital with $t < 0$ would be

a bad idea for workers—a capital subsidy would be inefficient in this world of diminishing returns to capital. But to return to the central issue: Trying to tax capital in order to give the money to workers only makes the workers poorer in this world. Redistributing wealth from capitalists to workers in a way that makes workers better off is impossible in this world.

So what's really happening here?

When α is close to one, diminishing returns are weak, and hence the capital demand curve is flat, the RIT is almost obviously true: Any rise in capital taxes creates an enormous capital outflow, since it takes a huge decline in the amount of capital to push the pre-tax rate of return high enough to compensate for the tax. And that huge decline in capital is going to hurt both worker wages and the tax base, making it impossible to raise the disposable income of workers.

When α is close to zero, the RIT still holds: The capital demand curve is steeper, so the capital stock doesn't fall as much, but since workers now get almost the entire economic pie anyway, the transfers from capital don't add up to much. And again, since a 1% rise in the tax rate causes a greater than 1% fall in the capital stock, there's less capital to tax when the tax goes up. Yes, the fall in wages due to a capital tax hike is small, but the rise in tax revenue is even smaller. *When capital is important, the global market is extraordinarily sensitive to the tax rate, and when capital is unimportant, the tax can't generate enough revenue to help out workers.* Either way, raising workers' disposable income through capital taxation is impossible, and the RIT holds.

We learn from microeconomics that it's unwise to tax an input that has perfectly elastic supply, but it took macroeconomics to show us just how unwise that decision could be. From the point of view of a worker in an open economy with rapid global capital flows, the optimal tax rate on capital is zero.