ALEXANDER TABARROK

# BELIEVE IN PASCAL'S WAGER?
# HAVE I GOT A DEAL FOR YOU!

## 1. THE WAGER

Pascal's wager is one of the most famous arguments in philosophical theology. The wager is a decision theoretic argument for believing in God. Either God exists or God does not exist and either you believe in God (bet on his existing) or you do not believe in God. According to Pascal, the utility of the outcome under each of these possibilities is as in Table 1. (My account of the wager follows that of Hajek (1998)).[1]

TABLE 1

|  | God exists | God does not exist |
|---|---|---|
| Believe in God | $\infty$ | $f_1$ |
| Do not believe | $f_2$ | $f_3$ |

If God exists and you believe then you have an "infinitely happy life to gain" but if God does not exist or you do not believe then your utility level ($f_1$, $f_2$ or $f_3$) is finite. Let $p$ be the probability that God exists then the expected value of believing in God is:

$$E(\text{Believe in God}) = \infty \times p + f_1(1 - p) = \infty$$

On the other hand, the expected utility of not believing in God is:

$$E(\text{Do not Believe}) = f_2 \times p + f_3(1 - p) = f_4$$

Since $f_4 < \infty$ a rational individual should believe in God.

## 2.  THE OFFER

The following offer should be irresistible to anyone who accepts
Pascal's wager as sound. Many philosophers do in fact think that
Pascal's wager is sound but for those who do not think so I suggest
below why the offer should be accepted by anyone who thinks there
is some probability that Pascal's wager might be sound, no matter
how small the probability.

The offer is as follows. For a fee of all your wealth I will use my
line to God to put in a word on your behalf. I assert that individuals
for whom I put in a good word are more likely to enter heaven and
receive everlasting joy than are other individuals. Table 2 shows
the different possible outcomes, their utilities, and in brackets some
probabilities to be discussed below.

TABLE 2

|                | God exists & line open | God exists & line broken | God does not exist |
| --- | --- | --- | --- |
| Pay fee        | $\infty$ ($p_1$) | $\infty$ ($p_3$) | 0 ($p_4$)   |
| Do not pay fee | $\infty$ ($p_2$) | $\infty$ ($p_3$) | $w$ ($p_4$) |

Since my offer applies to those who accept Pascal's wager as
sound they expect an infinite utility from believing in God whether
or not my offer is accepted. The table suggests, therefore, that
paying the fee is dominated by not paying the fee. The inference,
however, is incorrect. Paying the fee is infinitely better than not
paying the fee as becomes clear once we introduce probabilities.
If God exists and my line to God is open someone who pays the fee
enters heaven with probability $p_1$ but someone who does not pay
the fee enters heaven with only probability $p_2$ with $p_2 < p_1$. If God
exists but my line to God is broken then with probability $p_3$ both the
payer and non-payer enter heaven. With probability $p_4$ neither the
payer nor the non-payer enter heaven but the non-payer is better off
by amount $w$. The expected utility of paying the fee is:

$$E(\text{pay fee}) = p_1 \times \infty + p_3 \times \infty + p_4 \times 0$$

and of not paying:

$$E(\text{not pay fee}) = p_2 \times \infty + p_3 \times \infty + p_4 \times w$$

Now note that if $X > 0$ and $p_1 > p_2$ then $p_1 X - p_2 X = (p_1 - p_2)X > 0$. Subtracting the utility of not paying from that of paying we have:

$$(p_1 - p_2) \times \infty - p_4 w$$

but since $(p_1 - p_2) \times \infty = \infty$ it follows that the expected utility of paying the fee is infinitely greater than not paying the fee, no matter how small the probability is of my offer being true or how large your wealth.

Before turning to objections. Let us further examine who should accept this offer. Pascal's wager is an argument of extraordinary power. For suppose that we think the argument false yet place some probability, $p_w$, on the idea that we might be wrong and the argument true after all. No matter how small $p_w$, we should accept Pascal's argument as true. Consider Table 3.

TABLE 3

|                              | Pascal is correct | Pascal is incorrect |
| ---------------------------- | :---------------: | :-----------------: |
| Believe in Pascal's Argument | $\infty$          | $f_1$               |
| Do not believe               | $f_2$             | $f_3$               |

Following exactly the same reasoning as before we can show that a rational individual should believe that Pascal's wager is true. The offer I have given above, therefore, should apply to anyone who accepts that Pascal's wager is true which, as I have just shown, is anyone who thinks Pascal's wager has any positive probability of being true.

## 3. OBJECTIONS

Pascal's wager goes through no matter how small the subjective probability of God existing. The offer goes through no matter how small the subjective probability is of my having a line to God (or how large the probability is of my being a charlatan). The wager will

fail if the subjective probability of God existing is zero and similarly the offer fails if the subjective probability of my having a line to God is zero (i.e. $\Pr(Charlatan) = 1$).[2] For someone who accepts the wager, however, it seems unreasonable not to accept a positive probability of my having a line to God. Certainly the probability of my having a line is significantly less than say the Pope having a line but given that the offeree ascribes positive probability to God existing it seems unreasonable not to believe in the possibility of *someone* having a line to God. And if someone has a line to God why not me?

One could object that anyone offering to sell God's favor may by that reason alone be presumed not to have a line to God.[3] The offer can be recast to defeat this objection. Suppose that Mother Theresa claims that by abandoning one's current existence and devoting one's life to the poor of Calcutta, one can increase the probability of ascending to heaven and attaining infinite utility. Mother Theresa's offer cannot be dismissed as self-serving and indeed is rather plausible. Many people who give money to Mother Theresa's organization surely believe this offer has been made even if the terms of the offer are not explicit. The Ayatollah Khomeni made a similar offer to anyone willing to sacrifice themselves in the Iran-Iraq war. Many people willingly accepted the Ayatollah's offer (although not everyone had the opportunity to decline). A rational believer in Pascal's wager must be willing to accept the original or revised offer.[4]

The argument that $(p_1 - p_2) * \infty = \infty$ is at first surprising, but expresses the idea that when offered two gambles each with infinite expected utility one should choose the gamble with the higher probability of success. Imagine that at St. Peter's Gate, Saint Peter offers you either, infinite utility with probability $\frac{999,999}{1,000,000}$ and high but finite utility with probability $\frac{1}{1,000,000}$, or infinite utility with probability $\frac{1}{1,000,000}$ and very negative but finite utility with probability $\frac{999,999}{1,000,000}$. Despite the fact that these gambles have the same expected utility few people would express indifference between them. Indeed we can go further. Imagine that St. Peter offers you infinite utility with probability 1 or infinite utility with probability $\varepsilon$ where $\varepsilon$ is very small. These two gambles have the same expected utility, but to choose the latter gamble would surely be unreasonable.[5]

## 4. CONCLUSIONS

Pascal's wager is initially compelling because "believing" in God appears to be costless.[6] Believing in the possibility of infinite utility, however, implies that the believer is willing to accept *any* finite cost to achieve *any* positive probability, no matter how small, of attaining infinite utility. The offer makes this latter point particularly clear.

The offer does not prove that Pascal's wager is false, although some who have heard the offer accept it as a reductio. Those who do not believe the offer is a reductio are invited to send their checks to the address given below.

## ACKNOWLEDGMENTS

## NOTES

1. Many of the classic articles on Pascal's wager are collected in Jordan (1994).
2. A helpful referee points out that the principle of strict coherence requires that degrees of belief about factual, as opposed to purely logical, matters never be zero. Some theists and also some atheists, however, argue that God's existence is not a factual matter.
3. Laplace (1951, [1886], ch.11) makes a similar argument against Pascal. Laplace suggests that the more the offeror promises, the lower should be the probability ascribed to the offeror's promise being true. Since God promises infinite utility the probability of his promise being true (ie. God existing) should be zero. Laplace's "inverse probability" argument is well taken as a rule of thumb for avoiding crooks, but is otherwise vacuous. One would not want to be committed to the proposition that gods who promise little exist with high probability, but this is implied by Laplace's argument (Rescher, 1985). More generally, let $X$ be the promised reward and $p(X)$ the probabilty the reward is actually delivered. Then according to Laplace $p(X) * X \rightarrow 0$ (or some constant) as $X \rightarrow \infty$. But why should $p(X)$ decline in just the way Laplace requires? It's quite possible that $p'(X) < 0$ and yet $p(X) * X \rightarrow \infty$ as $X \rightarrow \infty$. And why must $X$ be the only indicator of truthfulness? Surely Mother Theresa is not to be disbelieved solely because she holds out the promise of a rewarding afterlife.

4. The offer promises an increased probability of ascending to heaven in return for all of the believer's wealth. The offer cannot be refused on the grounds that Mother Theresa (or someone else) makes a cheaper counter-offer. After accepting Mother Theresa's counter-offer the believer should still be willing to accept my offer. It may be rational to accept every cheaper counter-offer before accepting the offer but until all wealth is exhausted the believer should accept each and every offer.

5. See also Schlesinger (1994) who offers another reason for accepting that $(p_1 - p_2) * \infty = \infty$. On infinite utility more generally see for example, Cowen and High (1988), Sorensen (1994), and McClennen (1994).

6. When belief is costless Pascal's wager becomes a dominance argument. Tabarrok (1995) shows how Pascalian like dominance arguments can be quite important in the field of macro-economic policy.

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*Address for correspondence:* Dr A. Tabarrok, Director of Research, The Independent Institute, 100 Swan Way, Oakland, CA 94621, USA
Phone: 510 632 1366; Fax: 510 568 6040;
E-mail: atabarrok@independent.org